Maths Progress Second Edition 2019

Scheme of work overview

Year 1

Unit	Strand	Teacher	Hours
Autumn half term 1			
Baseline test	Assessment		1
Unit opener: Analysing and displaying data	Y1 Unit 1		0
1.1 Mode, median and range	Y1 Unit 1		1
1.2 Displaying data	Y1 Unit 1		2
1.3 Grouping data	Y1 Unit 1		2
1.4 Averages and comparing data	Y1 Unit 1		1
1.5 Line graphs and more bar charts	Y1 Unit 1		1
1 Check, Strengthen and Extend	Y1 Unit 1		2
End of Unit 1 tests: Analysing and displaying data	Assessment		1
Unit opener: Number skills	Y1 Unit 2		0
2.1 Mental maths	Y1 Unit 2		1
2.2 Addition and subtraction	Y1 Unit 2		1
2.3 Multiplication	Y1 Unit 2		1
2.4 Division	Y1 Unit 2		1
2.5 Money and time	Y1 Unit 2		1
2.6 Negative numbers	Y1 Unit 2		1
2.7 Factors, multiples and primes	Y1 Unit 2		1
2.8 Square numbers	Y1 Unit 2		1
2 Check, Strengthen and Extend	Y1 Unit 2		2
End of Unit 2 tests: Number skills	Assessment		1

Autumn half term 2

Unit opener: Expressions, functions and formulae	Y1 Unit 3	0
3.1 Functions	Y1 Unit 3	1
3.2 Simplifying expressions 1	Y1 Unit 3	1
3.3 Simplifying expressions 2	Y1 Unit 3	1
3.4 Writing expressions	Y1 Unit 3	2
3.5 Substituting into formulae	Y1 Unit 3	1
3.6 Writing formulae	Y1 Unit 3	1
3 Check, Strengthen and Extend	Y1 Unit 3	2
End of Unit 3 tests: Expressions, functions and formulae	Assessment	1
Unit opener: Decimals and measures	Y1 Unit 4	0
4.1 Decimals and rounding	Y1 Unit 4	1
4.2 Length, mass and capacity	Y1 Unit 4	2
4.3 Scales and measures	Y1 Unit 4	1
4.4 Working with decimals mentally	Y1 Unit 4	1
4.5 Working with decimals	Y1 Unit 4	1
4.6 Perimeter	Y1 Unit 4	1
4.7 Area	Y1 Unit 4	1
4.8 More units of measure	Y1 Unit 4	1
4 Check, Strengthen and Extend	Y1 Unit 4	2
End of Unit 4 tests: Decimals and measures	Assessment	1
Revision	Revision	2
End of Term tests: Units 1 to 4	Assessment	1

Spring half term 1

Unit opener: Fractions and percentages	Y1 Unit 5	0
5.1 Comparing fractions	Y1 Unit 5	1
5.2 Simplifying fractions	Y1 Unit 5	1
5.3 Working with fractions	Y1 Unit 5	2

5.4 Fractions and decimals	Y1 Unit 5	1
5.5 Understanding percentages	Y1 Unit 5	1
5.6 Percentages of amounts	Y1 Unit 5	1
5 Check, Strengthen and Extend	Y1 Unit 5	2
End of Unit 5 tests: Fractions and percentages	Assessment	1
Unit opener: Probability	Y1 Unit 6	0
6.1 The language of probability	Y1 Unit 6	1
6.2 Calculating probability	Y1 Unit 6	1
6.3 More probability calculations	Y1 Unit 6	1
6.4 Experimental probability	Y1 Unit 6	2
6.5 Expected outcomes	Y1 Unit 6	1
6 Check, Strengthen and Extend	Y1 Unit 6	2
End of Unit 6 tests: Probability	Assessment	1

Spring half term 2

Unit opener: Ratio and proportion	Y1 Unit 7	0
7.1 Direct proportion	Y1 Unit 7	2
7.2 Writing ratios	Y1 Unit 7	1
7.3 Using ratios	Y1 Unit 7	2
7.4 Ratios, proportions and fractions	Y1 Unit 7	1
7.5 Proportions and percentages	Y1 Unit 7	1
7 Check, Strengthen and Extend	Y1 Unit 7	2
End of Unit 7 tests: Ratio and proportion	Assessment	1
Revision	Revision	2
End of Term tests: Units 5 to 7	Assessment	0

Summer half term 1

Unit opener: Lines and angles	Y1 Unit 8	0
8.1 Measuring and drawing angles	Y1 Unit 8	2
8.2 Lines, angles and triangles	Y1 Unit 8	1

8.3 Drawing triangles accurately	Y1 Unit 8	2
8.4 Calculating angles	Y1 Unit 8	1
8.5 Angles in a triangle	Y1 Unit 8	1
8.6 Quadrilaterals	Y1 Unit 8	1
8 Check, Strengthen and Extend	Y1 Unit 8	2
End of Unit 8 tests: Lines and angles	Assessment	1
Unit opener: Sequences and graphs	Y1 Unit 9	0
9.1 Sequences	Y1 Unit 9	1
9.2 Pattern sequences	Y1 Unit 9	1
9.3 Coordinates and midpoints	Y1 Unit 9	1
9.4 Extending sequences	Y1 Unit 9	1
9.5 Straight-line graphs	Y1 Unit 9	2
9.6 Position-to-term rules	Y1 Unit 9	1
9 Check, Strengthen and Extend	Y1 Unit 9	2
End of Unit 9 tests: Sequences and graphs	Assessment	1

Summer half term 2

Unit opener: Transformations	Y1 Unit 10	0
10.1 Congruency and enlargements	Y1 Unit 10	2
10.2 Symmetry	Y1 Unit 10	1
10.3 Reflection	Y1 Unit 10	2
10.4 Rotation	Y1 Unit 10	1
10.5 Translations and combined transformations	Y1 Unit 10	2
10 Check, Strengthen and Extend	Y1 Unit 10	2
End of Unit 10 tests: Transformations	Assessment	1
Revision	Revision	2
End of Term tests: Units 8 to 10	Assessment	1
Revision	Revision	2
Revision	Revision	2

End of Year tests: Units 1 to 10	Assessment		1

Year 2

Unit	Strand	Teacher	Hours
Autumn half term 1			
Unit opener: Number	Y2 Unit 1		0
1.1 Calculations	Y2 Unit 1		2
1.2 Divisibility and division	Y2 Unit 1		2
1.3 Calculating with negative numbers	Y2 Unit 1		2
1.4 Powers and roots	Y2 Unit 1		2
1.5 Powers, roots and brackets	Y2 Unit 1		2
1.6 More powers, multiples and roots	Y2 Unit 1		2
1 Check, Strengthen and Extend	Y2 Unit 1		2
End of Unit 1 tests: Number	Assessment		1
Unit opener: Area and volume	Y2 Unit 2		0
2.1 Area of a triangle	Y2 Unit 2		1
2.2 Area of a parallelogram and trapezium	Y2 Unit 2		1
2.3 Volumes of cubes and cuboids	Y2 Unit 2		2
2.4 2D respresentations of 3D solids	Y2 Unit 2		2
2.5 Surface area of cubes and cuboids	Y2 Unit 2		2
2.6 Measures	Y2 Unit 2		2
2 Check, Strengthen and Extend	Y2 Unit 2		2
End of Unit 2 tests: Area and volume	Assessment		1
Autumn half term 2			
Unit opener: Statistics, graphs and charts	Y2 Unit 3		0
3.1 Pie charts	Y2 Unit 3		1
3.2 Using tables	Y2 Unit 3		2
3.3 Stem and leaf diagrams	Y2 Unit 3		1
3.4 Comparing data	Y2 Unit 3		2

3.5 Scatter graphs	Y2 Unit 3	1
3.6 Misleading graphs	Y2 Unit 3	1
3 Check, Strengthen and Extend	Y2 Unit 3	2
End of Unit 3 tests: Statistics, graphs and charts	Assessment	1
Unit opener: Expressions and equations	Y2 Unit 4	0
4.1 Algebraic powers	Y2 Unit 4	1
4.2 Expressions and brackets	Y2 Unit 4	2
4.3 Factorising expressions	Y2 Unit 4	1
4.4 One-step equations	Y2 Unit 4	2
4.5 Two-step equations	Y2 Unit 4	1
4.6 The balancing method	Y2 Unit 4	1
4 Check, Strengthen and Extend	Y2 Unit 4	2
End of Unit 4 tests: Expressions and equations	Assessment	1
Revision	Revision	2
End of Term tests: Units 1 to 4	Assessment	1

Spring half term 1

Unit opener: Real-life graphs	Y2 Unit 5	0
5.1 Conversion graphs	Y2 Unit 5	1
5.2 Distance-time graphs	Y2 Unit 5	1
5.3 Line graphs	Y2 Unit 5	1
5.4 More line graphs	Y2 Unit 5	1
5.5 Real-life graphs	Y2 Unit 5	2
5.6 Curved graphs	Y2 Unit 5	1
5 Check, Strengthen and Extend	Y2 Unit 5	2
End of Unit 5 tests: Real-life graphs	Assessment	1
Unit opener: Decimals and ratio	Y2 Unit 6	0
6.1 Ordering decimals and rounding	Y2 Unit 6	2
6.2 Place-value calculations	Y2 Unit 6	2

6.3 Calculations with decimals	Y2 Unit 6	2
6.4 Ratio and proportion with decimals	Y2 Unit 6	2
6 Check, Strengthen and Extend	Y2 Unit 6	2
End of Unit 6 tests: Decimals and ratio	Assessment	1

Spring half term 2

Unit opener: Lines and angles	Y2 Unit 7	0
7.1 Quadrilaterals	Y2 Unit 7	1
7.2 Alternate angles and proof	Y2 Unit 7	1
7.3 Angles in parallel lines	Y2 Unit 7	1
7.4 Exterior and interior angles	Y2 Unit 7	1
7.5 Solving geometric problems	Y2 Unit 7	1
7 Check, Strengthen and Extend	Y2 Unit 7	2
End of Unit 7 tests: Lines and angles	Assessment	1
Revision	Revision	2
End of Term tests: Units 5 to 7	Assessment	1

Summer half term 1

Unit opener: Calculating with fractions	Y2 Unit 8	0
8.1 Ordering fractions	Y2 Unit 8	1
8.2 Adding and subtracting fractions	Y2 Unit 8	1
8.3 Multiplying fractions	Y2 Unit 8	1
8.4 Dividing fractions	Y2 Unit 8	1
8.5 Calculating with mixed numbers	Y2 Unit 8	1
8 Check, Strengthen and Extend	Y2 Unit 8	2
End of Unit 8 tests: Calculating with fractions	Assessment	1
Unit opener: Straight-line graphs	Y2 Unit 9	0
9.1 Direct proportion on graphs	Y2 Unit 9	2
9.2 Gradients	Y2 Unit 9	2
9.3 Equations of straight lines	Y2 Unit 9	2

9 Check, Strengthen and Extend	Y2 Unit 9	2
End of Unit 9 tests: Straight-line graphs	Assessment	1

Summer half term 2

Unit opener: Percentages, decimals and fractions	Y2 Unit 10	0
10.1 Fractions and decimals	Y2 Unit 10	1
10.2 Equivalent proportions	Y2 Unit 10	2
10.3 Writing percentages	Y2 Unit 10	2
10.4 Percentages of amounts	Y2 Unit 10	2
10 Check, Strengthen and Extend	Y2 Unit 10	2
End of Unit 10 tests: Percentages, decimals and fractions	Assessment	1
Revision	Revision	2
End of Term tests: Units 8 to 10	Assessment	1
Revision	Revision	2
End of Year tests: Units 1 to 10	Assessment	1

Baseline test

Pearson published resources

Test and markscheme

- Baseline test (including mark scheme)

Unit opener: Analysing and displaying data

Pearson published resources

Objectives

Core objectives

- Find the mode, median and range for a set of data

Support objectives

- Find the mode, median and range for a set of data

Depth key concepts

- Understand what an average is a measure of, and what it does and doesn't represent

Key points

- **Data** is a set of information. Each piece of information is called a **value**. The **range** is the difference between the smallest and largest values. The larger the range, the more spread out the values.
- The **mode** is the most common value. It is also called the **modal** value.
- The **median** is the middle value when the data is written in order.

Common errors and misconceptions

- Students often forget to order data before finding the median. A concrete way to address this is to ask them to find the middle one in a set of towers made from cubes.
- Students may describe the range using everyday language, e.g. as 5–12 cm, instead of calculating it. Address this by emphasising that in maths, the range is a single number that measures the spread of the data.

Question guidance

Core

Q4 This question has been carefully constructed so that students work with increasingly challenging numbers.

Q7b Use this question to reinforce that range is a number, so can only be calculated for numerical data.

Q14–Q19 Check that students order the values before finding the median.

Q14 This question has been carefully constructed so that all data items are ordered, with the 'gaps' between the middle two numbers increasing.

Q15e Draw a vertical line through ordered data set to show half the values are less than the median.

Q16b Use this question to establish that the median doesn't have to be a value in the data set.

Support

Range Q1, Q2 Use these questions to highlight that, in maths, range is a single number to be calculated.

Mode Q1, Q2 Use these questions to highlight that, unlike other measures of average, the mode can be non-numerical.

Median General Encourage students to always copy out sets of data, order them and then cross off pairs of highest and lowest values as they work their way towards the middle value(s).

Purposeful practice

Purposeful practice 3 Q1 demonstrates the effect on the median of including different numbers in a set of data, and that including 0 cannot be ignored.

Reflect

Core

In this lesson, you ordered numbers to work out the median. What other maths skills did you use to work out the median? What maths skills did you use for other topics in this lesson?

Copy and complete this sentence to help you list them all:

I used ____ to work out the ____.

This task emphasises how students have already used their Key Stage 2 maths knowledge and applied it to learn new things at Key Stage 3. It aims to build their awareness of their thinking skills and their confidence in maths. It also helps students recognise the links between topics in maths.

Examples of completed sentences are: 'I used subtraction to work out the range' or 'I used counting to work out the mode'.

Support

Why is it helpful to order numbers from smallest to largest before finding the median, mode and range?

Writing the numbers enables you to visualise the data helping you to identify the middle value (median), most common value (mode) and the difference between the highest and lowest value (range).

Depth

What was the same and what was different about your method for answering parts **a** and **b** of **Q12**?

*In part **a**, the median is the 3rd value = 7.5, so you can write this first and write numbers around it. These could be all 7.5, or you could write any two values less than or equal to 7.5, and any two values greater than or equal to 7.5.*

*In part **b**, you start by writing 2nd and 3rd values that have 7.5 as 'middle value'. These could both be 7.5, they could be 7 and 8, or 6 and 9 – any two values that add to 15. Then you write one less than or equal to the 2nd value, and one greater than or equal to the 3rd value.*

Same: You could write all values 7.5, but you can also write two values less than the median and two values greater than the median.

*Different: For an odd number of values, the median value is the middle data value (as in part **a**). For an even number, you need two numbers that have the median value as a 'half-way' value.*

Purposeful practice Reflect and reason

Purposeful practice 1

Are these whole numbers or decimal numbers?

- The median between two even numbers.
- The median between two odd numbers.
- The median between one odd and one even number.

Give examples from Purposeful practice 1 for each statement.

*The median between two even or two odd numbers is always an integer (whole number), e.g. **Q3** parts **b**, **d** (because the difference between two odd numbers is an even number so divisible by 2, and likewise for two even numbers).*

*The median between one even and one odd number is always a decimal ending in .5 (e.g. **Q3** parts **a**, **c**, **e**, **f**, **g**, **h**) (because the difference between an odd and an even number is odd, so half this difference is a decimal ending in .5).*

Purposeful practice 2

When you add a value between 5 and 22 to the set 5, 9, 16, 22, the range does not change. Explain why.

To calculate the range you use the highest and lowest values. Adding a value between the highest and lowest values does not change the highest and lowest values, so the range does not change.

Purposeful practice 3

When you add a value higher than the original median, is the new median higher or lower than the original median?

When you add a value higher than the original median, the new median is higher than the original median. (This also means that when you add a value lower than the original median, the new median will be lower than the original median, and that adding a value the same as the median leaves the median unchanged.)

Objectives

Core objectives

- Find information from tables and diagrams
- Display data using tally charts, tables, bar charts and bar-line charts

Support objectives

- Find information from tables and diagrams
- Display data using tally charts, tables and bar charts

Depth key concepts

- Understand how to choose the best representation for different sets of data

Key points

- A **pictogram** uses pictures to show data. The **key** shows what each picture represents.
- A **bar chart** uses bars of equal width to show data.
- A **bar-line chart** is like a bar chart but uses lines instead of bars.
- You can record data in a **tally chart**. Use a tally mark | for each value.
- Group tally marks in fives like this 𝍸. The **frequency** of a value is the number of times it occurs.
- A **frequency table** shows how many of each value there are in a set of data.

Common errors and misconceptions

- Students often read frequencies as values and vice-versa in a table where 'frequency' is not labelled. Ask them focused questions to help address this. For example in **Q7**, ask *Which column tells you 'how many' or 'the frequency' of each type of house?*
- Students often confuse frequency and mode, giving the frequency of the highest bar rather than the mode. If so, discuss whether their answers make sense. For example, in **Q6**, if a student says the mode is 25, ask *How many Year 9 girls have size 25 shoes?*

Question guidance

Core

Q6e This draws out the common error of giving the frequency of the highest bar rather than the modal value.

Q7c Check that bars have equal width and are equally spaced and have the same shading. Help students draw and label axes. Students could also draw a horizontal bar chart for the same data.

Q8b This also addresses the common confusion of frequency with modal value.

Q10 Practise converting tallies to numbers and vice versa. Make sure that students understand the term 'frequency'. Explain it in different ways, e.g. the number of times something happens, how many times a value occurs/happens. Differentiate between a

tally chart and a frequency table. You could also ask students to draw a frequency table without the tally column.

Support

Bar charts Q1 Use this question to emphasise the importance of looking carefully at the scale on a bar chart.

Tally charts and frequency tables Q3 Use this question to check that students do not confuse frequencies (not labelled on the chart) with values.

Purposeful practice

Purposeful practice 2 Q1, Q2 provide practice finding mode and range for different types of data display.

Reflect

Core

Think carefully about what *frequency* and *mode* tell you. Write definitions in your own words.

This reflective task encourages students to think carefully about their understanding of frequency and mode. Likely responses are: the frequency of a data item is the number of times it occurs; the mode is the most common (frequent) value (data item).

Support

Look back at Pictograms **Q1** and Bar charts **Q3**. How do you find the mode from a pictogram? How do you find it from a bar chart?

The mode is the most common (frequent) value, so in a pictogram, it is the value with the most pictures and in a bar chart it is the value with the highest bar.

Depth

In **Q4** and **Q5**, which type of chart did you choose to draw? Explain why. What are the advantages of each type?

*For **Q4** students could choose any diagram that represented the data clearly; a pictogram would be suitable. For **Q5** they are instructed to draw either a bar chart or a bar-line chart.*

Bar-line graphs are easiest to draw because you only draw a line. Bar charts and pictograms are more colourful and eye-catching, e.g. for a newsletter report. Bar-line and bar charts may be easier to interpret than pictograms as you can read the value off the scale, instead of counting symbols and multiplying/dealing with half symbols.

Purposeful practice Reflect and reason

Purposeful practice 1

What is the same and what is different about the charts and tables you drew in **Q1–Q4**?

Same: they all show the data visually, using tallies, symbols, bars, lines. They are all grouped/go up in 5s. A pictogram is like a bar chart on its side.

Differences: Pictograms and tally charts have categories vertically/up the side. Bar-line and bar charts have them along the bottom.

Purposeful practice 2

What type of data does not have a range? Can this type of data have a mode? Use examples from this page to explain.

Data that is not numerical (i.e. categorical data) does not have a range (e.g. PP2 ***Q1*** *has range,* ***Q2*** *does not). Categorical data can have a mode (e.g. PP2* ***Q2****).*

Objectives

Core objectives

- Interpret simple charts for grouped data
- Find the modal class for grouped data

Support objectives

- Interpret simple charts for grouped data
- Find the modal class for grouped data

Depth key concepts

- Understand different averages and what they represent

Key points

- Data is sometimes organised into **groups** or **classes**, such as 1–5, 6–10, 11–15, ... The **modal class** is the one with the highest frequency.
- A **grouped frequency table** gives the frequency for each group.
- For data that comes from measuring, such as height, there are no gaps between the bars of a bar chart.

Common errors and misconceptions

- Students often do not understand they need to add individual frequencies to work out the number of people in a survey. A concrete way to address this is to use cubes or other countable items. For example, in **Q4b**, use cubes to represent numbers of students in each group and then ask, *How many altogether?*

Question guidance

Core

General This lesson informally introduces discrete and continuous data, which are met formally in Core 3.

Q3 Ask students to think about the effect on Eduardo's results if his classmates had exercised before he took their pulse rates. Agree that the classes would probably start with a higher value, e.g. 80–89.

Q5d and **Q8c** Use these questions to illustrate how some of the information can be lost when grouping data into classes. For example, in **Q5d**, the grouped data can only tells us that 6 students had 6, 7 or 8 coins on them – we can't tell if any had 8 coins. They could all have had 7 coins on them.

Q8 Ensure there are no gaps between the bars in students' bar charts.

Support

Bar charts for grouped data Q2b Check that students understand they need to add the frequencies of two bars.

Purposeful practice

Purposeful practice 2 Q1, Q2 These questions appear to show the same data. Use them to show why there should be gaps between the bars drawn in **Q1** but not in **Q2**.

Reflect

Core

Which did you find easier: working out the modal class from a table or from a bar chart?

Think carefully about how you learn in all your subjects. Do you understand things better when there is a diagram?

This task involves higher-order thinking skills, asking students to think about how different representations help them to identify the modal class. It also encourages them to reflect more widely about whether pictures and diagrams help them learn in maths, as well as other subjects.

Support

Look at **Q3** with a partner. Which group would the value 19 be in?

19 would be in the group 15–19.

Depth

Look back at the investigation. When do you think it is sensible to use grouped data? What do you need to think about, to decide how many groups there should be?

Use grouped data when the set of data is large; for example, it would not be suitable to group data if there are only five values. It can make sense to have different class widths if the data is not evenly spread across the range; for example, you could have a group of 60–99 for the investigation.

Things to think about:

- *how many items of data there are, e.g. 10 groups for 10 data items is pointless*
- *how many equal class intervals there will be*
- *what the boundaries of the class intervals will be, e.g. if the data is from 10 to 20 g, five class intervals are 10–12, 12.1–14, etc., whereas four class intervals are 10–12.5, etc.; avoid non-integer class boundaries.*

Purposeful practice Reflect and reason

Purposeful practice 1

Daisy wants to record her test marks in groups like this: 0–5, 5–10, 10–15, 15–20, ... Explain why this will not work. Suggest how Daisy could group her results.

This will not work because the groups overlap. Daisy should avoid overlapping her groups and group her results in either of these two ways: 0–5, 6–10, 11–15, ... or 0–4, 5–9, 10–14, ...

Purposeful practice 2

Class 7 draws bar charts for these sets of data.

A Numbers of leaves **B** Lengths of leaves **C** Weights of animals
D Speeds of animals **E** Numbers of animals

Which bar charts should have gaps between the bars? Explain.

*Bar charts **A** and **E** should have gaps because they are numbers, not measures. **B**, **C** and **D** are measured data so no gaps.*

Objectives

Core objectives

- Calculate the mean of a set of data
- Compare sets of data using their ranges and averages

Support objectives

- Calculate the mean of a set of data
- Compare sets of data using their ranges and averages

Depth key concepts

- Understand averages and what they represent
- Understand how to use the range to compare data
- Understand which average is most appropriate

Key points

- The **mean** of a set of values is the total of the set of values divided by the number of values.
- The **average** of a set of data gives a typical value for the data. The mode, median and mean are different ways of describing the average of a set of data.
- To **compare** two sets of data, find and compare an average (the mode, median or mean) and the range.

Common errors and misconceptions

- Students often confuse the different types of average – mode, median and mean. Help them remember the differences using these strategies: **mo**de is **mo**st; **med**ian is **mid**dle; **mean** is the most difficult to work out so is the **meanest**.
- Students often do not understand the need to use context when comparing data sets. Emphasise that comparisons must use the context of the question rather than pure numerical comparisons of the range and average. For example, in **Q9** they should mention men and women in their comparisons. Ask students to read out their answers. *Would someone understand what your comparison is about?*

Question guidance

Core

Q4 This question is carefully constructed with minimally varying data sets to reinforce that you divide by the number of values each time and that 0 (zero) counts as a value. The more zero values there are, the smaller the mean.

Q6 Check that students are pressing the + key on their calculators before pressing the ÷ key. Ask students how they could check that their calculated mean is sensible, and agree that the mean must lie between the smallest and greatest values.

Q11c Give those who need help some scaffolding to construct their sentences, e.g. The _____ had the higher median score, so they did better on average. The _____ had the smaller range, so their scores were more consistent.

Support

Comparing sets of data Q2, Q3 Use these questions to check that students understand and can meaningfully use *mean* and *range* when making comparisons.

Purposeful practice

Purposeful practice 1 and 2 deepen students' understanding of relationships in mean, total and number of values.

Reflect

Core

You often see the word 'average' in headlines. For example, 'Average screen size of TVs grows again'. 'Average' in everyday language could be the mean, median or mode.

Write notes in your own words on the differences between these.

This reflective task asks students to think about their understanding of the word 'average'. 'Average' in maths may refer to mean, median or mode, but in daily life is commonly used for 'mean'. In the headline 'Average screen size of TVs grows again', it is likely that the 'average' referred to is the mean.

Support

Here is Sophie's method for finding the mean:

Step 1: Count the values (for example, in the set of data 1, 2, 9, there are 3 values).

Write Sophie's step 2 and 3.

Compare your steps with a partner.

Likely answers are:

Step 2: Add the values to find their total (for example, 1 + 2 + 9 = 12).

Step 3: Divide the total of the values by the number of values (for example, 12 ÷ 3 = 4).

Depth

You often see the word 'average' in headlines:

Average screen size of TVs grows again

The changing face of the average American

Think about what you know about the mean, median and mode to describe averages. Are any of these types of averages appropriate for each of these headlines?

You could use the median or mean for the TV screen size. The 'average American' is probably not about any particular value or quantity, and so the phrase probably means 'most Americans'. We can't sensibly use mean, median or mode to describe it.

Purposeful practice Reflect and reason

Purposeful practice 1

Does adding another value to a set of data always change the mean? Explain how the mean changes when you add values less than or greater than the mean.

Not always; it depends on the size of the value being added. If you add the mean value to the data set the mean does not change. If you add a value lower than the original mean, the new mean is lower. If you add a value higher than the original mean, the new mean is higher.

Purposeful practice 2

20 people raise money for charity. The mean amount raised is £5. Who has correctly worked out the total amount raised? Explain your answer.

Sam	**Alex**	**Jo**
20 × £5 = £100	20 ÷ £5 = £4	5 ÷ 20 = £0.25

Sam is correct. Alex and Jo have both divided instead of multiplying. Mean represents amount raised by each person, so 20 × 5 represents the total.

Objectives

Core objectives

- Understand and draw line graphs
- Understand and draw dual and compound bar charts

Support objectives

- Understand and draw line graphs
- Understand dual and compound bar charts

Depth key concepts

- Understand how to choose the best representation for different types of data

Key points

- A **line graph** shows how quantities change.
- On a line graph showing changes over time, put time on the horizontal axis.
- A **dual bar chart** compares two sets of data.
- A **compound bar chart** combines different sets of data in one bar.

Common errors and misconceptions

- Students often think that every value on a line in a line graph has meaning. Encourage them to understand that only the measured and plotted values have meaning – the lines simply join those points. Refer to Rikki's temperature chart in **Q3**. *Does the point on the line above 8.15 am show Rikki's temperature at 8.15 am?* We cannot tell as only the values measured at 8 am and 9 am are certain. The actual temperature at 8.15 might have been higher than at 8 am.
- Students may use a dot rather than a cross to mark a point. Explain that crosses are better as they are easier to see.

Question guidance

Core

Q6 Use this question to probe students' understanding of a line graph. Ask *Does every point on the graph line mean something?* On this graph, it doesn't make sense to say that Michelle spent a certain number of hours on Facebook halfway between Friday and Saturday.

Q7d Students need to justify their answer numerically.

Q10 A complete two-way table like this one is preparation for Core 2, where tables have Totals and students have to fill in the missing values.

Q10a Students can divide the bars in two ways, e.g. photos of Gareth either at the top or the bottom of the bars. Check that students use the same order for all of their bars and write a key.

Support

Dual bar charts Q2 Check that students correctly read the values of bars that show odd frequencies.

Compound bar charts Q2 Make sure that students draw bars for July and August in the same order as June).

Purposeful practice

Purposeful practice 2 deepens students' understanding of compound bar charts and common errors.

Reflect

Core

Make a list of the questions from this lesson that you found easiest to answer. What made them easier for you?

This task involves higher order thinking, asking students to identify the questions they found easier to answer, and to think about why that may be. They may consider the different representations of data, but they may also take into account the simplicity or the wordiness of questions, or the amount of information included in questions.

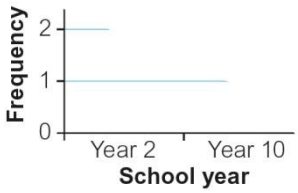
Support

Look back at Dual bar charts **Q1** and Compound bar charts **Q1**. Discuss with a partner the difference between a compound bar chart and a dual bar chart.

A dual bar chart shows two sets of data on the same graph. A compound bar chart combines two (or more) sets of data in a single bar.

Depth

For **Q4**, Tim drew these axes for his bar chart.



a How many bars will he have for each year group?

b What mistake has Tim made?

a *5 bars*

b To draw a bar chart he needs two bars for each group. He should put Car, Walk, Bicycle, ... along the horizontal axis and then draw a bar for Year 2 and a bar for Year 10 for each transport category.

Purposeful practice Reflect and reason

Purposeful practice 1

Was it easier to answer **Q2–5** using your dual bar chart or the separate bar charts? Give reasons for your answer.

*Students' own answers, e.g. it is easier to use the dual bar chart because you can see easily which days have equal bars (**Q4**), which have the dinner bar greater than the lunch bar (**Q3**), which have the lunch bar greater than the dinner bar (**Q2**) and which day has the greatest total (**Q5**)*

Purposeful practice 2

Lucy says, 'The vet saw 7 dogs and 10 cats on Monday.' What mistake has she made?

Lucy has read the number of cats from the top of the bar. She has not worked out that 'cat bar' is only from 7 to 10, so represents 3 cats.

Check, Strengthen and Extend

Core Check up

Q2 This question is designed to test if students realise that 0 (zero) values need to be included when calculating averages, especially the mean.

Q4b This question tests if students confuse values (children) with frequencies (families).

Q7 This question is designed to test that students understand how a compound bar chart displays data and that they need to use subtraction to work out the actual data values.

Q8c This question assesses students' ability to compare two sets of data using mean and range.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Write down the names of all the different charts you have learned about in this unit.

Beside each chart, write *one* thing that makes it different from the others.

Write *two* things that are the same for all charts.

This reflective task encourages students to identify the differences and similarities between charts for displaying data. It is intended to raise awareness of what makes each one unique, and help students identify and remember them in the future. For differences, students may state that a pictogram uses pictures, a bar chart shows data in bars, a bar-line chart shows data in vertical lines, and so on. For the two things that make all charts the same, students may state that they all have a title and axes that are labelled.

Extend

The Government's Office for National Statistics uses all the skills you have learned in this unit to interpret data about Britain today (for example, jobs, salaries and how we live).

What other organisations might use these skills?

What do you think they might use them for?

This reflective task aims to demonstrate to students the usefulness of the skills they have learned, and the prevalence of data collection and analysis among all kinds of organisations. For example, the NHS interprets data so that they know when someone's health is normal or not, so indicating illness. The Broadcasting Audience Research Board (BARB) interprets data on television viewing figures, so that TV stations can decide whether to make another series of a programme, or how much to charge for adverts in the middle of programmes. Google interprets data, including searches for 'flu remedies', to indicate where in the World there may be a flu epidemic.

Depth

Extend

Write down three things you have learned in this unit that you did not know before. Compare your list with a classmate's list.

Students' own answers. Did students write similar statements? Possible comments include having a better idea of what an average measures, how to choose the best representation for different data sets, understanding which average is the most appropriate to use for different data sets, how to use range to compare data sets.

End of Unit 1 tests: Analysing and displaying data

Pearson published resources

Support/Core

- Year 7 End of Unit 1 Support/Core test
- Year 7 End of Unit 1 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 1 Core/Depth test
- Year 7 End of Unit 1 Core/Depth mark scheme

Unit opener: Number skills

Pearson published resources

Objectives

Core objectives

- Use the priority of operations, including brackets
- Use multiplication facts up to 10 × 10 and the laws of arithmetic to do mental multiplication and division
- Multiply by multiples of 10, 100 and 1000

Support objectives

- Use multiplication facts up to 10 × 10 and the laws of arithmetic to do mental multiplication and division
- Multiply and divide by 10, 100 and 1000
- Use the priority of operations

Depth key concepts

- Understand how multiplying by 10, 100, 1000, etc. relates to our place value system and why this means we have a decimal system

Key points

Partitioning splits the bigger number to make some easier multiplications.

You must use the **priority of operations** to do calculations. Use **BIDMAS**:

- **B**rackets
- **I**ndices (powers)
- **D**ivision and **M**ultiplication
- **A**ddition and **S**ubtraction

When you have only × and ÷, or only + and - then work from left to right.

Common errors and misconceptions

- Students often confuse the rules for multiplying by 0 and adding 0, so could write 3 × 0 = 3. A concrete way to address this is to display three plastic sandwich bags with one counter in each (3 × 1 = 3). Display three empty sandwich bags and ask *How many counters?*

Question guidance

Core

Q4–Q7 These questions have been carefully constructed to give students practice rearranging numbers being multiplied, understanding commutativity, and partitioning to make multiplications easier.

Q4b This question shows that changing the order of numbers in a multiplication doesn't change the answer. A concrete way to illustrate this property is to use cubes or counters to form arrays.

Q8, Q10–Q13 These questions have been carefully constructed to show students how to apply the priority of operations.

Q14 Use this question to illustrate the importance of applying the correct priority of operations; by placing brackets in different places to achieve different priorities, different answers are obtained.

Challenge It would be useful to have a selection of scientific and non-scientific calculators to highlight that not all calculators use BIDMAS.

Support

Multiplying and dividing by 10, 100 and 1000 Q1, Q2 These questions offer intelligent practice and the opportunity to check that students understand that when multiplying/dividing by 10, 100 or 1000 you do not simply add/remove zeroes but that numbers are made 10, 100 or 1000 times bigger/smaller.

Purposeful practice

Purposeful practice 3 These questions use the same numbers in the same order but with different operations to demonstrate the importance of applying the priority of operations.

Reflect

Core

Write the new mathematical terms and words you have learned in this lesson.

Use your own words to write a definition for each of them.

For each definition, make up a calculation to show what it means.

This task encourages students to revisit new terms and words (such as BIDMAS and partitioning) and check their understanding by writing definitions in their own words, accompanied by their own calculations to illustrate them.

Support

Why do you think it is important to have a rule for which operation to calculate first?

Calculate 10 × 3 – 2 by doing the multiplication first. Then calculate it again doing the subtraction first. What do you notice?

It's important to have rules about the priority or order in which operations need to be done so that everyone follows the same rules and can get the same (consistent) answers.

If you do the operations in 10 × 3 – 2 in a different order, you get different answers. Do the multiplication first: 10 × 3 – 2 = 30 – 2 = 28; do the subtraction first: 10 × (3 – 2) = 10 × 1 = 10.

Depth

We use the decimal system for our numbers. This is a system based on the number 10. Describe ways that you think place value and arithmetic are based on the number 10.

Students' own answers. For example, the digits in a number correspond to the number of 1s, 10s, 100s in that number; when raising 10 to a power, the index number indicates the number of zeros in the answer, such as $10^2 = 100$.

Purposeful practice Reflect and reason

Purposeful practice 1

Why are the answers to **Q3–6** the same?

Why are the answers to **Q7–9** the same?

Q4 *is the same as* ***Q3*** *because 2 × 10 = 20;* ***Q5*** *is the same as* ***Q4*** *because 20 × 10 = 200;* ***Q6*** *is the same as* ***Q5*** *because 100 × 2 = 200*

***Q8** is the same as **Q7** because 8 × 100 = 800; **Q9** is the same as **Q8** because both are 3 × 8 × 100 and it doesn't matter what order you multiply (i.e. commutativity).*

Purposeful practice 2

Explain why the answers to **Q4** and **Q5** are different to the answer to **Q3**.

Explain why the answers to **Q8** and **Q9** are different.

*In **Q3**, both 2 tens (20) and 4 ones (4) must be multiplied by 7. However, in **Q4**, only the 2 tens (20) are multiplied by 7; and in **Q5** only the 4 ones (4) are multiplied by 7.*

*In **Q8**, the answer is 5 tens (50) is multiplied by 6 add 2 ones (2) multiplied by 6; in **Q9**, 5 tens (50) is multiplied by 6, then multiplied by 2.*

Purposeful practice 3

When did you use priority of operations? When did you work from left to right?

Use priority of operations where a calculation involves multiplication or division and addition or subtraction, or where there are brackets; work from left to right where a calculation involves multiplication and division only, or addition and subtraction only.

Objectives

Core objectives

- Make an estimate to check an answer
- Use inverse operations to check an answer
- Use a written method to add and subtract whole numbers of any size
- Round whole numbers to the nearest 10 000, 100 000, 1 000 000.

Support objectives

- Use a written method to add and subtract whole numbers
- Round whole numbers to the nearest 10, 100 and 1000

Depth key concepts

- Understand inverse operations (addition and subtraction)

Key points

An **approximation** is a number that is not exact. It is close enough for it to be useful though. Use approximations to **estimate** the answer to calculations.
≈ means 'approximately equal to'.

You can check a subtraction calculation using the inverse operation of addition.

For rounding to the nearest 10 000

- 5000 and above rounds up
- 4999 and below rounds down

Common errors and misconceptions

- Students often read column calculations from left to right. To address this, use expanded forms of the written method to build on students' understanding.

Question guidance

Core

Q5 This is an opportunity to assess whether students make the common error of reading column calculations from left to right. Consider whether there is an 'appropriate' level of rounding. *Would it be useful to round to the nearest 1000 to check 1375 + 2148? Why/why not?*

Q6 Introduce the bar model as a visual way to solve word problems involving addition and subtraction.

Q12–Q14 These questions have been carefully constructed to give students practice at rounding to increasing sizes of numbers.

Q15 Highlight the importance of reading word problems carefully to correctly identify what information is given and what is being asked for (and that several steps may be needed before the answer is arrived at). Demonstrate how bar models help with visualising the problem.

Support

Addition Q2 Encourage students to realise that, when finding pairs of 2-digit numbers that add to 100, the tens digits total 9 (making 90) because the ones digits sum to 10, and 90 + 10 = 100.

Subtraction Q3 Reinforce the idea of taking 1 from a column to the left and moving it to the adjacent column; stress place value and why the '1' in the tens column becomes a 10 in the ones column.

Purposeful practice

Purposeful practice 3 These questions give students practice at rounding the same numbers to the nearest 10, 100, 1000 and 10 000.

Reflect

Core

This lesson used some bar models for problem-solving. Would you use bar models to help you solve mathematics problems in future? Explain why, or why not.

This task encourages students to think about the purpose of bar models, and reflect on whether they found them useful in their own mathematical problem solving, and how.

Support

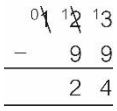
Draw a diagram or write a calculation to show how you would work out 123 – 99.

Students' own answers. Possible answers include:

using jumps on a number line to show 99 + 1 + 23 = 123 so 123 – 99 = 1 + 23 = 24

rounding 99 to 100 then adjusting: 123 – 100 = 23; 23 + 1 = 24

using the column method to show



Depth

Do the words in a problem always tell you exactly what calculation you need to do? Explain.

The words in a problem won't always tell you exactly what calculation you need to do. However, it should be possible to work out what calculations are required from the information given in the question.

Purposeful practice Reflect and reason

Purposeful practice 1

724 + 265 = 989

How can you use this addition calculation to work out

a 724 + 275 **b** 824 + 265?

a *There is one more ten (in 275) than in the addition calculation, so the answer must have one more ten (i.e. 999).*

b *There is one more hundred (in 824) than in the addition calculation, so the answer must have one more hundred (i.e. 1089).*

Purposeful practice 2

Write two related calculations for

a 867 + 429 **b** 902 – 471

a *867 + 429 = 1296; 1296 – 867 = 429; 1296 – 429* = 876

b 902 – 471 = *431; 902 – 431 = 471; 471 + 431 = 902*

Purposeful practice 3

Why did **Q3** parts **b**, **c** and **d** give the same answer?

Because the nearest 100, 1000 and 10 000 are all the same for 9984, i.e. 10 000.

Objectives

Core objectives

- Use an estimate to check an answer to a multiplication
- Use a written method to multiply whole numbers

Support objectives

- Use a written method to multiply whole numbers

Depth key concepts

- Knowing what it means to multiply, e.g. by comparing grid method and long multiplication and explaining why they are the same

Key points

- You can use estimation to check the answer to a multiplication calculation. Round the numbers and then multiply.
  In the **column method** you write the numbers in the calculation in their place value columns like this:

  $$\begin{array}{r} 242 \\ \times \quad 3 \\ \hline \\ \hline \end{array}$$

- **Long multiplication** is a written method to multiply by numbers with two or more digits.
- If money received is greater than money spent, then you make a **profit**. If money spent is greater than money received, then you make a **loss**.

Common errors and misconceptions

- Problems with place value can cause difficulties with written work. Encourage mental/jotting approximation before starting written work. Revisit partitioning numbers and revert to the grid method if the column method is continuing to cause confusion.

Question guidance

Core

Q5, Q10 These questions are carefully structured to give students practice using the column method for multiplying by single- and 2-digit numbers.

Q5 Show both column multiplication and grid method. Encourage students to use column multiplication because it is more efficient.

Q6 Use approximations to check which calculations are incorrect. Students need to identify the errors made. Discuss whether there is an 'appropriate/acceptable' level of rounding to be used when estimating an answer.

Q8 Recap the usefulness of bar modelling as a means of solving word problems.

Q10–Q13 Extension of multiplication work to TU × TU and HTU × TU. Stress the need to estimate the answer first.

Q13 Draw students' attention to the key point explaining profit and loss.

Challenge Provide students with concrete materials such as a set of digit and shape cards. Students can then use these cards to build the calculation given and use trial and error to replace each shape with a numbered card to find the missing digit. This could be used as a pair activity to allow students to discuss and try different methods and approaches.

Support

Multiplying by a single digit using the grid method Q2 and Q3 These questions demonstrate how partitioning the multiplier leads on to using the grid and eventually column methods of multiplication. Ensure students can partition numbers before attempting these questions. Encourage students to estimate their answers first.

Purposeful practice

Purposeful practice 1 Q1 Use this question to check that students multiply from right to left when doing column multiplication.

Reflect

Core

Look at your working for part **a** of the Challenge. Did you use a strategy to make sure you found all the possible answers?

If you answered 'Yes', what strategy did you use, and why?

If you answered 'No', what strategy could you have used, and why?

Could you have used a different strategy? Would this have been better?

This task aims to raise students' awareness of their problem-solving strategies and to encourage them to evaluate them in an effort to develop better strategies.

Support

Josh draws a grid to work out 421 × 6. What is wrong with his grid?

×	4	2	1
6	24	12	6

Draw the correct grid and work out the answer.

He has multiplied 6 × 4 instead of 6 × 400, and 6 × 2 instead of 6 × 20. The correct grid is

×	400	20	1
6	2400	120	6

2400 + 120 + 6 = 2526

Depth

Ali says that 452 × 87 = 17 176. Olivia says she has two simple checks to explain why Ali's answer is incorrect. One check uses estimating and the other check uses 'final digits'.

Give more details of what Olivia's checks mean. What are the advantages and disadvantages of each method?

Estimating means working out 450 × 100 = 45 000, so the answer needs to be much larger. 'Final digits' means working out 2 × 7 = 14, so the final answer needs to end with a '4'.

Estimating:

- *Advantage: Shows if the answer is of about the right size, or that it is not.*
- *Disadvantage: Doesn't show that the answer is wrong if, e.g. only the final digit is incorrect.*

Final digits:

- *Advantage: Quick way of checking that an answer is not correct (if final digit is wrong).*
- *Disadvantage: Doesn't tell you anything about the accuracy of the non-final digits.*

Purposeful practice Reflect and reason

Purposeful practice 1

Adrian worked out the answer to **Q2** by adding 3 to his answer to **Q1**. Is he correct? Explain why.

How can you use your answer to **Q7** to work out the answers to **Q8** and **Q9**?

*Yes he is correct because in **Q1** you have 313 lots of 3 and in **Q2** you have one more (314) lots of 3.*

*In **Q7** there are 619 lots of 5. In **Q8** there is one more (620) lot of 5 than in **Q7**. So, add 5 to the answer to **Q7**. In **Q9** there are two more lot of 5 (621) than in **Q7**. So, add two lots of 5 (10) to your answer to **Q7**. (Or, you may notice that in **Q9** there is one more lot of 5 (621) than in **Q8**. So, add 5 to the answer to **Q8**.)*

Purposeful practice 2

Lisa worked out the answer to **Q2** by subtracting 7 from her answer to **Q1**. Is she correct? Explain why.

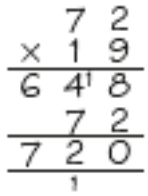
Look carefully at **Q6**, **Q7** and **Q9**. How can you use your answer to **Q6** to work out the answers to **Q7** and **Q9**?

*Yes, she is correct because in **Q1** you have 152 lots of 7 and in **Q2** you have one less (151) lots of 7.*

*You can subtract 479 from your answer to **Q6** to get the answer to **Q7** and you can add 479 to your answer to **Q6** to get the answer to **Q9**.*

Purposeful practice 3

Rohil works out this multiplication.



What mistake has he made?

What is the correct answer?

He has multiplied 72 by 1 instead of 10.

The correct answer is 1368.

Objectives

Core objectives

- Use a written method to divide whole numbers
- Use inverse operations to check an answer

Support objectives

- Use a written method to divide numbers

Depth key concepts

- Knowing what it means if a division calculation has a remainder
- Understand inverse operations (multiplication and division)

Key points

- You can check a division calculation using the inverse operation of multiplication.

- **Long division** is a written method to divide by numbers with two or more digits. It breaks down the calculation into smaller steps than short division.

Common errors and misconceptions

- Students often neglect to preserve place value in the answer, e.g. writing $3\mathop{\left){\vphantom{1{303}}}\right.\!\!\!\overline{\,\,\,\vphantom 1{{303}}}}\limits^{\displaystyle\,\,\,{11}}$ instead of $3\mathop{\left){\vphantom{1{303}}}\right.\!\!\!\overline{\,\,\,\vphantom 1{{303}}}}\limits^{\displaystyle\,\,\,{101}}$, or forget to do the subtraction when using long division. Model both carefully when showing examples.
- Students often fail to understand the process of division. Explain that division is sharing or grouping, and is the inverse of multiplication.

Question guidance

Core

Q4, Q6, Q9 These questions have been carefully structured to give students practice in increasingly complex short division calculations.

Q10, Q11 These questions have been carefully structured to give students practice in increasingly complex long division calculations.

Q14 Recap the use of bar modelling as a way to solve word problems. The use of inverse operations as a checking method needs to be considered. An understanding of inverse operations could be encouraged through the use of a concrete activity involving numbered cards and arrows which could be used to form 'number machines'.

Support

Short division without remainders Q1 Some students may find this visual structure helpful, but for others you may prefer to move straight on to the more formal method of division. It is important that students link the idea of division to 'sharing'.

Purposeful practice

Encourage students to ensure they are careful about lining up digits correctly when doing long division.

Reflect

Core

In lesson 2.3 you learned about multiplication and in this lesson you learned about division.

Which was more difficult? What made it more difficult?

Do you need more practice on any kinds of question? If so, which kinds?

This task asks students to reflect on their understanding of multiplication and division, and encourages them to take responsibility for their own learning by identifying where they may require more practice.

Support

The first numbers in the 4 times table are: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

What do the numbers in the 4 times table all end in?

Is the rule for dividing by 4 the same as the rule for dividing by 2?

All the numbers in the 4 times table end in the digits 4, 8, 2, 6, 0; they are all even digits.

You use short division to divide by 4 in the same way as for dividing by 2. Dividing by 4 is the same as dividing by 2 and then dividing by 2 again.

Depth

How often is multiplication useful when answering a division question?

Never? Sometimes? Often? Always? Explain.

Always. You can use multiplication to check the answer to a division calculation.

Purposeful practice Reflect and reason

Purposeful practice 1

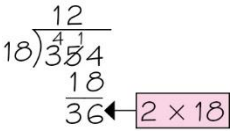
a $6\left){\vphantom{1{2754}}}\right.\!\!\!\!\overline{\,\,\,\vphantom 1{{2754}}}$ **b** 9 × 37

a *6 × 459 = 2754, 459 × 6 = 2754*

b $9\mathop{\left){\vphantom{1{333}}}\right.\!\!\!\overline{\,\,\,\vphantom 1{{333}}}}\limits^{\displaystyle\,\,\, {37}}$, *37 × 9 = 333*

Purposeful practice 2

Sally works out 354 ÷ 18



What mistake has Sally made? What is the correct answer?

Sally subtracts 18 from 54 instead of 35. Correct answer is 19 remainder 12.

Objectives

Core objectives

- Round money to the nearest pound or penny
- Interpret the display on a calculator in different contexts
- Use a calculator to solve problems involving money and time

Support objectives

- Round money to the nearest whole pound or penny
- Solve problems involving money and time

Depth key concepts

- Understand how multiplying by 10, 100, 1000 etc relates to our place value system and why this means that we have a decimal system. (NOTE: For this lesson this should be specifically in the context of money.)

Key points

- To **round** an amount to the nearest pound, look at the pence.

£12.61 rounds up to £13

- On a calculator

= £2.05

= £2.50

- Change is the money you get back after paying for something with more money than it costs.

Common errors and misconceptions

Students often interpret a calculator display incorrectly, either

- reading 3.5 as 'three pounds and five pence'. Encourage students to use a zero placeholder after the five to read as '3 pounds fifty'.
- seeing 3.40 as 3 hours 40 minutes. Stress the need to take care when calculating with time, as each hour has 60 minutes, not 100. Relating the decimal part of the amount to the equivalent fraction will help address this.

Question guidance

Core

Q6 This question demonstrates the need for rounding in a real-life context.

Q7 This question involves interpreting a calculator display in the context of money.

Q14 Use this question to reinforce the link between rounding to the nearest penny and rounding to 2 decimal places.

Q17 Address the errors that can arise when reading calculator display in the context of time, e.g. thinking that a display of 0.25 means 25 minutes rather than 0.25 of an hour.

Support

Working with money Q4 Students often struggle to round numbers correctly, for example, rounding £3.19 up to £4 because of the 9. Encourage them to use the number line provided to work out the position of each amount so it becomes obvious whether to round up or down.

Purposeful practice

Purposeful practice 3 Encourage students to learn how to recognise the common fraction and decimal equivalents ($\frac{1}{4} = 0.25, \frac{1}{2} = 0.5, \frac{3}{4} = 0.75$) and express them in minutes.

Reflect

Core

Write down two new things you have learned about using a calculator for money calculations.

Which questions used this new knowledge?

When you answered these questions, did you make any mistakes? If so, check that you understand where you went wrong.

This task encourages students to use higher-level thinking skills in order to identify important concepts they have learned, and to review and analyse whether they still do not understand, and where.

Support

When you enter time and money into a calculator you need to enter it in decimal form.

Explain why 3 hours and 40 minutes is not the same as 3.4 hours.

3.4 hours means 3 hours and 0.4 or $\frac{4}{10}$ of an hour. There are 60 minutes in 1 hour so

$\frac{4}{10} \times 60 = 24$ minutes. So 3.4 hours is 3 hours and 24 minutes.

Depth

Sarah says that calculations involving money are easier than calculations involving time.

Do you agree? Explain.

For example: Agree, because money calculations are based in 100s and time calculations are based in 60s.

Purposeful practice Reflect and reason

Purposeful practice 1

How do you know when to round up or down when working with money?

What was the same and what was different when you rounded the calculator money answers to the nearest penny in **Q2** parts **g** and **h**?

Round up to the nearest pound when there is 50p (£0.50) or more.

Round up to the nearest penny when the third number after the decimal point is 5 or more.

Round down to the nearest pound when there is 49p (£0.49) or less.

*Round down to the nearest penny when the third digit after the decimal point is less than 5 (**Q2** part **g**) and round up when it is 5 or more (**Q2** part **h**).*

Purposeful practice 2

Why do **Q1a**, **Q2a** and **Q2b** have answers that are the same amounts of money?

1.99[] *always rounds to £2 to the nearest pound whatever digit is in* []

1.99[] *always rounds to £2 to the nearest pence when 5 or more is in* []

Purposeful practice 3

Kamal uses his calculator to work out the answer to a time calculation in hours.

His calculator display shows 6.25.

He says. 'The answer is 6 hours 25 minutes.'

Why is this wrong? What is the correct answer?

0.25 in the calculator display means $\frac{25}{100}$ minutes, not $\frac{25}{60}$ minutes

$\frac{25}{100} = \frac{1}{4}$ so the correct answer is $6\frac{1}{4}$ hours

f

Objectives

Core objectives

- Order positive and negative numbers
- Add and subtract positive and negative numbers
- Begin to multiply with negative numbers

Support objectives

- Order positive and negative numbers
- Add and subtract positive and negative numbers

Depth key concepts

- Understand what negative numbers are and how they behave: where they fit into the ordering of the number line and how they multiply.

Key points

- The symbol > means greater than. The symbol < means less than.

Common errors and misconceptions

- Students often assume the absolute value when comparing or ordering negative numbers. For example, assuming that –4 is a 'bigger' number than –2. Address this by using number lines and negative numbers in context to show that a number's value is related to its position on the number line – the further to the left a number is on the number line, the smaller it is.

Question guidance

Core

Q4 and other questions involving addition and subtraction or comparison of positive and negative integers can be supported visually by a number line.

Q4–Q6 These questions introduce negative numbers in the familiar context of temperature.

Q9, Q10 These questions have been carefully constructed to show students the pattern created when subtracting the same number from numbers decreasing by 1, and when subtracting numbers decreasing by 1 from the same number.

Q11 This question uses the image of a number line and repeated addition to demonstrate multiplication of negative numbers.

Q11, Q12 These questions have been carefully constructed so that students see the effect of multiplying by negative numbers.

Support

For this lesson provide blank number lines for students to draw on. Students often find the concept of a 'larger' negative number difficult. Use the language of 'greater' and 'less than' and refer students to the thermometer in **Q2**. Ask questions such as *Which is the 'coldest' temperature?*

Ordering positive and negative numbers Q7 This has minimally different questions which encourage students to understand that as the number being subtracted increases the answer gets smaller.

Purposeful practice

Purposeful practice 2 Q4 Use this question to assess whether students can use their knowledge of positive and negative numbers and the relative sizes of the numbers to work out whether answers will be positive or negative.

Reflect

Core

Write this calculation: –1 + 2 – 3

Without looking at this book, imagine a number line and work out the answer to the calculation. Now check your answer by drawing a number line.

Did imagining a number line help you? If so, how? If not, do you have a different way of imagining negative numbers?

This reflective task encourages students to use visualisation to help them understand the concept of negative numbers, and any calculations that involve them.

Support

Work out 5 – 8.

Write down five other calculations that give the same answer. Compare them with a partner. Do the same for –2 – 5.

5 – 8 = –3; students' own answers for other calculations with –3 as the answer, e.g. 6 – 9, 7 – 10, 4 – 7, 3 – 6, 2 – 5.

–2 – 5 = –7; students' own answers for other calculations with –7 as the answer, e.g. –3 – 4, –4 – 3, –5 – 2, –1 – 6, 0 – 7.

Depth

Explain why you can't write a sensible question about a number of apples that uses the calculation in **Q13**.

The answer is negative. You can have negative temperature, but you can't have a negative number of apples (unless you owe someone some apples).

Purposeful practice Reflect and reason

Purposeful practice 1

Felix says, â€˜16 is greater than 15, so –16 is greater than –15.â€™

Is he correct? Explain your answer.

No, he's not correct. Writing the numbers on a number line demonstrate that 16 > 15 but –16 < –15.

Purposeful practice 2

Antony knows that 46 + 54 = 100. He writes

–46 + 54 = 100 and 46 – 54 = –100

Is he correct? If not, write the correct answers.

No, he is not correct.

–46 + 54 = 8

46 – 54 = –8

Objectives

Core objectives

- Find all the factor pairs for any whole number
- Identify common factors, the highest common factor and the lowest common multiple
- Recognise prime numbers

Support objectives

- Work out multiples and find the lowest common multiple
- Find all factor pairs of a number and the highest common factor of two numbers
- Recognise prime numbers

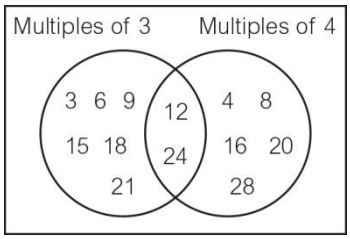
Depth key concepts

- Connecting remainders to factors and multiples

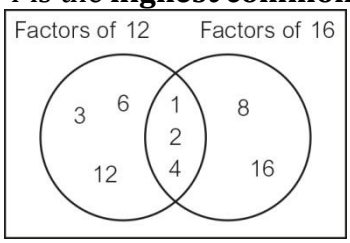
Key points

- A **multiple** of a number is in that number's multiplication table.
- A **Venn diagram** is a way of showing sets of numbers.
- Here are some multiples of 3 and 4.
  3: 3, 6, 9, **12**, 15, 18, 21, **24**, …
  4: 4, 8, **12**, 16, 20, **24**, 28, …
  12 and 24 are two **common multiples** of 3 and 4.

12 is the **lowest common multiple** of 3 and 4.



- A **factor** is a whole number that will divide exactly into another number. A **factor pair** is two numbers that multiply together to make a number.
- Here are the factors of 12 and 16.
  12: **1**, **2**, 3, **4**, 6, 12
  16: **1**, **2**, **4**, 8, 16
  1, 2 and 4 are the **common factors** of 12 and 16.
  4 is the **highest common factor** of 12 and 16.



- A **prime** number has exactly two factors, 1 and itself.

Common errors and misconceptions

- Students often fail to find all the factors of a number. Remind them that factors come in pairs, so can be written as $a \times b$. Encourage them to be systematic so they can be sure they record all the factors of a number.
- Students often think that 1 is a prime number and that 2 is not. Address this by focusing on the definition of a prime number – it has exactly two different factors. $1 = 1 \times 1$, and so has only one factor, therefore isn't prime. $2 = 1 \times 2$, and so has two factors, therefore is prime.

Question guidance

Core

Q12 Encourage discussion of how many multiples students need to write for each pair of numbers. To find the lowest common multiple, they need to find the first number that appears in both lists.

Q16b Ask *Have you found **all** the factors of 8? How do you know?* Emphasise that when finding factors of a number, they need to check every whole number up to half the value of the original number.

Q18b This question promotes discussion and understanding that 2 is the only even prime, as all other even numbers are multiples of 2.

Support

Multiples Q1–Q3 Students often confuse multiples and factors. Encourage them to connect 'multiple' to the meaning of multiplying (getting bigger) and link to the times tables, so they should expect multiples of a number to get bigger.

Multiples Q4, Q5 encourage students to realise that the LCM is not always found by multiplying the numbers together.

Purposeful practice

Purposeful practice 1 and 2 When working out all the factors of a number, students should be systematic and remember that each factor has a partner; square numbers are a special case because the partner of one of the factors is the same factor.

Reflect

Core

Write your own short definition for each of these mathematical words:

highest lowest common factor multiple

Now use your definitions to write (in your own words) the meaning of:

highest common factor lowest common multiple

This task encourages students to consider the individual words in 'highest common factor' and 'lowest common multiple', so that they can work out meaning for themselves, rather than view them as technical mathematical terminology that requires memorising.

Support

Are these statements true or false?

a 15 is a prime number as the only factors are 3 and 5.

b There are no even prime numbers.

c 1 is a factor of every number.

d 9 is a factor of 3.

a *False. The factors of 15 are 1, 3, 5 and 15.*

b *False. 2 is a prime number; it is the only even prime number.*

c *True. For every number you can write 1 × that number.*

d *False. 9 is a multiple of 3; 3 is a factor of 9.*

Depth

Write sentences explaining a connection between:

a factors and multiples

b primes and factors

c factors and remainders.

Compare your sentences with others in your class.

a *If a number is a factor of another number, then the other number is a multiple of the factor.*

b *Every number has factors that are prime.*

c *A number divided by its factor will give a remainder of 0; a number +1 divided by one of its factors will give a remainder of 1; a number +2 divided by one of its factors will give a remainder of 2, etc.*

Purposeful practice Reflect and reason

Purposeful practice 1

Crista lists all the factors of 15

3, 5

Is her list correct? If not, write the correct list.

Crista now lists the first six multiples of 30

1, 2, 3, 5, 6, 10

What mistake has Crista made? Write the correct list.

No, her list of factors is not correct. The correct list is of factors is: 1, 3, 5, 15

Crista has listed the factors of 30 instead of the multiples. The correct list of multiples is: 30, 60, 90, 120, 150, 180

Purposeful practice 2

When is the HCF one of the numbers in the pair? Is it the smaller or larger number in the pair?

When one number is a factor of the other. It is the smallest number in the pair.

Purposeful practice 3

When is the LCM one of the numbers in the pair? Is it the smaller or larger number in the pair?

When one number is a factor of the other. It is the largest number in the pair.

Objectives

Core objectives

- Recognise square numbers
- Use a calculator to find squares and square roots
- Use the priority of operations, including powers
- Use index form for powers
- Do mental calculations with squares and square roots

Support objectives

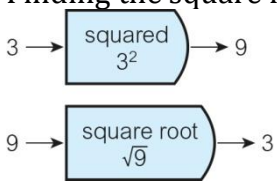
- Recognise square numbers
- Use a calculator to find squares and square roots
- Use the priority of operations, including powers

Depth key concepts

- Most square roots give decimal answers (Do not stray into surds!)

Key points

- **Square numbers** make a square pattern of dots. To find the square of a number, you multiply it by itself.
- You can write 3×3 as 3^2. You read this as '3 squared'. The '2' in 3^2 is called the power or **index**. The plural of index is **indices**.
- You can use the x^2 key on your calculator to work out a square.
- A **square root** is a number that is multiplied by itself to produce a given number. Finding the square root is the **inverse** of squaring.



- You can use the $\sqrt{\ }$ key on your calculator to find a square root.

Common errors and misconceptions

- Students often think that $\sqrt{n} = \frac{n}{2}$. To address this misconception, emphasise the link between squares and square roots, that is, they are inverses.
- Students often make errors with the order of operations. Emphasise BIDMAS and the priority of operations it describes; use counterexamples to show that the rules must be followed, so a consistent answer is always achieved.

Question guidance

Core

Q4 Use this question to reinforce pictorially the link between square numbers and the related square shapes.

Q11, Q12 Revisit BIDMAS including brackets and indices. Encourage students to write down each step in the calculation to avoid errors.

Q13 This question has been carefully constructed to show students what squared (^{2}) and cubed (^{3}) notation means.

Challenge part **c** If necessary, show that the number added to a triangle number increases by 1 each time. For part **f,** use isometric paper to show how the dot pattern grows to match the triangle numbers.

Support

In this lesson it may be helpful to give students counters to model making square numbers and taking square roots. This could help to address misconceptions that $\sqrt{n} = \frac{n}{2}$ and $n^2 = n \times 2$.

Purposeful practice

Purposeful practice 3 Remind students of the priority of operations and BIDMAS. Both **Q1** and **Q2** emphasise the effect of brackets.

Reflect

Core

Think about the *square* of 9 and the *square root* of 9. Which is 3 and which is 81? Make sure you know the difference between these two mathematical terms. Write down a hint in your own words to help you remember which is which.

This task gives students the opportunity to clarify (and so avoid confusion between) the terms square and square root.

Support

Erin says: 'The square root of 9 is 81. 9^2 is 3.'

Explain her mistakes. How should she work out $\sqrt{9}$ and 9^2?

Erin has confused squares and square roots. The square root of 9 is 3 and is shown as $\sqrt{9} = 3$ *whereas 9 squared is 81 and is shown as* $9^2 = 81$.

Depth

All square numbers have an odd number of factors. Explain why.

All factors of a number come in factor pairs. However, the 'pair' of the square root of a square number is itself, so this 'pair' only contributes 1 to the number of factors. Added to the even number of other factors, this means that the total number of factors is odd.

Purposeful practice Reflect and reason

Purposeful practice 1

Jean writes $15^2 = 30$.

What mistake has she made?

What is the answer to 15^2?

Jean has worked out 15 × 2 instead of 15 × 15.

$15^2 = 225$

Purposeful practice 2

Write 'square' or 'square root' to complete these sentences.

9 is the _____ of 3.

3 is the _____ of 9.

81 is the _____ of 9.

*9 is the **square** of 3. 3 is the **square root** of 9. 81 is the **square** of 9.*

Purposeful practice 3

Miguel says, '$6^2 - 5^2$ and $(6 - 5)^2$ give the same answer.' Explain why Miguel is wrong.

Miguel is not correct.

$6^2 - 5^2 = 36 - 25 = 11$ *and* $(6 - 5)^2 = 1^2 = 1$

Check, Strengthen and Extend

Core Check up

Q1 This question reinforces estimation as a checking method.

Q7 This question is designed to test the use of negative numbers in context.

Q8 Use this question to confirm students' understanding and ability to use the < and > signs.

Q9 This question requires recall of the definition of factor, multiple, prime and square numbers.

Q14 Look for evidence of sensible estimation rather than a calculation.

Q16 This question is designed to check students' efficacy and accuracy when using a calculator.

Q17 This question checks students' effective use of a calculator and also their understanding that there are 60 minutes in 1 hour.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Look back at the questions you answered in this lesson.

Some were

A calculations (for example, Mental work **Q4**)

B word problems (for example, Problem-solving **Q1**)

Sometimes you had to

C add (for example, Written methods **Q1**)

D subtract (for example, Written methods **Q2**)

E multiply (for example, Written methods **Q5**)

F divide (for example, Written methods **Q6**)

a Write down one other question number for each type of question: **A**, **B**, **C**, **D**, **E** and **F**.

b Which type of question did you find easiest?

c Which type of question did you find hardest? When you answered these questions, did you make some mistakes? If so, check that you understand where you went wrong.

This task encourages students to recognise different question types and then consider which type they find easiest and hardest. It encourages students to take responsibility for their own learning by reviewing and analysing if, and where, they still do not understand.

Extend

In this Extend lesson, do you think you did

A well **B** OK **C** not very well?

List three things that made a difference to how well you did.

They can be things that made you do better or worse.

Here are some things other students said, to give you some ideas.

'I didn't read some questions properly.'

‘It helped that I can do long multiplication.’

‘I chatted to John too much.’

How do you think you could have done even better in these lessons?

This task asks students to consider their performance and what affects it. This may relate to particular topics they find hard or easy, but it may also relate to the ways they like to work. For example, they may state that their performance was hindered by noise in the classroom, or benefitted from sitting next to someone who works at the same pace. Finally, students are asked to think of ways they may improve their performance.

Depth

Extend

Look back at the work you have done in this lesson.

Write a paragraph explaining the new things you have learned about numbers in Unit 2.

Write a sentence explaining anything new you have learned about solving maths problems.

Students’ own answers. Responses could refer to: working strategically to find missing digits in calculations, what the factorial symbol (!) means and how to use it, how to work out monthly instalments after paying a deposit, the means of the terms ***proper divisor****,* ***deficient number****,* ***perfect number*** *and* ***abundant number****.*

End of Unit 2 tests: Number skills

Pearson published resources

Support/Core

- Year 7 End of Unit 2 Support/Core test
- Year 7 End of Unit 2 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 2 Core/Depth test
- Year 7 End of Unit 2 Core/Depth mark scheme

Unit opener: Expressions, functions and formulae

Pearson published resources

Objectives

Core objectives

- Find outputs of simple functions written in words and using symbols
- Describe simple functions in words

Support objectives

- Find outputs of simple functions written in words and using symbols

Depth key concepts

- Understand that a function is a relationship that maps one set of numbers on to another, with each input mapping to exactly one output, and with the maths they know so far, it can use any of the four operations, and the order of the operations is important (e.g. ×3 + 1 is not usually the same as + 1 × 3)

Key points

- A **function** is a relationship between two sets of numbers. The numbers that go into a **function machine** are called the **inputs**. The numbers that come out are called the **outputs**.

Common errors and misconceptions

- Students sometimes decide what an unknown function is based on only one set of input and output values. You can resolve this using a pictorial activity. Ask students to draw what they think is the correct function machine with the inputs given, then work out all the outputs and check they match with those in the question.

Question guidance

Core

Q6 This question has been constructed to show students that they need to look at the relationship between every pair of inputs and outputs in order to work out the function that connects them.

Q11 Use this question to emphasise the importance of the order of the functions in a two-step function machine. Ask students to consider the following question. *If you input a number into the function machine 'add 2, then multiply by 2', will you get the same output number as from the function machine 'multiply by 2, then add 2'?* Encourage students to try different numbers to draw out an answer of 'no'. Can any of them work out the only number for which it is true? Zero.

Q12b This question highlights the important difference between dividing even numbers by 2 and dividing odd numbers by 2. The input numbers given are all odd. When 3 is added to an odd number, the answer is even, then an even number divided by 2 will always give a whole number. However, if the input number is even, even + 3 is odd, then odd ÷ 2 is not a whole number.

Support

Two-step function machines Q1 Encourage students to use the first input and output to check that when they input 3, they get an answer of 3, to confirm they are using the correct method.

Purposeful practice

Purposeful practice 2 This highlights the importance of the applying functions in the correct order. Understanding this is crucial when students apply inverse operations to work out missing inputs.

Reflect

Core

a Bella says, 'A function always has inputs and outputs. Is Bella correct?

b Kim says, 'Each input gives only one output.' Is Kim correct?

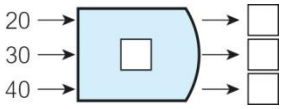
c What else can you say about functions? Discuss with a classmate.

This reflective task encourages students to review their work and begin to generalise and hypothesise about what a function is. It focuses students' attention back to the concept of functions rather than on the tool of a function machine.

a *Bella is correct.* ***b*** *Kim is correct.*

c *You could prompt students by asking them to choose a function from a question and test if it only works for the inputs shown, or if they think it would work for all possible input values.*

Support



Copy and create four different function machines with four different rules. Use +, –, × and ÷.

Bella says: 'There are only four rules you can write.' Is Bella correct? Explain your reasons.

Students' own examples, e.g. function of +10 giving outputs of 30, 40, 50; function of –5 giving outputs of 15, 25, 35; function of ×4 giving outputs of 80, 120, 160; function of ÷2 giving outputs of 10, 15, 20. Bella is wrong. Although there are four operations, they can be used in many different ways, e.g. any number can be added to, subtracted from, multiplied by or can divide the inputs.

Depth

Look back at the functions in this lesson.

a For each function, does each input map to exactly one output?

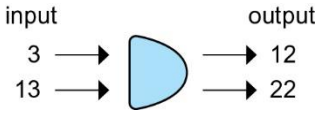
b Which function maps more than one input to the same output?

a *Yes* ***b*** *×0*

Purposeful practice Reflect and reason

Purposeful practice 1

Dan says, â€˜The function for this function machine is × 4.â€™ Explain why Dan is wrong

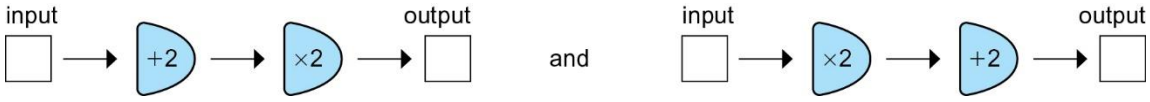


× 4 only works for 3 × 4 = 12. It does not work for 13 → 22. The correct function is + 9.

Purposeful practice 2

For any input number, do these function machines both give the same output number?

Choose three input numbers to show your working.



No; students' own three examples to show the different outputs, e.g. (5 + 2) × 2 = 14 and 5 × 2 + 2 = 12.

Objectives

Core objectives

- Use letters to represent unknowns in algebraic expressions
- Simplify linear algebraic expressions by collecting like terms

Support objectives

- Use letters to represent unknowns in algebraic expressions
- Simplify linear algebraic expressions by collecting like terms

Depth key concepts

- Know what an unknown is, how you can use any letter to represent an unknown number or quantity, and that as they represent numbers, you can add, subtract them in the same way as you do numbers

Key points

- In maths, if you do not know a value, you can use a letter to represent it.
- An **expression** contains **terms** using numbers and letters.
  Terms: $2m$ $13x$ y 7
  Expressions: $2m + 7$ $x + 2m$
- $1y$ is written as y.
- **Like terms** contain the same letter (or contain no letter). For example, $5x$ and $7x$ are like terms, but $4x$ and $3y$ are not like terms. You **simplify** an expression by collecting like terms.
- To find a **sum** you need to add. The sum of 2 and 7 is 2 + 7 = 9.

Common errors and misconceptions

- Students may write $5x - 4x = 1x$. Although it is not incorrect, explain to students that it is not necessary to write the 1. The convention is to write $5x - 4x = x$.

Question guidance

Core

Q4 To help students understand why $x + x + x + x$ is the same as $4x$, suggest they think of x as a distance, for example 1 km. If you add 4 of them together, the answer will be 4 km, or 4 lots of 1 km, hence $4x$.

Q6 Ask the students to think about whether $3x + 2x$ is the same as $2x + 3x$. How do you know? $3x + 2x$ and $2x + 3x$ both equal $5x$. Stress that adding with letters follows the same rules as if you were adding with numbers only.

Q8c Tell students to think of $2x + 2x - x$ as $2x + 2x - 1x$.

Q9 Make sure students read the strategy hint to help them. Make sure that students don't use the letter twice, e.g. $4x \times 2x$ for $8x$. Stress that the letter must only appear once.

Q12, Q16 These questions have been carefully constructed to give students practice at simplifying increasingly complex expressions involving multiple algebraic and number terms.

Support

General Encourage students to visualise or to draw and label each term as a bar to help them make sure they collect only like terms.

Purposeful practice

Purposeful practice 1 The terms in these questions vary minimally in order to give practice in adding terms then subtracting, then subtracting to give negative terms.

Purposeful practice 2 The expressions in these questions progress in complexity from 3 terms to more than 3 terms, and with increasing numbers of variables and/or numbers.

Reflect

Core

In algebra, letters are used to represent values you do not know.

This lesson might be the first time you have done algebra.

Choose **A**, **B** or **C** to complete each statement.

In this lesson, I did … **A** well **B** OK **C** not very well

So far, I think algebra is … **A** easy **B** OK **C** difficult

When I think about the next lesson, I feel … **A** confident **B** OK **C** unsure

If you answered mostly **A**s and **B**s, did your experience surprise you? Why?

If you answered mostly **C**s, look back at the questions you found most tricky.

Ask a friend or your teacher to explain them to you. Then complete the statements above again.

*This task aims to make conscious any negative thinking that has the potential to impede students' progress. This is particularly important for algebra, which has a reputation for being difficult. It is also intended to boost students' confidence, as hopefully most will answer with **A**s or **B**s.*

Support

Sanjay simplifies $10a + 5 - 4a - 2$. He writes the answer $9a$.

What mistake has Sanjay made?

Sanjay has not collected like terms – he has added and subtracted all the terms together. He should have written $10a + 5 - 4a - 2 = 10a - 4a + 5 - 2 = 6a + 3$.

Depth

What is the same about adding and subtracting expressions and adding and subtracting numbers? What is different?

Order is important in subtraction, but not in addition, which is the same as for numbers.

Same number rules apply: i.e. 3 + 5 = 8 whether they are just numbers or coefficients of like terms.

Inverse of + 3□ is â˜’ 3□ and vice versa, the same as + 3 is the inverse of â˜’ 3.

Different because you only get one term in the answer to a number addition/subtraction, but in algebra you may get one, two, three, etc. terms, with different unknowns.

Purposeful practice Reflect and reason

Purposeful practice 1

Is $2x + 5x$ the same as $5x + 2x$? Explain your answer.

Is $2x$ â˜’ $5x$ the same as $5x$ â˜’ $2x$? Explain your answer.

Yes, 2□ + 5□ = 5□ + 2□ because you can add in either order.

No, 2□ â˜’ 5□ ≠ 5□ â˜’ 2□ because 2□ â˜’ 5□ = –3□ and 5□ â˜’ 2□ = 3□.

Purposeful practice 2

Jay writes

$5x$ â˜’ $3 + 2x = 5x$ â˜’ $2x + 3$

Lou writes

$5x$ â˜’ $3 + 2x = 5x + 2x$ â€“ 3

Who is correct? Explain why.

Lou is correct. When you change the order of the terms you need to keep the signs with the terms.

Objectives

Core objectives

- Multiply and divide algebraic terms
- Use brackets with numbers and letters

Support objectives

- Multiply and divide algebraic terms
- Use brackets with numbers and letters

Depth key concepts

- Extend the understanding from 3.2 to include multiplying and dividing
- Understand that algebra uses the same arithmetic rules as number

Key points

- To find a **product** you need to multiply. The product of 7 and 2 is 14.
- When you multiply out a bracket, multiply every number inside the bracket by the number outside the bracket.
  $5 \times (\mathbf{3} + \mathit{12}) = 5 \times \mathbf{3} + 5 \times \mathit{12}$
  This is called the **distributive law**.
- $3(10 + 7)$ means $3 \times (10 + 7)$. You don't need to write the multiplication sign.
- To multiply out or **expand** expressions with brackets, multiply everything inside the bracket by the number outside.

Common errors and misconceptions

- Students often combine unlike terms, e.g. $2p + 3r = 5pr$. You can address this using a concrete activity using pens and rulers to represent p and r. Show that $2p + 3p$ simplifies to $5p$ and that $2r + 3r$ simplifies to $5r$, but that $2p + 3r$ cannot be simplified as you are not adding the same types of items.
- When expanding brackets, students often multiply only the first term in the bracket by the number outside the bracket, e.g. $4(x + 2) = 4x + 2$. Resolve this using a concrete activity such as using a box to represent a bracket. Write the number outside the bracket on a sticky note e.g. × 4 and stick it on the box. Then write x and +2 on different pieces of paper and put them inside the box. Tell students that when the pieces of paper are taken out of the box, both terms must be multiplied by 4.

Question guidance

Core

Q10–Q12 These questions have been carefully structured to show students how to use the distributive law.

Q14 This question addresses the common error of not multiplying all terms inside the brackets by the term outside the brackets.

Q13 Check students can remember how to work out the area of a rectangle. The width of this rectangle is the horizontal distance, i.e. $p + 3$.

Q15 If necessary, remind students to expand each bracket individually and then collect like terms. Take care with any 'â^' signs.

Support

Working with brackets Encourage students to use the grid method to support their expansion of brackets for as long as is needed to ensure they multiply all terms inside the brackets.

Purposeful practice

Purposeful practice 1 The terms in these questions vary minimally, offering practice in multiplying number and letter terms and writing answers using 'number first' and 'alphabetical order' convention.

Purposeful practice 2 The terms in these questions vary minimally, offering plenty of practice to make it clear you need to multiply both terms, and that the sign is important.

Reflect

Core

Lilina says, 'Simplifying algebra means make the expression so that it can't be added, multiplied, divided or subtracted any more.'

Howard says, 'Simplifying algebra means getting as few terms as possible.'

Do you agree with Lilina?

Do you agree with Howard?

Write a definition for yourself that begins 'In algebra, 'simplify' means ...'.

This reflective task addresses an important literacy, as well as mathematics point. This point is often overlooked when teaching expressions. What do we actually mean when we ask students to 'simplify'? In articulating a meaning for themselves, students not only reflect on how the word may be defined, but also on their own understanding.

Support

Mark simplifies $2(x - 3)$. He writes the answer $2x + 3$.

What mistake(s) has Mark made? What is the correct answer?

Mark has forgotten to multiply the second term in the bracket by 2, and he has not noticed that the sign in front of the second term is – and not +. The correct answer is 2□ – 6.

Depth

What is the same about multiplying and dividing expressions, and multiplying and dividing numbers? What is different?

The order of terms is important in division but not in multiplication, which is the same as for numbers.

3 × 5 = 15 whether they are just numbers or coefficients of like terms, i.e. same number rules apply.

The inverse of × 3□ is ÷ 3□.

Multiplying a bracket is the same as for a number because you multiply each term inside by each term outside.

Different because you can multiply and divide a term like 10□ by a letter, a number or a term involving a letter and a number.

Multiplying out brackets is different for a number because you end up with two terms (if there are two terms in the bracket), instead of a final number answer.

Purposeful practice Reflect and reason

Purposeful practice 1

Explain what is wrong with these simplifications.

$t \times 7 = t\,7$ $\quad$ $t \times m = t\,m$ $\quad$ $n \times n = 2n$

Write the number before the letter; it should be 7□

Write letters in alphabetical order; it, should be □□

□ × □ = □² whereas □ + □ = 2□

Purposeful practice 2

When you expand $2(x + 5)$ and $2(x - 5)$, what is the same and what is different?

When you expand $3(y + 2)$ and $3(y + 4)$, what is the same and what is different?

2(□ + 5) = 2□ + 10 $\quad$ *2(□ – 5) = 2□ – 10*

Same: 2□ and 10 terms. Different: sign before the 10.

3(□ + 2) = 3□ + 6 $\quad$ *3(□ + 4) = 3□ + 12*

Same: 3y term. Different: + 6 or + 12 term.

Objectives

Core objectives

- Write expressions from word descriptions using addition, subtraction, multiplication and division
- Write expressions to represent function machines

Support objectives

- Write expressions from word descriptions using addition, subtraction, multiplication and division
- Write expressions to represent function machines

Depth key concepts

- Begin to understand that an algebraic expression can represent a rule, and that writing an algebraic expression may be easier than explaining a rule in words, and easier for the reader to understand

Key points

- You write an algebraic expression by using letters to stand for numbers. The letter is called a **variable** because its value can change or vary.

Common errors and misconceptions

- When writing expressions, students often write subtractions the wrong way round, e.g. writing 3 less than x as $3 - x$. Suggest students say the sentence again replacing x with a number, e.g. 3 less than 5. The answer is 2. Then ask them to put the number into the expression, e.g. $3 - 5$. *Do you get the same answer?*

Question guidance

Core

General Encourage students to draw a function machine for as long as they need pictorial support to write expressions.

Q3 Students can use a bar model or a number line to help. They must treat parts **a** and **b** separately and compare Amber's age and Jenny's age to Danielle's, and not try to do part **b** using their answer to part **a**.

Q8 Students need to read the wording very carefully in parts **b** and **c** to get the subtractions the right way round.

Q14, Q15 These questions have been carefully constructed to ensure students read descriptions of expressions very carefully and to create expressions for increasingly complex situations.

Challenge Students should work with a partner to solve the problem. If they need help to start, suggest they write an expression for the normal time for Greg's evening journey, then create expressions for his weekly journey times in terms of the normal journey time.

Support

Writing expressions using function machines Q2, Q3 Use these questions to check that students are able to connect a description in words to the function machine that matches it.

Purposeful practice

Purposeful practice 1 The order of the calculations involved varies in each question but there is minimal variation in the numbers used, so students can focus on the meaning of the words and how to write the operations in algebra.

Purposeful practice 2 The numbers and terms in the questions vary minimally, offering intelligent practice with sum, difference and product of numbers, letters and numbers and two-letter terms.

Reflect

Core

This lesson suggested using bar models and function machines to help you with writing expressions. Did they help you? How?

Did you use any other methods? Explain the method(s) you used.

This reflective task asks students to consider how bar models and function machines help them to work with algebraic expressions. It also encourages them to reflect on other strategies they may use when writing expressions, e.g. it can be helpful to use the first letter of the name of an item or a property of the item to represent the item, e.g. p for pencil or y for yellow. (Make sure, however, that they do not think the letters actually ***mean*** *the item, as they could use any letter.)*

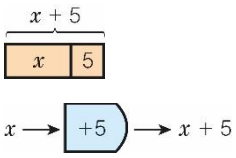
Support

This lesson used bar models and function machines.

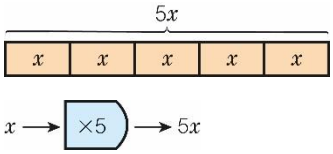
Draw a bar model and a function machine to represent each of these expressions.

a $x + 5$ **b** $5x$

a



b



Depth

Describe the effect of the function machine in **Q9** in words.

Explain how the expression you wrote in **Q9** part **a** gives the same information.

Why is it sometimes useful to write expressions with algebra rather than in words?

Function machine multiplies by 3 and then adds 2. Expression in ***Q9*** *part* ***a*** *is 3□ + 2, which shows that for any input value □ you multiply by 3 and then add 2. But 3□ + 2 is shorter. Algebra can save time and space, by avoiding writing lots of words. Also, algebra is an international language – you could understand 3□ + 2 in a textbook in any language.*

Purposeful practice Reflect and reason

Purposeful practice 1

For â€˜twice *x*â€™, Tim incorrectly writes the expression □ *+ 2*

Explain what Tim has done wrong.

Twice □ = 2 × □ = 2□. Tim has added instead of multiplying.

Purposeful practice 2

Liu writes *2□ + 9 = 11□*

Explain the mistake Liu has made.

She has added 2 and 9, but 2□ and 9 are not like terms, so you cannot add them.

Objectives

Core objectives

- Substitute positive whole numbers into simple formulae written in words
- Substitute positive whole numbers into formulae written with letters

Support objectives

- Substitute positive whole numbers into simple formulae written in words
- Substitute positive whole numbers into formulae written with letters

Depth key concepts

- Understand that the letters are called variables because they can change or vary, but the relationship between them given by the formula will always remain the same

Key points

- A **formula** shows the relationship between different variables, written as words or letters. You can use a formula to work out an **unknown** value by **substituting** the values that you do know into the formula.

Common errors and misconceptions

- Students often substitute incorrectly into a formula. For example, $v = at$ when $a = 5$ and $t = 6$, simply writing $v = 56$. Suggest that students always write the formula first with any missing multiplication signs put back in, so in this case, start by writing $v = a \times t$, then $v = 5 \times 6 = 30$.

Question guidance

Core

Q10 Tell students that they can follow the approach given in **Q9**.

Q10–Q16 These questions give students the opportunity to practise substituting given values into formulae of slightly increasing complexity.

Q17 Pose the following question based on the formula for predicting a girl's height. *Two parents have three daughters. Do you think they will all be the same height when adult? Is this a good model?* Establish that they will probably be different heights, so the model is not totally accurate. However, it is good to use it to estimate the height a child may be when they are fully grown.

Support

Formulae written in letters Encourage students to always write down what a variable represents, e.g. $W \rightarrow$ wages, to help prevent them from confusing the variables when writing expressions and formulae, and when substituting into formulae.

Purposeful practice

Purposeful practice 1 These questions vary minimally, with slow progression with one-step operations in formulae, and include substituting given values into each formula.

Purposeful practice 2 These questions provide practice with two-step formulae, including substituting zero using priority of operations.

Reflect

Core

Look back at the formula in **Q12**.

a Would it matter if this formula used the letters x and y instead of m and a?

b Do the letters help you to understand a formula? Explain.

This reflective task asks students to consider the letters used in formulae. It aims to address the misconception students often develop early on in their learning of algebra that letters stand for objects or abbreviations.

Support

Look back at **Q4** and **Q5**. Discuss with a partner how you know when to multiply or divide.

*In **Q4**, the formula is $□ = □□$, which is algebraic shorthand for $□ = □ \times □$, so you need to multiply $□$ and $□$. In **Q5**, the formula is $□ = □ \div 3$, so it tells you to divide $□$ by 3 in order to work out $□$.*

Depth

a Why are the letters in a formula called variables?

b Decide whether each statement is true or false.

i A formula uses letters to represent unknown values.

ii When you change the value of one variable in a formula, the values of the other variables change.

iii The relationship between the variables varies as their values vary.

a *They can take any value – i.e. can vary.*

b i *True* ***ii*** *True* ***iii*** *False*

Purposeful practice Reflect and reason

Purposeful practice 1

Sam substitutes $a = 3$ and $b = 5$ into $x = ab$. Sam writes $x = 35$.

Explain what Sam has done wrong.

Sam has forgotten that $ab = a \times b$, so $x = 3 \times 5 = 15$.

Purposeful practice 2

$z = 2x + 3y$

Explain how you use the priority of operations to find the value of z if $x = 4$ and $y = 6$.

$z = 2 \times 4 + 3 \times 6 = 8 + 18 = 26$; you need to do the multiplications before you add.

Objectives

Core objectives

- Write simple formulae in words
- Write simple formulae using letter symbols
- Identify formulae and functions
- Identify the unknowns in a formula and a function

Support objectives

- Write simple formulae in words
- Write simple formulae using letter symbols

Depth key concepts

- Understand that a formula can be a seen as a rule that tells you how to do a calculation (e.g. length × width) or how to work out the number of people when you know the number of tables, and writing it in algebra can save time drawing diagrams or writing out in words

Key points

- You can write a formula in words to work out an amount, then use letters to represent the variables.

Common errors and misconceptions

- Students often do not write a formula in its simplest form, e.g. writing $p = 5 \times t$. Suggest that once they have written a formula they check if there are any multiplication signs in the formula.
- When writing formulae, students often write the letters the wrong way round, e.g. $t = 5p$ instead of $p = 5t$. Stress the importance of checking their formula works by substituting in numbers and checking the answers are correct.

Question guidance

Core

Q4 Ask students to discuss whether it matters what letters you use in a formula. They should realise that, as long as any letters are clearly defined, it doesn't matter what letters they use.

Q7 This is the first question that contains two letters (unknowns). Suggest students use values for y and z to start with to see how to work out the answer, and then go back to algebra.

Q11b Students need to divide all the weights of the horses by 40 – discuss ways of doing this without a calculator. Draw out that they can divide each number by 10 , then divide the result by 4.

Challenge Students may find this task easier if they work with a partner so they can discuss ideas. If they are already working with a partner they can discuss methods between different groups. Some may set up a table to record their results. The number of faces increases by 3 each time, so they may have done 5 + 49 × 3, or 8 + 48 × 3, etc. or they may have come up with a formula.

Support

General A function machine is a helpful visual stepping stone to writing formulae using letters. Encourage students who need such visual support to draw a function machine each time.

Purposeful practice

Purposeful practice 1 These questions provide practice in writing formulae with one operation – multiplication or division – as a generalisation of calculations.

Reflect

Core

In lesson 3.5, you were given formulae to work with. In this lesson you wrote your own formulae.

a Which did you find more difficult?

b What made it more difficult?

c Are there particular kinds of questions you need more practice on? If so, what kinds?

This task asks students to reflect on their work substituting into, and then writing their own, formulae. It encourages them to take responsibility for their own learning by identifying where they may require more help or practice.

Support

In lesson 3.4 you looked at writing expressions. In this lesson you looked at writing formulae.

Discuss with a partner the differences between an expression and a formula.

A formula shows the relationship between different quantities, so it always has an = sign, e.g. □ = □□. *An expression contains terms and doesn't have an = sign, e.g.* $2a + 4$.

Depth

a In **Q1** and **Q3**, why is it useful to have a formula to work out the number of people or matchsticks, instead of drawing the diagrams?

b In **Q10**, why is it useful to have a formula for the area of a rectangle, instead of a written rule like 'multiply the length by the width'?

a *Saves time and space on the page.*

b *Saves time and is quicker to write and remember; others can understand it without having to read lots of words.*

Purposeful practice Reflect and reason

Purposeful practice 1

For **Q5**, Dom wrote this formula for number of legs on c cats: $L = 4c$

Jane wrote this formula for number of legs on n cats: $L = 4n$

Do both formulae give the same value of L for 10 cats?

Does it matter which letter you use in a formula?

Both give L = 40 for 10 cats. It does not matter which letter you use, as long as you explain what the letter stands for.

Purposeful practice 2

In a café, £T is the price of tea and £C is the price of coffee.

Which formula is correct for 4 teas, 1 coffee and a £2 tip? Explain your answer.

A $P = 4T + C - 2$ **B** $P = 4C + T + 2$ **C** $P = 4T + C + 2$ **D** $P = 4C + T - 2$

*C is the correct formula. **A** and **D** have '– 2' but you should add the tip, not subtract it, and **B** and **D** have 4 coffees instead of 1 coffee, and 1 tea instead of 4 teas.*

Check, Strengthen and Extend

Core Check up

Q2 This question ensures students are checking that their function works for all the input and output values, not just the first pair.

Q4c, d, Q7a These questions are designed to test students' knowledge that b means $1b$.

Q8 This question is designed to check that students remember to multiply every term within the brackets.

Q9 This question extends students' understanding of multiplying out brackets and collecting terms.

Q11 This question is designed to test students' understanding of the hidden multiplication sign, i.e. that $m = dV$ means $m = d \times V$.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Look back at the questions you answered in these lessons.

a Which hints were most useful? What made them more useful?

b Which hints were least useful to you? What made them less useful?

c What do your answers tell you about how you best learn maths?

This task aims to focus students' attention on the hints, how they use them, and what kinds of hints prove most useful. Then the final question encourages students to use their analysis of the hints to become more aware about their preferred ways of learning maths.

Extend

What kinds of jobs involve using formulae?

What careers are you interested in?

Do you think you will need to use formulae in your job? How?

What professionals are you likely to meet who might use formulae in their work?

This reflective task demonstrates to students the prevalence of formulae in many different jobs, and encourages them to think about where formulae may be used in their chosen careers. This is important, as algebra is often criticised by students for being too abstract and irrelevant.

Depth

Extend

In this unit, you have learned about expressions, functions and formulae.

What is the difference between them?

What advantages have you seen for using symbols instead of words to describe mathematical relationships?

An expression is just a collection of variables combined together in various ways. It doesn't have an equals sign. You can use it, for example, to give the length of the side of a shape, or the number of sweets that someone has.

A function gives a rule for turning one number into another. You can express it using algebra. For example, you could have a function which tells us to multiply a number by 3 and then add 2, and express this as the rule that □ gives 3x + 2.

A formula gives a rule for connecting one variable with another, and involves an equals sign. For example, you might have a formula for connecting a distance measured in miles with the same distance measured in km.

Symbols have an advantage of taking less effort to write, and also that it is much easier to manipulate symbols (for example, multiplying them together, or forming more complicated expressions) than it is to describe the same thing in words.

End of Unit 3 tests: Expressions, functions and formulae

Pearson published resources

Support/Core

- Year 7 End of Unit 3 Support/Core test
- Year 7 End of Unit 3 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 3 Core/Depth test
- Year 7 End of Unit 3 Core/Depth mark scheme

Unit opener: Decimals and measures

Pearson published resources

Objectives

Core objectives

- Measure and draw lines to the nearest millimetre
- Write decimals in order of size
- Round decimals to the nearest whole number and to 1 decimal place
- Round decimals to make estimates and approximations of calculations

Support objectives

- Measure and draw lines to the nearest millimetre
- Write decimals in order of size
- Round decimals to the nearest whole number

Depth key concepts

- Understand how to choose suitable numbers to round to when estimating an answer to a calculation (and this is not always rounding up or down to the nearest whole number).

Key points

Digits after the decimal point represent fractions. You can see the value of each digit in a **place value** table.

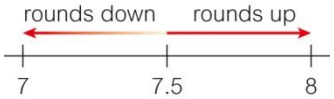
H	T	O	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
		0	•	1		
		0	•	0	1	
		0	•	0	0	1

$0.1 = \frac{1}{10}$ (one tenth)
$0.01 = \frac{1}{100}$ (one hundredth)
$0.001 = \frac{1}{1000}$ (one thousandth)

Numbers in **ascending** order go from smallest to largest. Numbers in **descending** order go from largest to smallest.

For rounding to the nearest whole number, look at the tenths.

- 5 tenths and above round up.
- 4 tenths and below round down.



A number rounded to **1 decimal place** (1 d.p.) has only one digit after the decimal point.

For rounding to 1 decimal place, look at the hundredths.

- 5 hundredths and above round up.
- 4 hundredths and below round down.

Common errors and misconceptions

- Students often do not measure from 0 on a ruler. Take every opportunity to model measuring from 0 correctly, demonstrating why they must not include the 'dead space' before the 0.
- Students often make errors with comparing and ordering numbers with different numbers of decimal places, e.g. writing 3.16 > 3.6. Encourage students to write numbers so they have equal numbers of decimal places, using zero placeholders where necessary.

Question guidance

Core

Q7 This may cause some confusion, so be prepared to provide some similar examples. Start with the value of digits in whole numbers if necessary.

Q8 Ask students *What is the easiest way to compare 5.24 and 5.248?* to see if they rewrite 5.24 as 5.240 so that both numbers have the same number of decimal places.

Q9 This question draws out the common misconception about place value and decimals.

Q17 Use this question to emphasise the need to look for relatively simple approximations that are easy to calculate. You could point out that **Q17a** uses the answers to **Q14a** and **c**.

Support

Understanding decimals with tenths Q3 Encourage pupils to use blocks as seen in the diagram for help with understanding place value and rounding in this lesson.

Understanding decimals with tenths Q4 This question reinforces that a decimal number with tenths only has 1 decimal place.

Understanding decimals with tenths Q5 To estimate, encourage students to think how long their ruler is and then think how much smaller each line is than their ruler. They can use answers in the different parts (so part **b** should be the longest) to help them find other answers.

Understanding decimals with hundredths Q3b Some students might want to say 0.74 as 'zero point seventy-four', so emphasise correct spoken language as well as written.

Ordering decimals Q3 A helpful way for students to remember which is greater and smaller is to say 'the crocodile eats the bigger number', where the symbol represents a crocodile's mouth.

Purposeful Practice

Purposeful practice 1 This draws out the common error of not understanding the place value of digits. Students could draw a place value table to help.

Reflect

Core

In this lesson you were rounding and estimating. Are rounding and estimating the same thing or different? Explain.

Could rounding and estimating be useful in other subjects?

This task encourages students to carefully consider the skills of rounding and estimating. Rounding and estimating are different. Rounding is always to a specified place value (e.g. to the nearest whole number, or to 1 d.p.), estimating can be to any place value that gives a good approximation for an answer to a calculation. Rounding and estimating are useful skills for any subjects involving large numbers (e.g. populations in geography), measurements (e.g. art and design) or calculations (e.g. science).

Support

Discuss these questions with a partner.

a Why is 2 the same as 2.0 and 2.00? **b** Which is larger, 2.02 or 2.20?

a *Students' own answers, but answers are likely to refer to the 2 as meaning 2 ones in all three instances and that the zeros show that there are no decimal parts, so 2, 2.0 and 2.00 are all ways of showing the integer (whole number) 2.*

b *2.20 is greater than 2.02. Students' explanations will vary but may describe 0.20 as being the same as* $\frac{2}{10}$ *whereas 0.02 is the same as* $\frac{2}{100}$*. They may use a place value table or a number line to support their explanation.*

Depth

When rounding numbers in real life, it is not always appropriate to round up or round down. Explain why.

Sometimes you will end up with fewer 'things' than you need if you round down. For example, you cannot fit all the people on the coach in ***Q8*** *if you round down.*

Purposeful Practice Reflect and reason

Purposeful practice 1

Copy and complete [] [].[] [] [] four times, using each of the digits 0, 1, 2, 3, 4 so that

a 2 has the greatest value **b** 2 has the smallest value

c your number is greater than 43.201 **d** your number is equal to 43.12

a any number 2 [].[] [] [] ***b*** *any number* [][].[] []*2*

c *43.210* ***d*** *43.120*

Purposeful practice 2

Which numbers give an answer of the same value when rounded to the nearest whole number as when rounded to 1 d.p.?

15.99 and 14.999

Purposeful practice 3

How did you decide what to round the square rooted numbers to in **Q2** parts **g**, **h** and **i**?

How did you decide what to round the dividend (first number) to in **Q2** parts **j**, **k** and **l**?

Nearest square number

*For **j** and **l**: the nearest number that is a multiple of the divisor rounded to the nearest whole number. It doesn't matter for **k** as its divided by 1.*

Objectives

Core objectives

- Multiply and divide by 10, 100 and 1000
- Convert measurements into the same units to compare them
- Solve simple problems involving units of measurement in the context of length, mass and capacity
- Convert between metric units of length, mass and capacity

Support objectives

- Multiply and divide whole numbers and decimals by 10, 100 and 1000
- Convert between metric units of length, mass and capacity

Depth key concepts

- Understand how all units in the metric system are multiples/divisors of a 'base' unit (metres, grams, litres, etc.), and explore the relationships between units not adjacent in size (e.g. mm and m, rather than only cm and m and mm and cm)
- Understand how decimal parts relate to each other, e.g. how tenths relate to hundredths; how hundredths relate to thousandths; how tenths relate to thousandths

Key points

- Metric units of **mass** include the **gram** (**g**) and **kilogram** (**kg**).
  1000 g = 1 kg
- Metric units of **capacity** include the **millilitre** (**ml**) and **litre**.
  1000 ml = 1 litre

Common errors and misconceptions

- Students often use the wrong operation (×10 instead of ÷10) when converting between units. Ensure, for example, that students understand that, because mm are smaller than cm, it takes more of them to make the same length; therefore to change from cm to mm multiply by 10.

Question guidance

Core

Q4–Q6 These questions are carefully structured to show students the effect on place value of multiplying by 10, 100 and 1000.

Q9 Clarify that students know they are expected to fill in the correct multiplier or divisor before calculating the answer.

Q10 Use diagrams and practical examples to show how to decide whether to multiply or divide and address the common error above.

Q13, Q17 If necessary, point out that all the measurements in a set need to have the same units, so students need to choose a suitable unit before multiplying/dividing. Encourage students to draw place value tables so they can write the converted measurements and compare them accurately.

Support

General Do not say to add zeros when multiplying by 10 or 100. Emphasise place value instead. Similarly, do not say to remove zeros when dividing by 10 or 100. Again, emphasise place value. When multiplying by 10, 100 and 1000, emphasise place value changing rather than the decimal point moving.

Purposeful practice

Purposeful practice 1 These questions offer intelligent practice showing students how digits move when a number is multiplied by 10, 100 or 1000.

Purposeful practice 2 These questions provide intelligent practice and address the common error by requiring students to decide whether to multiply or divide by 10, 100 or 1000.

Reflect

Core

Look back at **Q11**.

- For part **a**, did you convert 5 m to cm or 400 cm to m?
- For part **b**, did you convert 300 mm to cm or 15 cm to mm?
- For part **c**, did you convert 6 km to m or 8000 m to km?

How did you decide which measure to convert for each question?

Which is easier – converting smaller measures to larger measures (like cm to m) or converting larger measures to smaller measures (like cm to mm)? Explain.

*This task asks students to reflect on how they chose to convert measures in **Q11**: by dividing to convert from a smaller to a large measure, or multiplying to convert from a larger to a smaller measure. It asks students to be aware of whether their decision was based on finding it easier or harder either to multiply or divide by powers of 10.*

Support

Discuss with your partner

a how to convert from a smaller unit to a larger unit, for example: 450 cm to m

b how to convert from a larger unit to a smaller unit, for example: 3.7 km to m

a *To convert from smaller unit to a larger unit, divide by the correct power of 10.*

1 m = 100 cm so 450 cm = 450 ÷ 100 m = 4.5 m

Check: a metre is larger than a centimetre so there should be fewer of them.

b *To convert from a larger unit to a smaller unit, multiply by the correct power of 10.*

1 km = 1000 m so 3.7 km = 3.7 × 1000 m = 3700 m

Check: a metre is smaller than a kilometre so there should be more of them.

Depth

The measures you have used in this lesson are called 'metric' measures.

The metric system is described as a 'base 10' system.

What do you think this means?

The metric system is described as a 'base 10' system because all the units are based on powers of 10.

Purposeful practice Reflect and reason

Purposeful practice 1

When multiplying a number by 10, 100 or 1000, is your answer smaller or larger than you started with?

Larger

When dividing a number by 10, 100 or 1000, is your answer smaller or larger than you started with?

Smaller

Purposeful practice 2

Coral says, 'When converting from a larger to a smaller unit of measure, you always divide, but when converting from a smaller to a larger unit of measure, you always multiply.'

Is Coral correct? Write two examples to support your answer.

Yes, Coral is correct. For example,

22 mm = 22 ÷ 10 cm = 2.2 cm

larger → ÷ → *smaller*

22 cm = 22 × 10 mm = 220 mm

larger → × → *smaller*

Objectives

Core objectives

- Use scale diagrams
- Read scales
- Write decimal measures as two related units of measure
- Interpret metric measures displayed on a calculator

Support objectives

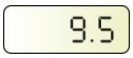
- Read scales
- Use scale diagrams

Depth key concepts

- Understand how different scales enable different levels of accuracy
- Understand why reading decimal fractions of metric measures on a calculator is easier than decimal fractions of measures of time

Key points

- A **scale drawing** shows a large real-life measure in a small space. It needs a **scale** to tell you what 1 cm on the drawing represents in real life.
- When you use a calculator to solve problems involving measures, make sure you understand the result on the calculator display.

  The result 

  can mean different things.
  - For money in pounds, it is £9.50.
  - For time in minutes, it is 9 minutes 30 seconds.
  - For length in metres, it is 9.5 m or 9 m 50 cm.

Common errors and misconceptions

- Students often misinterpret the divisions on a scale. Use a pictorial approach to address this. Suggest students draw the scale and label all the divisions when they have found out the value represented by one division.

Question guidance

Core

Q6 These are examples of measuring devices that might be typically seen in a science lesson and could also be used in the maths classroom.

Q7 The scales in this question have increasing numbers of unlabelled intervals or involve trickier values such decimals and negative numbers.

Q10 A typical misconception is the failure to realise, e.g. that –21 is smaller than –20. This question addresses the direction aspect of measuring below zero.

Q13 This question is carefully structured to give students practice in converting larger units to smaller units in order to write decimal measures as two related units of measure.

Support

The Reflect question provides students with an opportunity to verbalise their understanding of scales and the crucial feature of equally spaced and labelled intervals.

Purposeful practice

Purposeful practice 1 The Reflect and reason question gives students the opportunity to identify their step-by-step approach to reading scales.

Reflect

Core

Look back at **Q7**, **Q8**, **Q9** and **Q10**, about reading scales.

Write the steps you took to read the scales. Make sure you include every step.

You may begin with, 'Step 1: I looked at the values on the scale either side of the arrow.'

What maths operations (addition, subtraction, multiplication, or division) did you use?

This reflective task asks students to consider the steps they took to read a scale. They are also asked what maths operations they used. This emphasises the multiplicative reasoning required for reading scales, as students are likely to have used division to determine the value for each marker on the scale. This will also help students with reading axes on graphs.

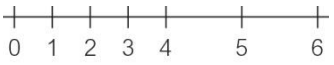
Support

Look at each of these scales. Discuss with a partner what is wrong with each scale.

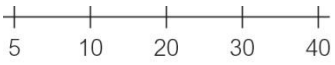
a

b

c



d



a *Each division is worth 1 but 4 is missing.*

b *The divisions are not equally spaced.*

c *The divisions are not equally spaced.*

d *The first division is worth 5 but the other divisions are worth 10.*

Depth

Look back at **Q5**. What type of measure is

a easiest to interpret when shown on a calculator

b hardest to interpret when shown on a calculator?

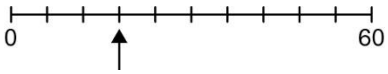
Explain your answers.

Students' own answers, for example: Pounds and pence is easiest to interpret when shown on a calculator; hours and minutes is hardest to interpret when shown on a calculator.

Purposeful practice Reflect and reason

Purposeful practice 1

Write instructions for a friend to help them find the value the arrow is pointing to on this scale.



Count the number of divisions → 10.

Divide 60 by the number of divisions to find the value of 1 division → 6.

Count up from 0, each division, to the arrow → 18.

Purposeful practice 2

Look at your answers to **Q2**, **Q4** and **Q7**. Explain why your answers are different each time.

The units are different in each question. 0.3 cm is 3 mm; 0.3 m is 30 cm; 0.3 km is 300 m.

Objectives

Core objectives

- Multiply decimals by multiples of 10, 100 and 1000
- Multiply decimals mentally
- Check a result by considering whether it is of the right order of magnitude

- Understand where to position the decimal point by considering equivalent calculations

Support objectives

- Multiply decimals by multiples of 10, 100 and 1000
- Multiply decimals mentally
- Understand where to position the decimal point by considering equivalent calculations

Depth key concepts

- Explore patterns in place value multiplication decimal calculations
- Understand the inverse operations of multiplication and division in relation to place value decimal calculations

Key points

- You can use multiplication facts to work out decimal multiplications.
- You can also use **partitioning** to work out decimal multiplications. You can check your answer by using an **approximate** calculation.
- You can use an answer to a decimal multiplication to work out the answers to related decimal multiplications.

Common errors and misconceptions

- Students often make errors identifying the number of decimal places when multiplying. Encourage students to estimate calculations first. Model various methods of estimating including rounding and show how to use these to check that answers are an appropriate order of magnitude.

Question guidance

Core

Q4, Q5, Q7 These questions have been carefully structured to give students practice at multiplying decimals first by single-digit integers and then by decimals.

Q10 If students are not yet clear on the partitioning process, model it for them using the grid method.

Q12 Explain that ore is the name for unprocessed rock containing metal.

Q15 This explores whether students have a mental sense of the size of an answer. Watch out for students who are unable to see David's answer as incorrect.

Challenge This task uses repeated calculations involving 1.1 and is a variation of questions involving 11. Students should be able to see that each answer involves 3s as the first and last digits, with 6s in between. Check the decimal points are in the right place.

Support

Multiplying decimals Q2 Encourage students to look at the patterns between pairs of related multiplications – what do they notice?

Multiplying decimals Q3 Encourage students to decide what they need to divide/multiply by, e.g. they could write out $3 \times 0.4 = 3 \times 4 \div 10$.

Using partitioning to multiply Q2 You could set this out as grid multiplication – students may find this easier to visualise the partitioning.

Purposeful practice

Problem-solving practice Q9 This question enables students to demonstrate their understanding of purposeful practice 3 by writing down equivalent calculations without doing any working out.

Reflect

Core

In this lesson you have learned two methods to help you multiply decimals mentally:

- multiplication facts
- partitioning

For each method, make up your own calculation to show how it works.

Work out the answer to each of your calculations. Show your working.

Explain how you chose the numbers for your calculations.

This task asks students to revisit the two methods they have learnt to mentally multiply decimals. For each, students are asked to make up a calculation and answer it, showing their working. In particular, students are asked to consider how they chose the numbers for the calculations, which should reveal to them when each method is most useful. For example, for multiplication facts, students are likely to choose to multiply a single-digit number by a decimal that has zero in all columns, except in tenths or hundredths; for partitioning, students are likely to choose to multiply a 2-digit number by a decimal with units and tenths.

Support

$34 \times 0.1 = 34 \times 1 \div 10 = 34 \div 10 = 3.4$

$1 \div 10 = 0.1$, so multiplying by 0.1 is equivalent to dividing by 10.

What is multiplying by 0.01 equivalent to?

Multiplying by 0.01 is equivalent to dividing by 100. For example, $34 \times 0.01 = 34 \times 1 \div 100 = 0.34$.

Depth

Write three things that you look for when using a calculation fact to work out other calculations.

Students' own answers, for example:

Numbers that are the same as in the original calculation.

The new calculation is a rearrangement of the original calculation.

A number in the new calculation is e.g. 10 times bigger than a number in the original calculation.

Purposeful practice Reflect and reason

Purposeful practice 1

Why do **Q1** parts **b** and **e** give the same answer?

Multiplication can be done in any order (is commutative).

0.4 × 0.2 = 4 ÷ 10 × 2 ÷ 10 = 4 × 2 ÷ 10 ÷ 10 = 8 ÷ 100

0.2 × 0.4 = 2 ÷ 10 × 4 ÷ 10 = 2 × 2 ÷ 10 ÷ 10 = 8 ÷ 100

Why do **Q1** parts **i** and **l** give the same answer?

0.08 × 3 = 8 ÷ 100 × 3 = 8 × 3 ÷ 100 = 24 ÷ 100

0.03 × 8 = 3 ÷ 100 × 8 = 8 × 3 ÷ 100 = 24 ÷ 100

Dan says, 'The answer to **Q2e** is 0.6.' What do you think he has done wrong?

Dan may have worked out 0.5 = 5 ÷ 10 and 0.12 = 12 ÷ 100, and used these to calculate 0.5 × 0.12 = 5 ÷ 10 × 12 ÷ 100 = 60 ÷ 1000. He may have incorrectly worked out 60 ÷ 1000 as 0.6.

Purposeful practice 2

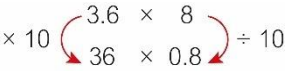
How did your answers to **Q1**, **Q3**, **Q5** and **Q7** help you find the answers to **Q2**, **Q4**, **Q6** and **Q8** respectively?

*To work out **Q2**, **Q4**, **Q6** and **Q8** you have to partition the first number into tens and ones then multiply. The calculation in **Q1**, **Q3**, **Q5** and **Q7** was the tens multiplication.*

Purposeful practice 3

Why do **Q2** parts **d** and **f** give the same answer as **Q2a**?

*Comparing **Q2a** and **d**:*



*Comparing **Q2a** and **f**:*

a
f

$$\times 100 \left(\begin{matrix} 3.6 \times 8 \\ 360 \times 0.08 \end{matrix} \right) \div 100$$

*The multiplication of the first number in **Q2d** and **f** (compared to **Q2a**) is cancelled out by the inverse operation of division of the second number in **Q2d** and **f** (compared to **Q2a**).*

Objectives

Core objectives

- Add and subtract decimals
- Multiply and divide decimals by single-digit whole numbers
- Divide numbers that give decimal answers

Support objectives

- Add and subtract decimals
- Multiply and divide decimals

Depth key concepts

- Understand when a mental method may be better than a written method (and vice versa)

Key points

- You can use a **zero place holder** when adding or subtracting decimals with different numbers of decimal places. For example, write 45.9 – 23.45 as

  $$\begin{array}{r} 45.90 \\ -\ 23.45 \\ \hline \\ \hline \end{array}$$

- You can use the column method to multiply a decimal by a whole number or vice versa.
- You can use short division to divide a decimal by a whole number.

Common errors and misconceptions

- Students often fail to line up decimal points when adding or subtracting. Model this consistently and explicitly in any calculation. Students could use squared paper to help them vertically align decimal points and digits of the same place value.
- Students often do not use estimation to check the results of multiplication. Encourage them to make an informal estimate before calculating, and again after calculating. *Does that answer seem to be about the right size?*

Question guidance

Core

Q9 *Which of the four operations will you need to calculate this question?*

Q10 This question has been structured carefully to give students practice in column addition and subtraction of numbers with different numbers of decimal places. Check whether students use zero place holders to align digits correctly.

Q11 These questions probe students' understanding of place value issues around 'carrying'. Encourage the use of zero place holders.

Q15, Q16 *How did you decide which calculation to do?*

Support

Adding and subtracting decimals Q1, Q2 Encourage students to use bar models when writing tenths and units. Concrete equipment such as linking cubes or diene blocks can be used to help students understand what they get when adding or subtracting.

Decimal column addition and subtraction Use the first Worked example to demonstrate and emphasise that the column method for adding decimals is the same as the method used for adding whole numbers. Similarly, use the second Worked example to demonstrate the column methods are the same for subtraction.

Multiplying decimals Q1 Students may prefer to use the grid method, or the lattice/gelosia method. These methods are often easier with decimal numbers and should be used as an alternative.

Purposeful practice

Problem-solving practice Encourage students to estimate answers before calculating and then use these to check that their answers are the right order of magnitude. This should highlight errors in alignment and /or mistakes in carrying/exchanging.

Reflect

Core

In this lesson, you added, subtracted, multiplied and divided with decimals.

Which did you find most difficult? What made it most difficult?

Do you need more practice on any kinds of question? If so, which kinds?

This task involving higher-order thinking skills asks students to reflect on their use of the four operations with decimals, and encourages them to take responsibility for their own learning by identifying where they may require more practice.

Support

For each of these calculations decide whether **i**, **ii** or **iii** is correct. Discuss with a partner how you got your answer.

a 1.2 × 9 **i** 1.08 **ii** 10.8 **iii** 1.8

b 9.6 ÷ 6 **i** 1.6 **ii** 1.06 **iii** 0.16

a ***ii*** *is correct; 12 × 9 = 108 and 1.2 = 12 ÷ 10 so 1.2 × 9 = 108 ÷ 10 = 10.8*

Check using estimation: 1.2 × 9 ≈ 1 × 10 = 10

b ***i*** *is correct; 96 ÷ 6 = 16 and 9.6 = 96 ÷ 10, so 9.6 ÷ 6 = 16 ÷ 10 = 1.6*

Check using estimation: 9.6 ÷ 6 ≈ 10 ÷ 5 = 2

Depth

Sometimes using a written method can make the calculation harder.

Create your own subtraction question where it would be more efficient to use a mental method.

Students' own answers. Something like ***Q13,*** *where subtracting £2.99 is easier done mentally.*

Purposeful practice Reflect and reason

Purposeful practice 1

How could you use your answer to **Q1a** to help you answer **Q1b**?

How could you use your answer to **Q1e** to help you answer **Q1f**?

Why are all the answers to **Q3** integers (whole numbers)?

The second number in ***Q1b*** *is the second number in* ***Q1a*** *+ 0.1, so you are adding an extra 0.1. This means the answer to* ***Q1b*** *is 0.1 bigger than the answer to* ***Q1a****.*

In ***Q1f*** *you are subtracting an extra 0.1 so the answer to* ***Q1f*** *is 0.1 less than the answer to* ***Q1e***

In ***Q3****, all the decimal number parts add to a whole number.*

Purposeful practice 2

Jon works out 92.7 – 3.68

```
  8 11
  9 2.17
- 3.6 8
  5 5.9
```

What mistake has he made?

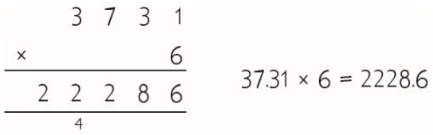
What is the correct answer?

Jon has not lined up the decimal point or the ones with the ones, tenths with tenths.

Correct answer is 89.02.

Purposeful practice 3

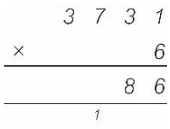
Jenny works out 37.31 × 6



What two mistakes has she made?

What is the correct answer?

Jenny has not carried 1:



Jenny has divided the answer by 10, not 100.

Correct answer is 223.86.

Objectives

Core objectives

- Work out the perimeters of composite shapes and polygons
- Solve perimeter problems

Support objectives

- Work out the perimeter of squares, rectangles and regular polygons
- Calculate the perimeter of shapes made from rectangles

Depth key concepts

- Understand how to deduce formulae for perimeters of different shapes

Key points

- The **perimeter** is the total distance around the edge of a shape. To work out the perimeter of any shape, add up the lengths of all the sides.

Common errors and misconceptions

- Students often use the wrong calculation for perimeter and area. Use a pictorial example such as the white lines around a football pitch for perimeter, and the amount of grass on a pitch to represent the area.
- Students often do not collect terms accurately when using algebraic expressions. Ensure they understand they can only collect like terms. Look out for

misconceptions such as $3x + 3 = 6x$. Stress that the letter represents a number – substitute a value for x to show why $3x + 3 \neq 6x$.

Question guidance

Core

Q2 *What is the quickest way to work out the perimeter of a square? What is the quickest way to work out the perimeter of a rectangle?*

Q9 Watch for students who fail to calculate x and y correctly.

Q10 Look out for errors in collecting terms in part **a**. Discuss the answer to part **b** with the whole class.

Challenge This task is about maximising and minimising the perimeter of shapes of given number of squares (area). The generalisation that should emerge is that a rectangle with a single unit width or length will give the longest perimeter. The shortest perimeter will come from the perimeter that has its sides closest in size (or a square if possible).

Support

Perimeter of squares and rectangles Q2 If students are finding it difficult to calculate then ask them to copy each shape onto square paper first, then they can count the length of the outside.

Perimeter of polygons Explain that equilateral triangles and squares are also regular polygons, but they have special names.

Reflect Encourage students to write down a method, or even a formula. Ask them if they can find the fastest method.

Purposeful practice

Purposeful practice 3 These questions provide practice in creating algebraic expressions for the sides of a rectangle and then calculating the perimeter.

Problem-solving practice Q1, Q5 These questions reverse the concept covered in Purposeful practice 2.

Reflect

Core

In this lesson, the hint for **Q10** suggests that you draw a diagram.

Why is this a good strategy for working out the perimeter of a shape?

What other kinds of diagrams have you used to help you do maths?

This task asks students to think about the purpose of diagrams for working out the perimeter of a shape. Students may recognise how a diagram makes it easier to visualise a problem, and to ensure they have captured all relevant information. Other areas of maths

where students may recognise diagrams as having been useful include bar modelling for number work and function machines for algebra.

Support

Discuss with a partner a method for finding the perimeter of

a a rectangle **b** a square **c** a regular pentagon

d a regular hexagon **e** any regular polygon

Students' own answers. Possible answers include

a–e *sketching a diagram of the shape*

a *adding 2 lengths and 2 widths; writing a formula such as* $P = 2l + 2w$ *or* $P = 2(l + w)$

b *multiplying the side length by 4; writing a formula such as* $P = 4l$

c *multiplying the side length by 5; writing a formula such as* $P = 5l$

d *multiplying the side length by 6; writing a formula such as* $P = 6l$

e *multiplying the side length by the number of sides; writing a formula such as* $P = nl$ *where n is the number of sides*

Depth

Write two advantages of using a formula to describe the perimeter of a shape.

You can substitute in different values for different measures on the shape. You can use the formula to help identify patterns.

Purposeful practice Reflect and reason

Purposeful practice 1

The shapes in **Q3** and **Q4** are both made up of the shapes in **Q1** and **Q2**. Why don't the shapes in **Q3** and **Q4** have the same perimeter?

*The shape in **Q3** is made by joining the shapes in **Q1** and **Q2**. The shape in **Q4** is made by overlapping the shapes in **Q1** and **Q2**, not joining them.*

Purposeful practice 2

What do you notice about the perimeters of the polygons in **Q1** and **Q5**; **Q2** and **Q6**; **Q3** and **Q7**; **Q4** and **Q8**?

Predict the perimeter of a pentagon with side length 16 cm.

*Perimeters of **Q5–Q8** are double perimeters of **Q1–Q4**.*

*The perimeter of a pentagon with side length 16 cm will be double the perimeter of **Q8**, i.e. 80 cm.*

Purposeful practice 3

Copy and complete this sentence for the rectangle in Purposeful practice 3.

If one side length of a rectangle stays the same, and the other increases by 1 cm, then its perimeter increases by ___ cm.

If one side of a rectangle stays the same length and the other increases by 1 cm each time, then its perimeter increases by 2 cm each time.

Objectives

Core objectives

- Find areas of irregular shapes by counting squares
- Calculate the areas of shapes made from rectangles
- Solve problems involving area

Support objectives

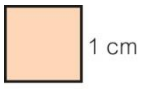
- Find areas of shapes by counting squares
- Find the area of rectangles and squares
- Calculate the area of shapes made from rectangles

Depth key concepts

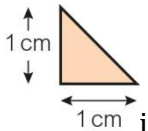
- Understand that shapes can have the same area, but different perimeters; and shapes can have the same perimeter but different areas
- Know why area is measured in square units, and length (perimeter) is measured in linear units

Key points

- The **area** is the total space covered by a shape. You can find the area of a shape drawn on squared paper by counting the squares inside it.
  The **units** used for area are square units, such as mm^2, cm^2, m^2 and km^2.



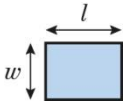
has area 1 cm^2.



is half a square. It has an area of 0.5 cm^2.

  Read cm^2 as 'square centimetres'.

- To work out the area of a rectangle or square, use area = length × width.
  $A = lw$



- To work out the area of a shape made from rectangles, it helps to split the shape into smaller rectangles.
- When finding the area of a shape, all dimensions must be in the same units of measure.

Common errors and misconceptions

- Students sometimes multiply by 2 when squaring. Use an image of squares to remind students that the equal sides are multiplied together, not by 2.
- Students often do not know or recognise square roots of square numbers. Encourage them to learn the square numbers up to 100 and the corresponding square roots.

Question guidance

Core

Q5 This question uses algebra which may cause confusion. Look out for students who struggle with this, and those who fail to simplify the expressions.

Q6 This question is constructed carefully to lead students to the formula for calculating the area of a square, $A = l^2$.

Q10c Students should be able to see that the area could be calculated in several ways, depending on how you break up the compound shape. Discussion should draw out the easiest calculation in this instance.

Q12, **Q14** These questions rely on students knowing the square root of a square number. Being able to spot square numbers and knowing their square roots is important time-saving knowledge when solving problems.

Support

Area of rectangles and squares Q2 Ask students to label the length and the width and then calculate. Discuss whether it matters which is the length and which is the width.

Purposeful practice

It may be helpful to revisit Purposeful practice 2 and 3 in lesson 4.6 to reinforce the difference between area and perimeter.

Reflect

Core

Here is a list of some jobs where you have to calculate and use area:

estate agent, gardener, festival organiser.

Copy the list and write one more job where calculating and using area is important.

Describe how you would use area in each job.

What career are you interested in? Do you think you will need to use area in your job? How?

This reflective task demonstrates to students how area is used in very different jobs. Students may consider: an estate agent measures the floor space in property; a gardener measures an area to scatter the right amount of seed; a festival organiser needs to know how many stalls or tents can fit in an area of a field. Students may think of other jobs, such as surveyor or carpet fitter. The task also encourages students to think about if and where area may be used in their chosen careers.

Support

What information do you need to work out the area of these shapes?

a rectangle **b** square

a *Need to know length and width of the rectangle; it doesn't matter which is which.*

b *Need to know length of one side.*

Depth

What is the same and what is different about perimeter and area?

The same: both are ways to describe and measure a shape.

Different: perimeter measures a length and its unit is in linear units (cm, m, km, etc.); area measures space covered and its unit is in square units.

Purposeful practice Reflect and reason

Purposeful practice 1

Write down the first four square numbers. How do the areas of these shapes relate to the square numbers? Explain why.

1, 4, 9, 16

The areas are double the square numbers. Each shape is made of an area equivalent to two squares.

Purposeful practice 2

How can your answers to **Q1** and **Q2** help you find the answers to **Q3**, **Q4** and **Q5**?

Why can't your answers to **Q1** and **Q2** help you find the answer to **Q6**?

*The shapes in **Q3**, **Q4** and **Q5** are all made up by joining the shapes in **Q1** and **Q2**. The shape in **Q6** is made by overlapping the shapes in **Q1** and **Q2**, not joining them.*

Purposeful practice 3

Copy and complete this sentence for the rectangle in Purposeful practice 3.

If one side of a rectangle stays the same length, b, and the other side increases by 1 cm each time, then the area increases by ___ cm^2 each time.

If one side of a rectangle stays the same length and the other side increases by 1 cm each time, then the area increases by b cm^2 each time.

Objectives

Core objectives

- Choose suitable units to measure length and area
- Use units of measure to solve problems
- Use metric and imperial units

Support objectives

- Choose suitable units to measure area
- Use units of measure to solve problems
- Use metric and imperial units

Depth key concepts

- Understand that to compare measures in different units, they must all be converted to the same unit
- Understand how to make choices about which unit to convert measures to

Key points

- It is important to be able to choose the most suitable **units** for measuring length, capacity and area. The metric units you already know are
  Length: mm, cm, m, km Mass: g, kg
  Capacity: ml, litres Area: mm^2, cm^2, m^2, km^2
- Some more metric units that you need to know are
  Mass: 1 tonne (t) = 1000 kg Area: 1 hectare (ha) = 10 000 m^2
  Capacity: 1 millilitre (ml) = 1 cm^3 Volume: 1 litre (l) = 1000 ml
- You need to know these conversions between **metric** and **imperial units**.
  1 foot (ft) ≈ 30 cm 1 mile ≈ 1.6 km

Common errors and misconceptions

- Students often do not understand the relationship between metric and imperial units. You could address this either pictorially or in a concrete way, by giving

students the opportunity to compare units using scales on measuring equipment, such as rulers, tape measures, etc. (for cm / feet) or using maps with both scales indicated (for km / miles). This lesson covers units of length but you could also introduce other units if appropriate.

Question guidance

Core

Q5 Suggest students refer back to the first Key point to help them decide the most sensible unit for measuring each quantity.

Q6, Q7 Remind students of earlier lessons in this unit on converting between larger and smaller units of measure and how they decided whether to multiply or divide.

Q11c Encourage students to think carefully about how much space is not taken up by cars in a car park.

Q13 Use this question to check students' understanding of the relative distances of a mile and a kilometre.

Support

Choosing suitable units of measure Q2 Ask students to recall what they know about units in length and extend this to area.

Converting metric units Q5 Ask students to consider how many fields they think the average farm has.

Purposeful practice

Problem-solving practice Q4–Q8 When converting between metric and imperial units, students can use the same 'common sense' check to make sure they've multiplied/divided correctly as when converting between smaller and larger metric units.

Reflect

Core

Write two headings, 'metric' and 'imperial'.

Write down as many different measures as you can under each heading. Do not use this book to help you.

When you were trying to remember all the measures, what were you thinking or imagining?

Discuss your way of remembering with other students.

This reflective task asks students to remember and list measures and identify which ones are metric and which are imperial. Then they are asked to reflect on what they were doing when trying to remember them. Were they picturing measuring equipment or pages of their student book? Were they thinking methodically about the questions they have

answered, or working through the alphabet thinking about whether there is a measure beginning with...? Not only does this make students aware of how they remember, but, by discussing their method with other students, it makes them aware of other methods of remembering that they may find helpful.

Support

Write a list of the units of measure you have come across and group them.

Discuss with a partner

a which of the units are new to you and which you knew before

b how you grouped your units and why.

Students' own answers. Possible answers include

a *new units: tonne, hectare, cubic centimetre (cm^3), foot*

b *grouping according to: quantity being measured (i.e. length, mass, capacity, area); type of unit (e.g. metric or imperial); relative size of unit, indicated by prefix (e.g. large units such as km or kg and small units such as cm, mm, mg, ml, etc.).*

Depth

Why can you compare two measures given in feet and centimetres, but not litres and centimetres?

Is it ever possible to compare a litres and a centimetres measure? Explain.

Litres are a unit of volume whereas centimetres and feet are both units of length. You can compare two lengths, but you cannot compare a length to a volume. Litres can be compared to cm^3.

Purposeful practice Reflect and reason

Purposeful practice 1

How did you decide which unit of measure was suitable?

Match the type of measure (length, mass, capacity or area) to each situation. Then consider its size to decide on the magnitude of the measure, for example, short length can be measured in feet; long length can be measured in km.

Purposeful practice 2

Why do you need 0 in your answers to **Q1**, **Q2** and **Q3** but not **Q4**?

*0 is used to hold the place value of a digit, e.g. 7100 kg in **Q2** without zeros would be 71 kg.*

Purposeful practice 3

How did you use your answer to **Q2a** to answer **Q2b**?

How did you use your answer to **Q2c** to answer **Q2d**?

*The measure in feet in **Q2a** and **b** was the same, so you can convert the answer to **Q2a** in cm into m for the answer to **Q2b**.*

*The measure in **Q2c** was in cm and the measure in **Q2d** was in mm. Therefore, the answer to **Q2d** was the answer to **Q2c** divided by 10.*

Check, Strengthen and Extend

Core Check up

Q2 This question is designed to test students' ability to read a scale accurately when scale divisions are not labelled.

Q6 This question checks the common misconception that 23.45 is bigger than 23.8.

Q8 This question is designed to test students' ability to round to 1 decimal place. Ensure that students understand that rounding 13.96 to 1 decimal place gives 14.0 and that it is necessary to include the zero.

Q10, Q13 These questions are designed to check that students can extract key information in a word problem to create an appropriate calculation and work out the answer.

Q11, Q12, Q14 These questions check that students can multiply and divide decimals, obtaining answers with the decimal point positioned correctly.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

These lessons used place value tables to help solve problems with decimals and measures. Did the place value tables help you? Explain why or why not.

Will you use place value tables to help you solve mathematics problems in future?

This task encourages students to think about the purpose of place value tables, and to reflect on whether they found them useful in their own mathematical problem solving, and how.

Extend

Write a definition of a decimal, in your own words. Be as accurate as you can.

This task encourages students to think carefully about what a decimal is, and provide as accurate a definition as possible, even taking account of the notation (the decimal point).

Students are introduced (perhaps for the first time) to the term 'notation', and its meaning in maths.

Depth

Extend

Write five new things about measures that you have learned in this Depth unit.

Students' own answers. Answers might mention

- *how to choose suitable numbers to round decimals when estimating answers*
- *that different scales allow different levels of accuracy when measuring*
- *understanding how to work out formulae for perimeters of regular polygons*
- *understanding that shapes can have the same area, but different perimeters, and also that shapes can have the same perimeter but different areas*
- *understanding why area is measured in square units, and length (perimeter) is measured in linear units.*

End of Unit 4 tests: Decimals and measures

Pearson published resources

Support/Core

- Year 7 End of Unit 4 Support/Core test
- Year 7 End of Unit 4 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 4 Core/Depth test
- Year 7 End of Unit 4 Core/Depth mark scheme

Revision

End of Term tests: Units 1 to 4

Pearson published resources

Support/Core

- Year 7 Autumn End of Term Support/Core test
- Year 7 Autumn End of Term Support/Core mark scheme

Core/Depth

- Year 7 Autumn End of Term Core/Depth test
- Year 7 Autumn End of Term Core/Depth mark scheme

Unit opener: Fractions and percentages

Pearson published resources

Objectives

Core objectives

- Use fraction notation to describe parts of a shape
- Compare simple fractions
- Use a diagram to compare two or more simple fractions
- Order fractions

Support objectives

- Use fraction notation to describe parts of a shape
- Compare simple fractions

Depth key concepts

- Know that, for unit fractions, the larger the denominator, the smaller the value of the fraction

Key points

- A **fraction** is part of a whole. The number above the line in a fraction is the **numerator**. The number below the line is the **denominator**.

  $\frac{1}{2}$ ← numerator
  ← denominator

- A **unit fraction** has numerator 1. For example, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ are unit fractions.

Common errors and misconceptions

- When working out the fraction shaded, students often give an answer of number shaded (as the numerator) and number unshaded (as the denominator), e.g. if 2 out of 5 are shaded, writing an answer of $\frac{2}{3}$. Resolve by emphasising that the fraction is 'number of parts shaded' over 'number of parts in the whole'.
- When comparing unit fractions, students assume that the fraction with the bigger denominator is the bigger fraction. Address this in a concrete way by using fraction strips or rods to show which fraction is bigger.

Question guidance

Core

Q5 This question has been carefully constructed to demonstrate that the shaded and unshaded fractions sum to 1.

Q7 Students can approach this in two different ways. They can count the number of green squares then count the white squares by adding together the triangles to make squares. Alternatively, they can divide the shape up into triangles that are the same size as either the corner triangles (24 green and 24 white) or the side triangles (12 green and 12 white).

Q9 *When the denominators are the same, how can you decide which fraction is larger without drawing a diagram?* Establish that the larger the numerator the larger the fraction.

Q11 *When you are given two unit fractions, how can you decide which is larger without drawing a diagram?* The larger the denominator, the smaller the fraction. This is harder for students to understand when the denominators start to get quite large, so refer back to common fractions such as $\frac{1}{2}$ and $\frac{1}{4}$. Also relate to division, so, for example, if you divide something into 12 parts, each part will be smaller than if you divided it into 5 parts.

Support

Using fractions to describe parts of shapes Q1 This question demonstrates visually that the denominator is unchanged when the number of parts the bar is split into is unchanged but the numerator changes to show the number of parts shaded. It also demonstrates that, when the denominator is constant, the fraction increases in size as the numerator increases.

Using fractions to describe parts of shapes Q2, Q3 These questions demonstrate that shaded parts do not need to be next to one another.

Ordering fractions Q5 In parts **a–d**, the bar is the same length and 3 sections are shaded, but each bar is divided into a different number of equal-sized pieces. Students should spot that, as the denominator increases, the size of the fraction decreases.

Purposeful practice

The fraction walls provide decreasing amounts of visual support in these questions. In Purposeful practice 1, students compare pairs of fractions with the same denominator whereas in Purposeful practice 2, they compare pairs of unit fractions with different denominators, and in Purposeful practice 3 they compare mixed pairs of fractions.

Reflect

Core

This may be the first time you have studied fractions since primary school.

Choose **A**, **B** or **C** to complete each statement.

In this lesson, I did … **A** well **B** OK **C** not very well

So far, I think fractions are … **A** easy **B** OK **C** difficult

When I think about the next lesson, I feel … **A** confident **B** OK **C** unsure

If you answered mostly **A**s and **B**s, did your experience surprise you? Why?

If you answered mostly **C**s, look back at the questions you found most tricky.

Ask a friend or your teacher to explain them to you.

Then complete the statements above again.

*This reflective task aims to make conscious any negative thinking that has the potential to impede students' progress. This is particularly important for fractions, which many students perceive as being difficult. It is also intended to boost students' confidence, as hopefully most will answer with **A**s or **B**s.*

Support

Two fractions have different denominators. Both have numerator 1. How can you decide which is larger?

When comparing unit fractions, the larger fraction is the one with the smaller denominator.

Depth

Copy and complete this sentence with the word larger or smaller.

For unit fractions, the larger the denominator, the _____ the fraction.

Give examples to support your answer.

Smaller, e.g. $\frac{1}{{10}}{\rm{ }} < {\rm{ }}\frac{1}{8}$

Purposeful practice Reflect and reason

Purposeful practice 1

When the denominators are the same, how can you decide which fraction is larger?

Look at the numerators. When denominators are equal, a larger numerator gives a larger fraction.

Purposeful practice 2

For unit fractions, how can you decide which fraction is larger?

For unit fractions, a larger denominator gives a smaller fraction.

Purposeful practice 3

Look at your answers to parts **b**, **c** and **o**. Does the larger fraction always have a larger numerator?

No, $\frac{3}{4} > \frac{4}{6}$ even though $\frac{4}{6}$ has a larger numerator.

Look at your answers to parts **f** and **o**. Does the smaller fraction always have a smaller numerator?

No, $\frac{1}{3} > \frac{2}{7}$ even though $\frac{1}{3}$ has a smaller numerator than $\frac{2}{7}$

Objectives

Core objectives

- Change an improper fraction to a mixed number
- Identify equivalent fractions
- Simplify fractions by dividing numerator and denominator by common factors

Support objectives

- Change an improper fraction to a mixed number
- Identify equivalent fractions
- Simplify fractions by dividing the numerator and denominator by common factors

Depth key concepts

- Understand that simplifying fractions can make them easier to visualise

Key points

- **Equivalent fractions** have the same value.
- An **improper fraction** has a numerator that is bigger than its denominator, for example $\frac{3}{2}$
  A **mixed number** has a whole number part and a fraction part, for example ${\rm{1}}\frac{1}{2}$
- You can find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.
- You can **simplify** fractions by dividing the numerator and denominator by a common factor. To write a fraction in its **simplest form**, divide the numerator and denominator by their highest common factor (HCF).

Common errors and misconceptions

- Students often write mixed numbers incorrectly. For example, writing $\frac{5}{3}$ as ${\rm{1}}\frac{1}{2}$ (because 5 ÷ 3 = 1 r 2). Resolve this pictorially by suggesting students draw the mixed number so they can see how many whole parts there are, and the correct fraction remainder.

- Students often fail to simply fully when writing a fraction in its simplest form. Address this by suggesting students do a final check each time to see if both the numerator and denominator can be divided by 2, 3, or 5 (and any more of the prime numbers if applicable).

Question guidance

Core

Q6 This question has been carefully constructed to give students practice in converting improper fractions to mixed numbers. Students could draw bar models to help.

Q7 Explain that if there are 10 pieces of chocolate in each bar and Ossian has eaten 17 pieces, then he must have eaten more than one whole bar. It may help students if they copy the diagram of the bar and then draw the extra 7 pieces.

Q9 Encourage students to draw the × and ÷ arrows onto parts **c**, **d**, **g** and **h**.

Q14 This question has been carefully constructed to give students practice in the skill of simplifying fractions with a denominator of 100.

Q15 Encourage students to explain using the correct mathematical vocabulary, i.e. numerator, denominator, simplest form, cancel, factor. *Does it matter how many steps it takes to cancel a fraction to its simplest form?* Establish that it doesn't matter, but it is a lot quicker if you use the HCF rather than take one step at a time.

Support

Changing improper fractions to mixed numbers Q2 If students struggle with $\frac{10}{5}$, encourage them to think about how many wholes this represents and refer to the diagram in part **e**.

Changing improper fractions to mixed numbers Q3 Suggest students draw diagrams like those in Q2 to help.

Equivalent fractions Q2 Encourage students to discuss which bars/fractions they think will be equivalent before shading, then compare their answers after shading.

Purposeful practice

Purposeful practice 2 These questions offer structured practice to show that order of multiplication or division does not matter in simplifying and that division and/or multiplication by two factors is equivalent to dividing and/or multiplying by their products.

Reflect

Core

After this lesson, Lucy said, 'Fractions are not really like whole numbers.'

Kala said, 'Yes! Fractions can be written in many different ways.'

Look back at your work on fractions.

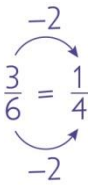
a What do you think Lucy means? **b** What do you think Kala means?

This reflective task encourages students to reflect on what a fraction is and its notation. It also gives students an opportunity to begin to use the language of fractions for themselves. Students may write:

- *Fractions are parts of a whole.*
- *Fractions are written in two parts, with a denominator and a numerator.*
- *A fraction can be written as an improper fraction or as a mixed number if it is more than 1.*
- *All fractions can be written as an equivalent fraction.*

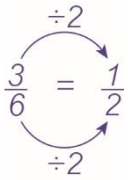
Support

Tia tries to simplify $\frac{3}{6}$ like this:



Draw diagrams to show that she is incorrect. Explain in words what she has done wrong.

To simplify a fraction, you must divide the numerator and the denominator by the same number (not subtract). Tia should have simplified $\frac{3}{6}$ like this:



Depth

a Which of these fractions is easiest to visualise?

$\frac{49}{63}$ $\frac{2}{10}$ $\frac{42}{72}$ $\frac{24}{32}$

Explain why.

b Simplify the fractions. Does this make them easier to visualise, or not?

a $\frac{2}{10}$*; the denominator is smallest.*

b $\frac{49}{63} = \frac{7}{9}, \frac{2}{10} = \frac{1}{5}, \frac{42}{72} = \frac{7}{12}, \frac{24}{32} = \frac{3}{4}$*; yes*

Purposeful practice Reflect and reason

Purposeful practice 1

Three students convert $\frac{10}{3}$ to a mixed number.

Sam writes $\frac{10}{3} = 1\frac{7}{3}$

Liz writes $\frac{10}{3} = 2\frac{4}{3}$

Jeff writes $\frac{10}{3} = 3\frac{1}{3}$

Explain why their mixed numbers are all equivalent. Whose answer is best? Why?

Students' explanation, possibly using diagrams, that $\frac{10}{3} = 1\frac{7}{3} = 2\frac{4}{3} = 3\frac{1}{3}$

Jeff's answer is best because the fraction part of his mixed number is a fraction less than 1.

Purposeful practice 2

Matt says, 'To simplify a fraction, it doesn't matter which factor you divide by first.'

Use your answers from Purposeful practice 2 to show that Matt is correct.

What single multiplication is equivalent to ×2 then ×3? Which two divisions are equivalent to ÷15?

Q3 *parts* ***a*** *and* ***b*** *show that whether you divide by 2 and then divide by 3, or divide by 3 and then divide by 2,* $\frac{12}{30}$ *simplifies to* $\frac{2}{5}$*. Similarly for* ***Q4*** *parts* ***a*** *and* ***b****.*

×6 is equivalent to ×2 and ×3.

÷3 and ÷5 are equivalent to ÷15.

Both can be done in either order.

Objectives

Core objectives

- Add and subtract simple fractions
- Calculate simple fractions of quantities

Support objectives

- Add and subtract fractions
- Calculate fractions of a quantity

Depth key concepts

- Understand inverse operations relating to fractions

Key points

- When you add or subtract fractions with the same denominator, you add or subtract the numerators. Then write the result over the same denominator.
- To work out a fraction of a quantity, divide the quantity by the denominator, and then multiply the answer by the numerator.

Common errors and misconceptions

- When adding fractions, students often add the denominators as well as add the numerators. Resolve this pictorially using bar modelling or use a real-life context such as a pizza. Draw a pizza and cut it into 8 equal slices. *If I eat* $\frac{3}{8}$ *then eat another* $\frac{1}{8}$*, how much have I eaten altogether? What fraction is left?* etc.

Question guidance

Core

Q4, Q5 These questions have been carefully constructed to give students practice in adding or subtracting fractions with the same denominator.

Q10 If students need a hand to get started, explain that the obvious equivalent to $\frac{1}{4}$ is $\frac{2}{8}$, but the only addition is $\frac{1}{8} + \frac{1}{8}$ which is not allowed, as the fractions are the same. Suggest they look at other equivalent fractions. Discuss students' different answers, as there are many answers to this question depending on the equivalent fraction that students use, e.g. $\frac{2}{12} + \frac{1}{12}$, $\frac{4}{24} + \frac{2}{24}$

Q12–Q14 These questions have been carefully structured to give students practice in finding unit fractions of quantities and of numbers and then finding non-unit fractions of quantities.

Q16 First of all, students need to realise that the gauge is divided into eighths.

Q18 The easiest approach is to work out $\frac{2}{3}$ of all the B values and then compare them with the A values. They do not have to match exactly for it to be a good model, but they need to be close.

Challenge Students can work in pairs to solve this problem. They need to link the dominoes by matching questions that give the same answer. There are 2 different ways of linking the dominoes.

Support

Adding fractions When adding fractions with the same denominator, students should see that the denominator doesn't change but the numerator increases.

Subtracting fractions Q2 Encourage students to write out the calculations as shown in the Worked example.

Purposeful practice

Purposeful practice 2 Students can use their answers to **Q1** to help work out some of the answers to **Q2**. There is minimal variation in both **Q1** and **Q2** so students can focus on practising adding to make 1 and subtracting from 1.

Reflect

Core

a Write down an easy fraction. **b** Write down a difficult fraction.

c What makes one fraction easier or harder than the other?

This reflective task encourages students to consider what makes one fraction more difficult than another. In doing so, they may think about the notation and the size of fractions, as well as the operations they must carry out when working with fractions. For example, they may perceive $\frac{5}{8}$ *to be a harder fraction than* $\frac{1}{2}$*, because* $\frac{5}{8}$ *is not a unit fraction, it has a larger denominator than* $\frac{1}{2}$*, and finding* $\frac{5}{8}$ *of a quantity requires dividing by 8, which is harder than dividing by 2.*

Support

How do you find $\frac{1}{6}$ of an amount?

Complete this sentence. To find a unit fraction of an amount, divide by its _____.

To find $\frac{1}{6}$ *of an amount, divide the amount by 6.*

To find a unit fraction of an amount, divide by its denominator.

Depth

On Monday Nina completed some addition calculations with fractions.

Here are three of her answers.

a $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$ **b** $\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$ **c** $\frac{3}{11}+\frac{6}{11}=\frac{9}{11}$

On Tuesday Nina was attempting some subtraction calculations.

Explain how her answers from Monday would help her correct these.

a $\frac{3}{5}-\frac{2}{5}=\frac{2}{5}$ **b** $\frac{5}{7}-\frac{2}{7}=\frac{4}{7}$ **c** $\frac{9}{11}-\frac{3}{11}=\frac{3}{11}$

a $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}, so\ \frac{3}{5}-\frac{2}{5}=\frac{1}{5}$ **b** $\frac{2}{7}+\frac{3}{7}=\frac{5}{7}, so\ \frac{5}{7}-\frac{2}{7}=\frac{3}{7}$

c $\frac{3}{11}+\frac{6}{11}=\frac{9}{11}, so\ \frac{9}{11}-\frac{3}{11}=\frac{6}{11}$

Purposeful practice Reflect and reason

Purposeful practice 1

Lin adds $\frac{2}{5}$ and $\frac{1}{5}$ like this:

$$\frac{2}{5} + \frac{1}{5} = \frac{2 + 1}{5 + 5} = \frac{3}{10}$$

Explain what Lin has done wrong.

Lin has added the numerators and added the denominators. The correct answer is $\frac{2+1}{5} = \frac{3}{5}$

Purposeful practice 2

Sam subtracts $\frac{3}{4}$ from 1 like this:

$$1 - \frac{3}{4} = \frac{1 - 3}{4} = \frac{-2}{4}$$

Explain what Sam has done wrong.

$\frac{3}{4}$ *is less than 1 so* $1 - \frac{3}{4}$ *cannot be negative. Sam needs to write 1 as quarters first, as* $\frac{4}{4}$*. Then*
$\frac{4}{4}{\rm{ }}\; - {\rm{ }}\frac{3}{4}{\rm{ }} = {\rm{ }}\frac{1}{4}$

Purposeful practice 3

Sara and Lee work out $\frac{4}{5}$ of £15.

Sara	Lee
15 ÷ 5 = 3	15 ÷ 5 = 3
4 × 3 = 12	*3 + 3 + 3 + 3 = 12*

Explain why they are both correct. Which is the better method and why?

Both find that $\frac{1}{5}$ *of 15 is 3, then they both find 4 lots of 3.*

Sara's method is better because 4 × 3 is quicker to write and calculate than 3 + 3 + 3 + 3.

Objectives

Core objectives

- Work with equivalent fractions and decimals
- Write one quantity as a fraction of another

Support objectives

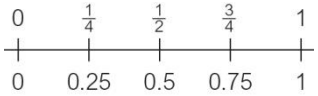
- Work with equivalent fractions and decimals
- Write one number as a fraction of another

Depth key concepts

- Understand that all 1, 2 and 3 place decimals are also fractions

Key points

- You can write fractions as decimals.
  Three important examples are $\frac{1}{4}$= 0.25, $\frac{1}{2}$= 0.5, $\frac{3}{4}$= 0.75.



- You can convert a decimal to a fraction by looking at the place value.
- You can convert a fraction to a decimal by writing an equivalent fraction with a denominator of 10 or 100 and then using place value.
- You can write one quantity as a fraction of another.

$\frac{{{\rm{number \ of \ black \ squares}}}}{{{\rm{total \ number \ of \ squares}}}} = \frac{4}{9}$

Common errors and misconceptions

- Students often do not use the total as the denominator when writing one number as a fraction of another. For example, when there are 3 boys and 7 girls, they write the number of boys as $\frac{3}{7}$ instead of $\frac{3}{10}$. Resolve this by telling students to write the fractions as they would *say* them, i.e. 3 out of a total of 10 children were boys and 7 out of a total of 10 children were girls.

Question guidance

Core

Q5, Q6 These questions have been carefully constructed to give students practice in writing decimals as fractions in their simplest form, and converting fractions to decimals. Suggest they use a place value table to make sure their digits have the correct value.

Q9b Yes, there are different ways to convert $\frac{12}{20}$ to a decimal, e.g. $\frac{12}{20} = \frac{6}{10}$ = 0.6, $\frac{12}{20} = \frac{120}{200} = \frac{60}{100}$ = 0.6, etc.

Q11–Q15 Make sure students write each fraction in its simplest form.

Q16 Students need to add together all the stages first to find the denominator of the fraction.

Challenge Students can work in pairs on this task which informally introduces them to recurring decimals. For part **d**, first ask students to predict $\frac{8}{9}$ and then $\frac{9}{9}$. They may say 0.999 99… Look again at the fraction and read it as 9 ÷ 9. Discuss the fact that 0.999 99… is almost 1. Now predict $\frac{10}{9}$, $\frac{11}{9}$ and so on. Also talk about how to write recurring decimals using the recurring dot instead of writing lots of repeating digits.

Support

Converting between fractions and decimals Students could draw up place value tables to help. Encourage students to use the correct language, stressing the difference between tenths/tens and hundredths/hundreds.

Converting between fractions and decimals Q2 Discuss with students how each time the numerator is 10 times larger the 6 (or 8) moves one place left in the place value table.

Writing one number as a fraction of another Q1 Students need to read the questions carefully in order to identify the total (the denominator) and the number of parts (the numerator).

Purposeful practice

Purposeful practice 1 Students practise converting between decimals and fractions in order to reinforce common fraction/decimal equivalents.

Purposeful practice 2 Q2 is structured to lead students to understand that they can find multiples of fractions as decimals.

Reflect

Core

After this lesson, Faiz says, 'Decimals are just another way to write fractions.'

Do you agree with Faiz? Explain.

This reflective task encourages students to understand that fractions and decimals are not two distinct mathematical topics, but are instead different ways of representing the same thing. Students may use a generic statement to explain their answer, or give specific examples.

Support

Eight people go for a meal. Three of them have fish.

Jack says: $\frac{8}{3}$ of the people have fish. What has he done wrong?

What fraction of people do have fish?

Jack has confused the numerator and the denominator – the total (denominator) is 8 so he should have said, '$\frac{3}{8}$ of the people have fish.'

Depth

Luke says,

'Give me any decimal with 1, 2 or 3 decimal places and I can write it as a fraction.'

Is Luke correct? Give some examples.

Yes. For example: $0.7 = \frac{7}{10}$, $0.23 = \frac{23}{100}$, $0.561 = \frac{561}{1000}$

Purposeful practice Reflect and reason

Purposeful practice 1

Tracy writes $0.03 = \frac{3}{10}$

What mistake has Tracy made?

$0.03 = \frac{3}{100}$ *because the 3 is in the hundredths column, not the tenths.*

Purposeful practice 2

$0.04 = \frac{1}{25}$

How can you use this fact to work out decimal equivalents for these fractions?

$\frac{2}{25}$ $\frac{3}{25}$ $\frac{6}{25}$ $\frac{12}{25}$ $\frac{24}{25}$

$\frac{2}{25} = 2 \times \frac{1}{25} = 0.08$, *and so on. Multiply both sides by the same number.*

Objectives

Core objectives

- Understand percentage as 'the number of parts per 100'
- Convert a percentage to a fraction or decimal
- Work with equivalent percentages, fractions and decimals

Support objectives

- Understand percentages as 'the number of parts per 100'
- Convert a percentage to a fraction or decimal

Depth key concepts

- Understand when it is easier to compare proportions when using fractions, decimals or percentages, e.g. comparing marks in a test
- Working with fractions and percentages that are >1 and what this means (laying ground work for percentage increase)

Key points

- **Per cent** means 'out of 100'. % stands for 'per cent'. 50% means '50 out of 100', which is $\frac{50}{100}$
- You can write any **percentage** as a fraction with denominator 100.
- You can write a percentage as a fraction and then convert to a decimal. You can write a decimal as a fraction with denominator 100 and then convert to a percentage.

Common errors and misconceptions

- Students often write 0.8 as 8% or similar. Resolve this pictorially by using a place value table. Some students may find it easier to change a decimal into a percentage if they convert to a fraction first. Encourage students to use the table to write 0.8 as a fraction, i.e. $\frac{8}{10}$, and then change it to a fraction with a denominator of 100, i.e. $\frac{80}{100}$. They can then see that the correct percentage is 80%, as it is the number 'out of 100'.
- Students often write 8% as 0.8 or similar. Again resolve this pictorially by using a place value table and suggest students change a percentage to a decimal by converting it to a fraction first. They can write 8% as $\frac{8}{100}$, then use the place value table to write the 8 in the hundredths column. They can then see that the correct decimal is 0.08.

Question guidance

Core

Q4 Each block is 100 squares, so students need to count the number of shaded and unshaded squares to work out the percentage. For part **d** they should realise that the pairs of answers in parts **a–c** total 100%.

Q5b If students add their answers to part **a**, then subtract the total from 100% this will give them the amount of empty floor space.

Q8 This question is carefully constructed to give students practice in converting percentages to fractions and recognising when a fraction can be simplified.

Q11 The answer is no, as the original price is 100%. If you give a 100% discount the item will be free, so you cannot discount more than 100%.

Q12–Q14 These questions have been carefully constructed to give students practice in converting decimals and fractions into percentages so they become fluent in the methods.

Q17 Students can either cancel down the fraction first, or build it up, e.g. $\frac{18}{40} = \frac{9}{20} = \frac{45}{100} =$ 0.45 or $= \frac{18}{40} = \frac{180}{400} = \frac{45}{100} =$ 0.45.

Challenge This task uses a grid of 20 squares to represent 100%. Students can work in pairs or individually, but they will definitely need a partner for part **d**. Students need to make three copies of the grid. Encourage them, before they even read parts **b** and **c**, to think of the 20 squares as 100%, and so work out the percentage that one square represents. They should then be able to tackle the rest of the task.

Support

Understanding percentages You could hand out 100 grids and challenge students to shade different percentages of the grid.

Converting percentages to fractions and decimals Q1 Challenge students to suggest ways in which you could easily write a percentage as a decimal without writing it as a fraction first.

Converting percentages to fractions and decimals Q3 If necessary, remind students divide both the numerator and denominator by the same number in order to simplify the fraction.

Purposeful practice

Purposeful practice 2 Students convert percentages to decimals and vice versa, to tackle the common error that 3% = 0.3 and to build fluency.

Problem-solving practice Q6, Q7 Students need to add the numbers of animals and swimming races to find the totals in each question before they can answer the questions.

Reflect

Core

After this lesson, Alex says, 'Percentages are just another way to write fractions.'

Do you agree with Alex? Explain.

This reflective task encourages students to consider the relationship between fractions and percentages as different ways of representing the same thing. Students may use a generic statement to explain their answer, or give some specific examples.

Support

Jamal writes: $30\% = \frac{30}{100} = \frac{6}{20}$

Explain why he hasn't written the percentage as a fraction in its simplest form.

What does he need to do to write it in its simplest form?

Jamal hasn't written $\frac{6}{20}$ in simplest form because both 6 and 20 can be divided by 2.

In simplest form $\frac{6}{20}$ *is* $\frac{3}{10}$

Depth

Kai gets $\frac{41}{50}$ in his Geography test.

Milo scores $\frac{22}{25}$ in his History test.

They wonder who got the higher mark.

a Convert both scores into percentages.

b Convert the scores into fractions with the same denominator.

c Did you find it easier to work out the percentage or the fraction?

Would your answer change if the Geography test was out of 45 instead of 50?

a 82%, 88%

b *For example:* $\frac{41}{50}$, $\frac{44}{50}$

c *Comparing out of 50 and out of 25 is fairly easy both ways. The equivalent fraction out of 50 seems easier. If the mark was out of 45 instead, then the percentage is probably easier.*

Purposeful practice Reflect and reason

Purposeful practice 1

Write down the percentage/fraction equivalents you are fluent in. This means the ones you 'just know' without having to work them out.

Students' own answers, for example 50% = $\frac{1}{2}$

Purposeful practice 2

Lucy says, '8% is equivalent to 0.8.'

Explain Lucy's mistake.

8% = $\frac{8}{100}$ *= 0.08, not 0.8.*

Objectives

Core objectives

- Use different strategies to calculate with percentages
- Express one quantity as a percentage of another

Support objectives

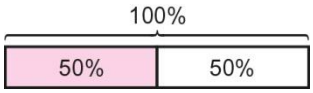
- Calculate percentages

Depth key concepts

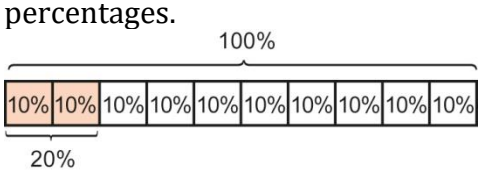
- Working with fractions and percentages that are >1 and what this means (laying ground work for percentage increase)
- When is it possible to have 110%? (what are realistic contexts? (revenue, exam marks, effort))

Key points

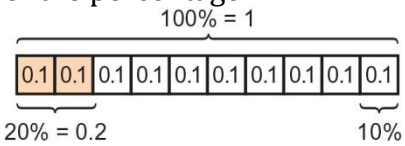
- 50% is the same as $\frac{1}{2}$
  To find 50% of an amount, divide by 2.



- 10% = $\frac{10}{100} = \frac{1}{10}$
  To find 10% of an amount, divide by 10. You can then use 10% to find other percentages.



- When you are working out more complicated percentages of amounts, you can make notes or use **jottings** to help you.
- You can use a **multiplier** to work out a percentage, by using the decimal equivalent of the percentage.



  10% = $\frac{10}{100} = \frac{1}{10} = 0.1$
  To find 10% you multiply by 0.1, to find 20% you multiply by 0.2, ...

Common errors and misconceptions

- Students think that, because you divide by 10 when finding 10%, therefore you divide by 20 when finding 20%. Resolve this with some concrete materials such as counters. For example, start with 30 counters. *What is 10% of 30?* 3. Arrange the counters into 10 piles of 3 counters. Demonstrate that 20% of the counters is 2 of the piles (i.e. 6 counters), 30% is 3 of the piles, etc., emphasising that each pile is 10%, so multiple piles must be multiples of 10%.

Question guidance

Core

Q6 This question has been carefully constructed to demonstrate that 25% + 75% = 100% (1 whole). Students may also notice that 75% = 25% × 3.

Q8d Stress the importance of the correct money notation.

Q12 This question has been carefully constructed to give students practice in finding 10% of an amount, and using this to find related percentages.

Q14 Discuss how students worked their answer to part **d**. There are many different methods. Discuss the advantages and disadvantages of each.

Q17 Discuss what the question means by 'a suitable amount'. *If you were sending some money to a charity, would you round to the nearest 1p, £1, £5…?*

Support

General Encourage students to draw bar models if they need them for support.

Calculating 50%, 25% and 75% Q3c, d These repeat quantities used in **Q2** parts c and **d**, so students can use their answers to these to work out **Q3** answers.

Calculating 10%, 20%, 30%, … Q4 Students should begin to see a pattern. Encourage them to discuss what each extra 10% adds and why – link this to knowledge of times tables. Challenge students to find two ways of finding 90% (10% × 9 or 100% – 10%).

Calculating 1% Make sure students do not think that to find 1% you divide by 1 (see Common errors) – model what happens when you do.

Purposeful practice

Purposeful practice 1 and Purposeful practice 2 Students practise calculating percentages by finding 10% first, then progress to finding other percentages using jottings based on finding 10%.

Problem-solving practice Students need to show their working to explain or justify their answers.

Reflect

Core

In this lesson, you found 20% of a quantity in two different ways.

Strategy 1: finding 10%, then multiplying by 2

Strategy 2: using the multiplier 0.2

a Look back at the questions where you used these different strategies.

b Which strategy did you like better? Explain.

c Which strategy would you use if an item was 20% off in a shop? Explain.

d Write down any advantages or disadvantages of each strategy.

This reflective task aims to raise students' awareness of different strategies for finding percentages of quantities, and reinforce that there is not always one way of doing maths. The task moves on to urge students to critique the strategies by considering advantages and disadvantages. For example, students may state that finding 10% then multiplying by 2 only works for percentages that are factors or multiples of 10, but it is easier to do mentally. They may also state that it partly depends on whether the 10% value is an easy number to double. Alternatively, they may state that using the multiplier is a quicker or more efficient strategy, but that more difficult decimals may require a calculator.

Support

Jessica says: 'To find 50% you need to find 10% and then multiply your answer by 5.'

Is Jessica correct? How else could she calculate 50%? Which method would you use?

Jessica is correct, because 50% = 10% × 5. She could also find 50% by dividing by 2 (halving).

The preferred method might depend on the numbers involved.

Depth

Which of these statements make sense? Explain why you think this.

A puppy's weight increases by 120%.

A company makes a 130% profit.

A person gives 110% effort.

The first two make sense. 100% effort means 'doing your best', and it is impossible to do more than your best.

Purposeful practice Reflect and reason

Purposeful practice 1

For **Q4a**, Melissa writes *10% of £42 is £4.2.*

Explain the mistake she has made.

2 decimal places are needed for money. £4.2 = £4.20

Purposeful practice 2

In **Q2**, how could you use the answers to earlier parts of the question to answer part **f**?

Write three more percentages of £250 that you could find using your answers to **Q2**.

*For part **f** 16% you could add 10%, 5% and 1% from parts **a–c** or add 10% and 6% from parts **a** and **e** or add 15% and 1% from parts **c** and **d**.*

Students' own answers, for example, 46% of £250, 7% of £250, 66% of £250.

Purposeful practice 3

Which of these methods calculates 20% of an amount? Choose all options that apply.

A Find 10% and double **B** Multiply by 20 **C** Multiply by 0.2

D Divide by 10 and double **E** Divide by 20 **F** Divide by 5

G Divide by 0.2

A, C, D, F

Check, Strengthen and Extend

Core Check up

Q2 This question tests students' understanding that for unit fractions, the greater the denominator, the smaller the fraction.

Q3 This question is designed to check students' ability to compare fractions using their knowledge of relative sizes of numerator (when denominator is the same), forming equivalent fractions, and comparing denominator (when numerator is the same).

Q7–Q9 These questions assess students' understanding of simplest form.

Q11–Q15 These questions test students' ability to convert between fractions, decimals and percentages.

Q17 This question tests students' knowledge of the fact that the denominator of a fraction must always be the total number of people (in this case).

Q21 This question tests students' ability to recognise a percentage as a decimal and not make an error in the place value of the digit.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Caspar says 'In the lesson I am doing lots of division.'

Look back at your work in this lesson. Where did you use division?

How did you use it?

This reflective task encourages students to recognise the relationship between fractions and division. It is also intended to build students' confidence as they become aware of how

they are building their mathematical skills by drawing on maths learned previously and applying it to new concepts. Students may give examples of finding fractions by dividing shapes into equal parts, or dividing linking cubes into groups that represent wholes with a remaining fractional part.

Extend

Look back at the questions you have answered in these lessons.

Which question(s) did you find easiest? What made them easy?

Which question(s) did you find most difficult? What made them difficult?

Are there particular kinds of questions that you need more practice with?

If so, which kinds?

This reflective task encourages students to reflect on the questions they find easy and hard, and so where they may require more help or work to improve understanding. In doing so, it urges students to take responsibility for their own learning.

Depth

Extend

Which is larger, $\frac{5}{24}$ or $\frac{6}{29}$?

What have you learned in this lesson that would help you answer the question?

You could make them into equivalent fractions $\frac{145}{696}$ and $\frac{146}{696}$, or decimals 0.2083 and 0.2069, or even percentages 20.83% and 20.69%. All these give the same conclusion, i.e. that $\frac{5}{24}$ is slightly larger.

End of Unit 5 tests: Fractions and percentages

Pearson published resources

Support/Core

- Year 7 End of Unit 5 Support/Core test
- Year 7 End of Unit 5 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 5 Core/Depth test
- Year 7 End of Unit 5 Core/Depth mark scheme

Unit opener: Probability

Pearson published resources

Objectives

Core objectives

- Use the language of probability
- Use a probability scale with words
- Understand the probability scale from 0 to 1

Support objectives

- Use the language of probability
- Use a probability scale with words
- Understand the probability scale from 0 to 1

Depth key concepts

- 'Unlikely' and 'Likely' have more precise meanings in probability than everyday language
- Assigning numerical values to probabilities can help us compare them more accurately

Key points

- **Probability** is the chance that something will happen.
  Even chance means that something is as likely to happen as not.
  In probability, an **event** is something that might happen.
- You can show a probability on a **probability scale**.
- All probabilities have a value between 0 and 1. You can use fractions, decimals and percentages to describe probabilities.
  An impossible event has probability of 0 or 0%. A certain event has probability of 1 or 100%.

Common errors and misconceptions

- Students may confuse equally likely with even chance. Clarify using spinner B from **Q6**.

Question guidance

Core

Q3 List a few possible and impossible events. Discuss colloquial sayings but emphasise that in maths we must use standard descriptions of probability.

Q5 The question introduces the term 'event'. After checking that students understand this, give some examples of impossible events, e.g. rolling a 7 with an ordinary six-sided dice.

Q6 Address the misconception using spinner B. *Why does blue have an even chance?* Half the spinner is blue. *Why are red and white equally likely?* Each has equal area of the spinner. *Why are red and blue not equally likely?* The blue segment is larger than red, so blue is more likely.

Q7 This question has been carefully constructed to encourage students to think about the probability of particular events, and to become fluent in expressing probabilities using fractions, decimals and percentages. Discuss the advantages of using numbers to describe probabilities. Explain that common words such as 'likely' mean different things to different people; they are not accurate. Sometimes we only need a rough idea, so words are acceptable. When we need more accurate comparisons, numbers are better.

Support

General Students often don't understand that a probability cannot be larger than 1.

Probability in words Q2 Discuss the probability words we use in everyday life – do we use them correctly?

Probability in words Q4 *If something has probability of* $\frac{1}{2}$*, what does this mean?*

Purposeful practice

The questions focus on the language of probability and how it can differ from everyday use, before moving to numerical probability. In particular, they draw out that 'likely' and 'unlikely' cover a large range of probabilities, and that 'likely' means more than 50% or even chance.

Reflect

Core

This lesson used a probability scale labelled in different ways.

a Did the scale help you to understand probability, or not? Explain.

b List at least two other areas of maths where you have used a scale.

c Do these scales help you to understand the maths?

This task asks students to think about the purpose of the probability scale, and reflect on whether they find it helpful for their understanding of probability. It then encourages students to consider the scale as a pictorial tool for visualising and understanding other areas of maths too. Students may mention using scales for axes, negative numbers, decimals or measures.

Support

In this lesson you have met some new words and learned about probability scales. Write down what you think it means for an event to be

a impossible **b** certain **c** likely.

Students' own answers. Possible answers are:

a *An event that is impossible cannot/will not happen. The probability of an impossible event is zero (0).*

b *A event that is certain will definitely happen. The probability of an event that is certain is one (1).*

c *An event that is likely will probably happen. The probability of a likely event is greater than half ($\frac{1}{2}$) but less than one (1).*

Depth

Lucy and Isla support the same football team.

After 10 matches, they look back on their team's performance.

Lucy says it is likely the team will win their next match.

Isla says the probability they will win is $\frac{7}{10}$

Who has made the more precise statement? Explain your answer.

Isla is more precise because she gave an exact value; Lucy's statement of 'likely' means that the probability could be anything between 0.5 and 1.

Purposeful practice Reflect and reason

Purposeful practice

Cara says, 'Event P and event R are both likely. This must mean they have the same probability.'

Is Cara correct? Explain your answer.

What is the advantage of using numbers to describe probabilities?

'Likely' and 'unlikely' cover a large range of probabilities so Cara is not correct. Numbers are more accurate and help distinguish between two 'likely' (or 'unlikely') probabilities.

Objectives

Core objectives

- Identify outcomes and equally likely outcomes
- Calculate probabilities
- Use a probability scale from 0 to 1

Support objectives

- Identify outcomes of an event
- Calculate probabilities

Depth key concepts

- Know that probability can be represented as a fraction, decimal or a percentage (and how you choose which to use for a given question)
- There may only be n outcomes, but the probability of each is only $\frac{1}{n}$ if the outcomes are equally likely

Key points

- **Outcomes** are all the possible results of an event. The possible outcomes of flipping a coin are 'heads' and 'tails'.
- **Successful outcomes** are the outcomes you want.
- Probability of an event happening = $\frac{{\rm{number \ of \ successful \ outcomes}}}{{\rm{total \ number \ of \ possible \ outcomes}}}$
- Picking an item **at random** means that each item has exactly the same chance of being picked.
- **P(*X*)** means 'the probability that *X* happens'.

Common errors and misconceptions

- Students may not include outcomes that appear identical. Spin a spinner such as the one in the Student Book Worked example to emphasise that outcomes may look the same but are in fact different. They must all be listed.
- Students may not understand the difference between an event and an outcome. To distinguish between the two, use the idea of successful outcomes, i.e. the successful outcomes make the event happen.
- Students may calculate probability using outcomes that are not equally likely. Use a circular spinner coloured 75% blue and 25% red to demonstrate that the formula for probability does not work for outcomes that are not equally likely.

Question guidance

Core

General Make sure that students understand the difference between an outcome and an event.

Q4 In part **c** the fewer competing numbers, the more likely a 1 is to occur. Once students have completed **Q4**, discuss how many possible outcomes there are for the first ball drawn in the UK National Lottery Lotto game.

Q5 The successful outcomes are the ones that satisfy the event in question. The formal definition of an event is a set of outcomes. Where necessary, practise defining events using the correct terms, e.g. *What are the successful outcomes for the event 'spinner lands on an odd number'? Which outcomes satisfy the event?*

Q9a iv *What does a multiple of 10 look like?*

Q10 Make sure students are familiar with raffles, playing cards, dominoes and bingo, using real equipment or images if available. Ask students for other ways of choosing something at random, including selecting random play on an mp3 player. Students should convert their answers to fractions. Once they have completed **Q10**, pose the following question. *What is the probability of a baby being born at the weekend? Explain your answer.*

Q14 Use the opportunity to test the definition of multiple, prime and square numbers. Other possible events include 'factor of 12' and 'number less than 13'.

Q16c *How many combinations are there?* $10 \times 10 \times 10 = 1000$

Challenge Students can discuss their approach to solving this problem.

Support

General Watch out for students writing fractions the wrong way around

Probability where some outcomes are the same Q1 Students will often assume that because there are two outcomes they are equally likely. Use the example of one chocolate in a bag of toffees – are you equally likely to pick the chocolate as the toffee?

Purposeful practice

Purposeful practice 2 There is minimal variation in the numbers of colours and numbers of counters in these questions, giving students practice in calculating all possible and all successful outcomes and noting what changes as numbers and colours change. Some students might notice that P(red) + P(blue) = 1 and therefore P(red) = 1 – P(blue), which is covered in the next lesson.

Reflect

Core

Andy and Kofi are playing a game with a fair six-sided dice.

Andy needs a 6 to win. He rolls a 2!

Andy says, 'It's not fair. It's harder to roll a 6 than a 2.'

Use what you have learned in this lesson to decide whether Andy is correct. Explain.

This reflective task involves a typical probability situation, which students may have met many times: having to roll a 6 on a fair dice in a game. It challenges the commonly held belief that rolling a 6 is more difficult. However, students should be able to use what they have learned in this lesson to work out that the probability of rolling a 6 on a fair dice is $\frac{1}{6}$*, and that this is the same as the probability of rolling a 2. Therefore, Andy is wrong.*

Support

Look at the spinner in **Q7**.

Which of these statements are true?

A The probability of landing on orange is $\frac{1}{2}$

B The probability of landing on yellow is $\frac{1}{3}$

C The probability of landing on yellow is $\frac{1}{6}$

***A** and **C** are true. P(orange) =* $\frac{3}{6} = \frac{1}{2}$*; P(yellow) =* $\frac{1}{6}$

Depth

Look back at your answers to **Q9–Q10**. How did you decide whether to write the probabilities as a fraction, a decimal or a percentage?

*Writing decimals or percentages makes sense when the number of objects is a factor of 10 or 100 (e.g. **Q9** is out of 5). Fractions make sense when this is not the case (e.g. **Q10** is choosing from 26 objects).*

Purposeful practice Reflect and reason

Purposeful practice 1

Why do **Q1i** and **Q2i** have the same answer?

Explain using the number of possible outcomes and number of successful outcomes.

Both have 4 possible outcomes and 3 of those are successful.

Purposeful practice 2

In which question were the probabilities for red and blue the same? Explain why.

Dom says, 'Each bag has only 2 colours of counters: red or blue. So the probability of picking red must be $\frac{1}{2}$.' Use your answers to **Q1** and **Q2** to explain why Dom is wrong.

***Q2c**, because there are equal numbers of red and blue, so there is an even chance or probability of $\frac{1}{2}$ of picking each colour.*

*The probability depends on the number of balls of each colour. In all parts of **Q1** and **Q2** except **Q2c** there are two colours but not equal probabilities.*

Objectives

Core objectives

- Calculate more complex probabilities
- Calculate the probability of an event *not* happening

Support objectives

- Use probability notation
- Calculate the probability of an event *not* happening

Depth key concepts

- Understand that when there are outcomes A, B and C, P(A or B) = P(A) + P(B), and that P(A) + P(B) + P(C) = 1, so P(C) = 1 – P(A or B)

Key points

- P(green or blue) means 'the probability of landing on green or blue'.
- P(event not happening) = 1 – P(event happening)
- Percentage probability of something *not* happening = 100% – percentage probability of event happening

Common errors and misconceptions

- When answering Depth questions students may count the same outcome twice. Demonstrate using playing cards by asking questions involving 'or', e.g. *How many cards are black or are an ace?*

Question guidance

Core

General When working out the probability of an event not happening, remind students how to subtract a fraction from 1 by writing 1 as a fraction with the same denominator.

Q5 If available, give a pack of playing cards to students who are unfamiliar with them, or display a set on the whiteboard.

Q6 Take the opportunity to remind students of the meaning of the term 'random'.

Q6a Students should give their answers as percentages.

Q6b Students need to cancel the probability fraction to its lowest terms, $\frac{1}{4}$, then write this as an equivalent percentage.

Q7–Q12 These questions have been carefully constructed to give students practice in calculating probabilities using decimals, fractions and percentages.

Q7 Discuss the following questions. *Can the probability of something happening be 0? Can the probability of something happening be the same as the probability of it* ***not*** *happening?*

Challenge Where necessary, remind students how to add fractions with the same denominator.

Support

General Students often fail to understand that probabilities must sum to 1. You may wish to spend some time addressing this fundamental aspect of probability.

Purposeful practice

Purposeful practice 2 These questions provide structured practice in finding numerical probabilities of P(A) and P(not A).

Reflect

Core

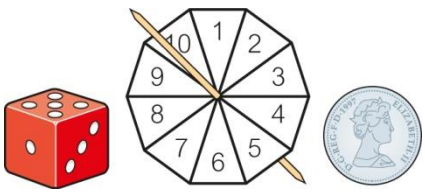
Would you find it easier to use a fraction, a decimal or a percentage to write the probability for

a rolling 1 on the dice

b landing on 1 on the spinner

c flipping the coin and getting a head?

Explain your answers.



This reflective task encourages students to recognise that they have learned many different ways to represent probability: as decimals, fractions and percentages. Students are asked which way they would find easiest for different situations. For example, they may state that, for the dice, it is easiest as a fraction, as there are 6 possible outcomes and this does not convert to an easy decimal or percentage. However, for the spinner, it may be

easiest as a decimal, as there are 10 possible outcomes. There are only 2 possible outcomes when flipping a coin so each format is equally convenient.

Support

The probability of winning a game is 20%. Ami says if she knows the probability of winning, she also knows the probability of losing. How can she work this out?

If winning and losing are the only two possible outcomes (a draw is not possible), then P(losing) = 1 – P(winning). In this case, P(losing) = 100% – 20% = 80%.

Depth

There are red, green and yellow counters in a bag.

One is selected at random.

Explain two ways to work out the probability of *not* choosing yellow.

Add up the probabilities for red and green or find the probability of yellow and then subtract from 1.

Purposeful practice Reflect and reason

Purposeful practice 1

Using the counters in Purposeful practice 1, work out P(R or Y or W).

Copy and complete this sentence: P(R or Y or W) = [] because it is _____ that you will pick red or yellow or white.

Next, work out P(not R or Y or W).

Copy and complete this sentence: P(not R or Y or W) = [] because it is _____ to pick a counter that is not red or yellow or white.

P(R or Y or W) = 1

*P(R or Y or W) = **1** because it is **certain** that you will pick red or yellow or white.*

*P(not R or Y or W) = **0** because it is **impossible** to pick a counter that is not red or yellow or white.*

Purposeful practice 2

Use appropriate words to complete these sentences.

When the probability of an event is zero, the probability that it does not happen is _____.

When an event is likely to happen, the probability that it does not happen is _____.

*When the probability of an event is zero, the probability that it does not happen is **certain**.*

*When an event is likely to happen, the probability that it does not happen is **unlikely**.*

Purposeful practice 3

Explain why the answers to **Q7** and **Q8** are the same.

In what two ways could you work out P(R or B or G)?

If the ball is red or white or blue, it is not green. So the probabilities are the same.

P(R or B or G) = P(not W) = 1 – P(W)

Objectives

Core objectives

- Record data from a simple experiment
- Estimate probability based on experimental data
- Make conclusions based on the results of an experiment

Support objectives

- Estimate probability based on experimental data

Depth key concepts

- Understand that experimental probability is always an estimate, and for some contexts you can only use experimental probability as it is not possible to calculate a theoretical probability
- Understand why more trials lead to better estimate of probability

Key points

- You can use the results of an **experiment** to estimate probabilities. This is called **experimental probability**.
  Experimental probability = $\frac{{{\rm{frequency \ of \ event}}}}{{{\rm{total \ frequency}}}}$
- You can estimate a probability from results, using experimental probability. Probability can be used to **predict** what may happen in the future.
- The more times you repeat an experiment, the more accurate the experimental probability.

Common errors and misconceptions

- Students may feel that estimated/experimental probability is inferior to 'exact' or theoretical probability based on equally likely outcomes. Explain that, in the real world, experimental/estimated probability is used the most. Calculating probabilities using the formula is based on equally likely outcomes, which are not

very common in the real world, but it is important to learn and be able to use the formula in order to understand more useful statistics.

Question guidance

Core

General Ensure students understand why an experiment is needed to estimate the probability of, for example, a drawing pin falling point up. There are only two outcomes but we do not know how likely each is just by looking at the drawing pin. Compare this with the theoretical probability of flipping a coin or rolling a dice.

Q7 Discuss why experimental probability is only an estimate of the probability.

Q9b Students need to add together the frequencies for 9, 10 and 11 months. Alternatively, they can add percentage probabilities calculated in part **a**.

When they have completed **Q9**, ask students to discuss whether, when you repeat an experiment, will you get the same results? Agree that you probably won't, although it is possible. The more times the experiment is carried out the more accurate the experimental probability becomes.

Q10b The probability estimate uses data from all months of the year and so will differ from data for a particular month. *Is the weather in July likely to be the same as in December?* No. *How could you estimate the probability of more than 10 cm rain in July?* Use only the recorded rainfall for July in previous years.

Q11a This question is about informally distinguishing between theoretical and experimental probability. Refer to the common misconception.

Support

General Students may think that when there are only two outcomes, each has an equal probability of happening.

Experimental probability as a percentage Encourage students to always write or convert fractions so they have a denominator of 100 to help write probabilities as percentages.

Purposeful practice

Purposeful practice 2 Students calculate and compare probabilities from different number of events, leading to the understanding that the more trials there are, the more accurate the experimental probability will be.

Reflect

Core

In the Challenge in this lesson, you collected your own data and worked out the experimental probability.

Other questions gave you data.

Which was easier? Explain.

This reflective task asks students to recognise whether they develop a better understanding from building a mathematical scenario for themselves, by collecting and working with their own data. Alternatively, they may find it easier to use others' data, which has already been organised for them into a table, and work with that. However, for some students, reading data from a table may prove to be difficult. Therefore, this task encourages students to recognise where data comes from and identify how they prefer to be offered it.

Support

If you flip a fair coin, the mathematical probability of it landing on heads is $\frac{1}{2}$.

If you do an experiment flipping a coin, is it certain to land on heads exactly $\frac{1}{2}$ of the time?

No, it is not certain to land on heads exactly $\frac{1}{2}$ of the time, but the number of heads will get closer to $\frac{1}{2}$ the more times the coin is flipped.

Depth

Julia says that tomorrow it will either rain or it won't rain, so the probability of it raining is $\frac{1}{2}$.

What have you studied in this lesson that might help you convince Julia she is wrong?

You should look at experimental data. You should also remember that even with two choices this does not mean they are equally likely.

Purposeful practice Reflect and reason

Purposeful practice 1

Deb says, 'In **Q1** and **Q2**, rolling a dice and flipping a coin are experiments, but in **Q3** and **Q4**, trains and people arriving are not experiments. So you cannot calculate experimental probability for **Q3** and **Q4**.'

Explain why Deb is wrong.

You can estimate probabilities from results on past data, using the same method as for experimental data.

Purposeful practice 2

Which person's results give the best estimate for the probability of rolling a six with this dice?

Why do you think that is?

Deepak's results give the best estimate because he rolled more times; the greater the number of trials, the more accurate the estimate of the probability.

Objectives

Core objectives

- Use probability to estimate the expected number of times an outcome will occur
- Apply probabilities from experimental data in simple situations

Support objectives

- Use probability to estimate the expected number of times an outcome will occur
- Apply probabilities from simple experimental data in simple situations

Depth key concepts

- Understand that if an event has probability $\frac{1}{3}$ then we expect it to happen 1 in 3 times, but that doesn't mean that it *will* happen 1 in 3 times

Key points

- You can use probability to estimate the **expected number** of times an outcome will occur.

Common errors and misconceptions

- When playing a game, students may forget to subtract the entrance fee from the prize. Point out the difference between 'prize' and 'winnings'.

Question guidance

Core

Q4c *Would your decision be the same if game 1 had a much bigger prize than game 2?*

Q10 Students should work out the total income from all 50 people and the total prizes given to the 13 winners.

Q12b Students should work out the total cost of playing 6 times and the total winnings for 6 spins.

Q13 *What size prize encourages people to buy tickets, but still makes a profit for the organiser?* This is subjective. Suggest a range of prizes and survey the class, e.g. 25p, 30p, 35p, 40p, 45p.

Challenge Students work in pairs. They repeatedly toss two coins, recording a win for two heads and a loss otherwise. Suggest students toss the coins a multiple of 12 times.

Support

General Students often struggle to multiply by a fraction.

Calculating probability Q1 If students find this concept challenging, discuss what would happen if you had 1 toffee in a box of 20 chocolates. *What would be the chances of picking the toffee? What happens if you have even more chocolates but still only one toffee?*

Calculating probability Q2 You many need to remind students of the types of number mentioned.

Calculating probability and expected outcomes Q1 Challenge student to make predictions (e.g. if you ask twice as many people you would expect twice as many would choose chicken, etc.).

Purposeful practice

Purposeful practice 2 Students calculate the expected number of wins for outcomes which vary minimally, leading to the understanding that the expected number might be accurate overall, but not necessarily for a small number of trials.

Reflect

Core

Look back at the questions in this lesson.

List all the mathematics needed to design a fundraising game that raises money.

This reflective task asks students to review what they have learned, and the maths that has been most helpful to them. It reinforces that designing a fundraising game that raises money requires a range of mathematical skills, including working with money (as learned in Unit 2) and understanding fractions in the context of probability (as learned in Unit 5).

Support

Will the expected outcome always be the same as the actual outcome? Try it – if you roll a dice 6 times do you get each number exactly once? If you did, will this happen every time?

Students' own answers, reflecting actual results. Answers are likely to indicate that when rolling a dice 6 times each number does not occur once. Although it is possible that each number might be rolled once in a set of 6 rolls, it will not happen every time.

Depth

Hiro rolls a dice 24 times and looks at his results.

He says, 'Getting a prime number has probability 3 out of 6, which is $\frac{1}{2}$, so I should expect to get a prime number 12 times in my results.'

When he looks at his results, he has rolled a prime number 10 times.

Is the dice biased?

No; this is a reasonable difference due to the dice landing randomly. He is correct to expect 12 out of 24, but actually obtaining 10 is very close and not a surprise.

Purposeful practice Reflect and reason

Purposeful practice 1

Use the words 'lower' or 'higher' to complete these sentences about **Q3** and **Q4**.

The lower the probability of red, the _____ the expected number of red counters picked.

The higher the probability of red, the _____ the expected number of red counters picked.

*The lower the probability of red, the **lower** the expected number of red counters picked.*

*The higher the probability of red, the **higher** the expected number of red counters picked.*

Purposeful practice 2

Emma says, 'My answers to **Q2** show I am unlikely to lose money with any of the spinners, but I could actually win a lot!'

Use your answer to **Q3** to explain why Emma could lose money on the spinners.

*Student answer based on list of W and L in **Q3**. For example, If Emma had paid for the first 5 spins, she would have paid £5 and won £3 – so would have a loss of £2.*

Check, Strengthen and Extend

Core Check up

Q1, Q2 These questions are designed to check students' understanding of the language of probability in the context of familiar events, and then to give these relative numerical values.

Q4 This question is designed to test students' ability to draw the relevant information from a worded question.

Q5 This question checks whether students include outcomes that appear identical.

Q7 This question asks students to calculate more complex probabilities.

Q8–Q10 These questions test whether students can calculate probability of an event not happening when given the probability that the event happens.

Q11b, Q12b These questions are designed to assess students' interpretation of their calculations and ability to justify their answers in a written format.

Challenge For part **b**, students need to think about how many sides their spinner should have. If necessary, prompt them to choose a number that makes calculating the probabilities easy, i.e. a multiple of 10.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

a Copy and complete this sentence with three different endings.

In this unit I learned to ___ .

b Copy and complete at least four of these sentences.

I showed I am good at ___ . I found ___ hard.

I got better at ___ by ___ . I was surprised by ___ .

I was happy that ___ . I still need help with ___ .

This reflective task asks students to write down what they have learned in these lessons, and then aims to raise awareness of how they felt about how they did. Students are encouraged to think about both their positive and negative reactions, and where they may still need some additional help.

Extend

Probability is used in many jobs, for example

sport medicine insurance forensic science weather forecasting

For each job, write down one way it uses probability.

What career are you interested in?

Do you think you will need to use probability in your job? If so, how?

This reflective task demonstrates to students the prevalence of probability in many different areas of life. Students may consider: in sport, the toss of coin at the beginning of a match, or the choice of player to take a penalty based on past performance; in medicine, the probability outcomes of pharmaceutical trials, or success of a medical procedure; in insurance, the risk of accident or flood; in crime, the chance of a crime happening in a particular location, based on past crime statistics; in environment, the probability of a hurricane hitting a coastline. The task also encourages students to think about if and where probability may be used in their chosen careers.

Depth

Extend

A class at school spend some time rolling dice and recording their results.

This is the data they collect.

Number	1	2	3	4	5	6
Frequency	95	84	89	111	116	105

Use what you have learned in this lesson to comment on the fairness of their dice.

We would expect 100 for each number. The number 2 looks suspiciously low, and the 5 looks suspiciously high.

End of Unit 6 tests: Probability

Pearson published resources

Support/Core

- Year 7 End of Unit 6 Support/Core test
- Year 7 End of Unit 6 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 6 Core/Depth test
- Year 7 End of Unit 6 Core/Depth mark scheme

Unit opener: Ratio and proportion

Pearson published resources

Objectives

Core objectives

- Use direct proportion in simple contexts
- Solve simple problems involving direct proportion
- Use the unitary method to solve simple word problems involving direct proportion

Support objectives

- Use direct proportion in simple contexts
- Solve simple problems involving direct proportion

Depth key concepts

- Understand when scaling/doubling/halving may be more or less efficient than the unitary method, to solve direct proportion problems

- Understand that when two quantities are in direct proportion, when one increases the other increases at the same rate

Key points

- When two quantities are in **direct proportion**, as one increases or decreases, the other increases or decreases at the same rate.
  This means that when one quantity is zero, so is the other; when one doubles, so does the other; when one is multiplied by 3, so is the other, etc.
- In the **unitary method**, you find the value of one item before finding the value of more.

Common errors and misconceptions

- Students sometimes do not find the value of one item first when answering a question that requires the use of the unitary method. Resolve this by emphasising the importance of breaking the question down to one item first, before building up.

Question guidance

Core

General This lesson goes straight into unitary method (which works in all cases) and then shows doubling and halving as a more efficient alternative method later in lesson.

Q6–Q14 These questions have been carefully constructed to give students practice in using the unitary method in a range of different contexts.

Q15 Suggest working out the time for one student first.

Q16 Students need to work out the cost of one mug first, and then they can either multiply by 35, or find the cost of 4 mugs and add that to £93. Discuss these different methods.

Q21 How can students use their answers to parts **a** and **b** to work out the answers to parts **c** and **d**?

Q22 There are several different ways to tackle this problem, e.g. work out the cost of 1 ticket then multiply by 14, work out the cost of 2 tickets then multiply by 7, etc.

Support

General Throughout this lesson encourage students to think logically – are they expecting the answer to be smaller or larger?

Working with one item Q2–Q6 Encourage students to say whether they think this will cost/weigh more or less – this will help them decide whether to divide or multiply.

Working out information about multiple items You may wish to explain to students that this is called the 'unitary method' because you work out the value of 1 item then multiply to find the value of more items.

Purposeful practice

Purposeful practice 3 These questions demonstrate that using doubling, halving or other simple multiplication may be more efficient than the unitary method.

Reflect

Core

Alan tells his mum he spotted two examples of direct proportionality at the petrol station:

A Mum's car: 30 litres of fuel costs £42

Next car: 10 litres of fuel costs £14

B In our queue six people are served in 10 minutes.

In the other queue, three people are served in 6 minutes.

His mum is impressed!

Should his mum be impressed? Explain.

Students often have difficulty discriminating proportional from non-proportional situations. This task encourages students to recognise direct proportionality and explain their reasoning. Note that fuel and price is the only example of direct proportionality. Students should be able to explain why the the time taken in the queue is not an example of direct proportion, and state what the time may have been if time and number of people were directly proportional.

Support

Jenny knows that 12 eggs cost £1.44. How can she work out how much 6 eggs cost? Can you think of more than one way?

- *Method 1: 6 eggs are half of 12 eggs so should cost half of £1.44, or 72p.*
- *Method 2: 1 egg costs £1.44 ÷ 12 = 12p, so 6 eggs cost 12p × 6 = 72p.*

Depth

a In which questions did you

i divide to find 1 item, and then multiply

ii use halving or doubling?

b How did you decide which method to use?

a *Answers will depend on the methods that students have used for each question.*

b *Use halving or doubling when the amount is double the other amount.*

Purposeful practice Reflect and reason

Purposeful practice 1

How can you use your answers to **Q1b** and **Q1c** to work out the answer to **Q1d**?

How can you use your answer to **Q1d** to work out the answers to **Q1f**?

*As **Q1b** gives the cost for 2 children and **Q1c** gives the cost for 4 children, then **Q1d** (the cost for 6 children) is the answer to **Q1b** + **Q1c**.*

*As **Q1d** gives the cost for 6 children, then **Q1e** (the cost for double 6, i.e. 12 children) is double. Then **Q1f** (24 children) is double **Q1e**.*

Purposeful practice 2

Sarah works out the answer to **Q1**.

1 person requires 6 ÷ 3 = 2 teaspoons

What mistake has Sarah made?

Sarah has worked out 6 ÷ 3 instead of 3 ÷ 6.

Purposeful practice 3

For **Q1**, Kamal first works out $6\mathop{\left)\vphantom{1{10.{}^40{}^40{}^40}}\right.\!\!\!\overline{\,\,\,\vphantom 1{{10.{}^40{}^40{}^40}}}}\limits^{\displaystyle\,\,\, {1 . 6 6 6}}$ then he writes

```
    1.6 6 6
×         3
_____________
_____________
```

What is an easier method for Kamal to use?

If 6 glow sticks cost £10 then 3 glow sticks are half of £10, i.e. £5.

Objectives

Core objectives

- Use ratio notation
- Reduce a ratio to its simplest form
- Reduce a three-part ratio to its simplest form by cancelling

Support objectives

- Use ratio notation

- Reduce a ratio to its simplest form

Depth key concepts

- Understand how to use ratios to make comparisons

Key points

- A **ratio** is a way of comparing two or more quantities.
  Ratios are written as numbers separated by a colon ':'
  For example, in this tile pattern there are 2 blue tiles for 1 red tile.

  The ratio of blue tiles to red tiles is 2 : 1.
- You can make the numbers in a ratio as small as possible by **simplifying**. You simplify a ratio by dividing the numbers in the ratio by the **highest common factor**.

Common errors and misconceptions

- Students sometimes write a ratio in the wrong order. Resolve this using concrete materials such as coloured counters or beads to demonstrate that a ratio of 1 : 4 is not the same as 4 : 1. Encourage students to write the items in the required ratio first (e.g. blue : red and then write the numbers underneath, to preserve the correct order.
- Students sometimes fail to write a ratio in its simplest form. Address this by suggesting that they check to see if numbers in a ratio are divisible by 2, 3, 5 or 7 (and possibly more prime numbers if necessary).

Question guidance

Core

Q6 No. Suggest students think of mixing 3 litres of white paint with 2 litres of blue paint and 2 litres of white paint with 3 litres of blue paint. *Will they be the same shade of blue? Which will be darker?*

Q8, Q9 These questions have been carefully constructed to provide visual support for the concept of simplifying ratios.

Q10, Q13 These questions have been carefully constructed to give students practice in simplifying ratios where one part is a factor of the other (Q10) before progressing to ratios which share common factors.

Q15 Suggest students write the ratio of staff to children at the nursery, then cancel it down, then compare it with 1 : 4. Make sure students explain their final answer.

Support

Ratios in tiles and bead patterns Q1 Encourage students to understand that no matter how long the pattern the ratio remains the same since the ratio represents the

relationship between the black and white beads. You may wish to create a pattern using tiles or counters and ask students to create their own with the same ratio of the colours.

Ratios in tiles and bead patterns Q3 Discuss with students whether the position of the green tiles affects the ratio.

Simplifying ratios Q2 You could challenge students to tell you how many beads there would be in each necklace if the ratio must remain constant.

Purposeful practice

Purposeful practice 1 Students need to write or draw the ratios in the correct order, so they need to read the questions carefully.

Purposeful practice 2 Check that students simplify fully, and preserve order in each ratio.

Reflect

Core

After this lesson, Miguel and Judith discussed what they noticed about ratios.

Miguel said, 'A ratio tells you how much of one thing there is compared to another.'

Judith said, 'The things must always be the same kind. So, they must all be numbers of objects, or all numbers of people, or all lengths.'

Look back at the questions you answered about ratios.

Is Miguel correct?

Is Judith correct?

What other quantities or amounts could you write in a ratio?

This task asks students to revisit questions and think more deeply about the meaning of the concept of ratio, in relation to comparison of quantities. If students are encouraged to consider the term 'ratio' and how it is used time and again, they will become more comfortable with the language associated with this new concept, and learn to recognise ratios in a range of situations.

Support

Jeremy says: 'Knowing my times tables really helps when simplifying ratios.'

Did knowing your times tables help you to simplify ratios? What other maths skills did you use?

Students' own answers. Knowing times tables should help students recognise when numbers in ratios share common factors and so can be simplified. Simplifying ratios involves division.

Depth

Tim has mixed red and white paint in the ratio 5 : 2 to create pink.

His sister Janet has mixed red and white paint in the ratio 7 : 3.

Their shades of pink look very similar, but are slightly different.

Use what you have learned in this unit to explain the difference.

Tim 35 : 14, Janet 35 : 15 so her paint has slightly more white and will look slightly paler.

Purposeful practice Reflect and reason

Purposeful practice 1

Which of your answers to **Q1** are the same ratio? Explain why.

A, ***B***, ***G*** *and* ***H*** *are the same (showing 1 : 1);* ***C*** *and* ***D*** *are the same (showing 2 : 1 for white : green);* ***E*** *and* ***F*** *are the same (showing 1 : 2 for white : green).*

Which of your answers to **Q2** are the same set of tiles? Explain why.

a *and* ***c*** *are the same (3 white and 2 grey);* ***b*** *and* ***d*** *are the same (2 white and 3 grey)*

Which of your answers to **Q3** are the same set of tiles? Explain why.

a *and* ***d*** *are the same (1 white, 2 black, 2 grey);* ***b*** *and* ***c*** *are the same (1 grey, 2 white, 2 black)*

Purposeful practice 2

Lucy says, '**Q1d, Q1g, Q1h** and **Q1j** all simplify to the same ratio, as all the answers give a ratio that includes the numbers 2 and 5.'

Is Lucy correct? Explain your answer.

No, Lucy is not correct as ***Q1d*** *and* ***Q1g*** *simplify to 2 : 5, which is a different ratio to* ***Q1h*** *and* ***Q1j*** *which simplify to 5 : 2.*

Objectives

Core objectives

- Find equivalent ratios
- Divide a quantity into two parts in a given ratio
- Solve word problems involving ratio
- Use ratios and measures

Support objectives

- Divide a quantity into two parts in a given ratio
- Solve word problems involving ratio

Depth key concepts

- Understand the multiplicative nature of ratio
- Know the relationship between km, metres (cm) and mm, litres and ml, kg and g, and understand how all units in the metric system are multiples/divisors of a 'base' unit (metres, grams, litres, etc).

Key points

- Multiplying all the numbers in a ratio by the same number gives an **equivalent ratio**.
- You can use **ratios** to convert between **metric units**.

Common errors and misconceptions

- When dividing an amount in a ratio, e.g. £12 in the ratio 2 : 3, students sometimes work out 12 ÷ 2 and 12 ÷ 3. Resolve this pictorially using a bar model to show that the ratio 2 : 3 means there are 2 + 3 = 5 parts in total, so start by sharing £12 into 5 equal parts.
- Students sometimes assume that questions which involve using a ratio to find one part when the other is known are 'share in a given ratio' questions. Address this by stressing the importance of reading the question thoroughly. Suggest students summarise the information in the question so they can then see what information they have and what they need to work out.
- Students sometimes do not know whether to multiply or divide when converting units. Resolve this by suggesting they ask themselves 'Am I going from a bigger unit to a smaller one?' If they are, there will be more of them, so they need to multiply. If they are going from a smaller unit to a bigger unit, there will be fewer of them, so they need to divide.

Question guidance

Core

Q9 Discuss whether there is more than one way to work out the answer.

Q10c Add together answers for parts **a** and **b**. The total should be 15.

Q12 This question has been carefully constructed to give students practice in working out the value of the parts when given a ratio and total.

Q14b The ratio of men to women given is for the whole of the UK. It will vary from place to place. Not all places will have this exact ratio so it can only be used as an estimate.

Q17, Q18 These questions have been carefully constructed to give practice in converting between larger and smaller units of measure in preparation for problems involving ratios of quantities.

Q22 Start by writing the ratio of Wilson : Jones for each year, and then use the ratios to work out the cost that each family pays for each year.

Support

Equivalent ratios Q1 Encourage students to recognise that two ratios are equivalent if you multiply or divide all the parts by the same value.

Using equivalent ratios to solve problems Q1a Ask students to look for patterns – is there anything they notice? Can they work out the answer without counting?

Using equivalent ratios to solve problems Q2 Challenge the students to say how they could work out how many children there are for *any* number of adults.

Purposeful practice

Purposeful practice 2 Students share totals in given ratios. Check that they write the pieces in the correct lengths according to the ratio given each time.

Reflect

Core

Look back at the questions you answered in this lesson.

For type A questions, you were given the ratio and one part and asked to find the other part (for example **Q6**, **Q7**). For type B questions, you were given the ratio and total and asked to find the parts (for example **Q10**, **Q11**).

Find two other questions for types A and B.

What maths operations (addition, subtraction, multiplication, or division) did you use to solve each type of question?

This task encourages students to identify the different types of information that may be given and asked for in ratio situations.

*Type A questions are **Q8**, **Q9**, **Q20**; type B questions are **Q12–Q14**, **Q19**, **Q22**.*

This reflective task also aims to develop students' multiplicative reasoning, by asking them to recognise the maths operations they used to answer each type of question. Students should recognise that they always multiply and divide, in particular:

- *given the ratio and one part and asked to find the other part (type A), they multiply each part by the same number to get an equivalent ratio*
- *given the ratio and total, and asked to find the parts (type B), they first of all divide (by the total number of parts), then multiply.*

Support

Write the ratio of adults to children in your classroom.

Add the parts of your ratio. What does this tell you?

Students' own answers. Check that the ratio is written in the correct order, i.e. adults : children. Adding the parts of a ratio gives the total number (of people / items / money).

Depth

Look again at the ratio in **Q16**.

Explain why it appears possible for Brian to have 6 sweets, but when looking at Amy and Chaya this is not possible.

It looks like the 4 part could increase by half to make 6, but this would turn Amy's part into 4.5 and Chaya's part into 7.5 but these are not whole numbers.

Purposeful practice Reflect and reason

Purposeful practice 1

Why does the gardener in **Q2** never plant an odd number of daffodil bulbs?

To plant an odd number of daffodil bulbs, the gardener would have to plant $\frac{1}{2}$ *a snowdrop bulb, for example, 1 daffodil bulb means planting* $3\frac{1}{2}$ *snowdrop bulbs, 3 daffodil bulbs means* $10\frac{1}{2}$ *snowdrop bulbs, etc. Ratios involve whole numbers not fractions.*

Purposeful practice 2

Sam says, 'I can use the answers to **Q2** parts **a**–**d** to predict the two lengths when 35 m is cut in the ratio 2 : 3, without having to do a calculation.'

Explain the pattern Sam has noticed.

What is 35 m cut in the ratio 2 : 3?

Each increase in length by 5 m increases the lengths of the cuts by 2 m and 3 m.

Continuing the pattern from 20 m cut in the ratio 2 : 3:

25 m cut in the ratio 2 : 3 is 10 m and 15 m

30 m cut in the ratio 2 : 3 is 12 m and 18 m

35 m cut in the ratio 2 : 3 is 14 m and 21 m

Objectives

Core objectives

- Use fractions to describe and compare proportions
- Understand and use the relationship between fractions, ratio and proportion

Support objectives

- Use fractions to describe proportions
- Understand the relationship between ratio and proportion

Depth key concepts

- Understand the relationship between ratio and proportion (what is the same; what is different)
- Understand that a ratio is simply another way of comparing parts – and how this relates to comparing parts written in fraction form

Key points

- A **proportion** compares a part with a whole. You can write a proportion as a fraction, a decimal or a percentage.
- A ratio compares a part with a part. A proportion compares a part with a whole.

Common errors and misconceptions

- When changing a ratio to a fraction, students sometimes do not write the total as the denominator, e.g. when given a ratio of boys : girls = 2 : 3, they write $\frac{2}{3}$ are boys. Address this using concrete materials such as coloured counters or beads to represent the items in the ratio.

Question guidance

Core

Q7b No; the proportion is $\frac{12}{30}$ ($\frac{2}{5}$) because there are 12 boys out of the whole class of 30 students.

Q8b, **Q9b**, **Q10b** and **Q11b** Add together the fractions in parts **i** and **ii**. They should total 1.

Q17 Students need to start by writing the fraction of albums sold on each website, i.e. $\frac{5}{14}$ and $\frac{2}{7}$. They can then change $\frac{2}{7}$ to $\frac{4}{14}$ in order to compare it with $\frac{5}{14}$.

Q18b Use the numbers given in the question and work out that there are 208 – 16 = 192 students. Write the ratio of staff : students as 16 : 192 or 1 : 12. Alternatively, write the number of staff as a fraction, $\frac{16}{208}$ which simplifies to $\frac{1}{13}$. From this students should see

that there is 1 member of staff for every 13 people, and then be able to write the staff : student ratio of 1 : 12.

Q19 Students can use either ratios or fractions to compare the squash mixes. The stronger squash is the one with the bigger proportion of squash, or the smaller proportion of water.

Challenge Students will need a partner to complete part **b**.

Support

Proportions as fractions These questions progress from dealing with one colour as a proportion of a whole (**Q1** and **Q2**) to dealing with two colours and two proportions (**Q3**) and then finding the whole as well without pictorial support (**Q4**).

Proportions as fractions Q3 Emphasise that the numerator is the number of the particular colour but the denominator is the total number of counters.

Ratios and proportions as fractions Q1b Invite students to look at how the ratio and the fractions are related.

Purposeful practice

Purposeful practice 1 and Purposeful practice 2 These questions give students practice in writing proportions which first require them to work out totals.

Reflect

Core

Write in your own words the difference between a ratio and a proportion.

Use this bar as an example to write some statements about the ratio of red to yellow and the proportion of red and yellow.

Students' own answers, but should reflect the following.

A ratio compares two or more quantities, or one part with one or more parts.

A proportion compares one part with the whole; a proportion can be written as a fraction, a decimal or a percentage.

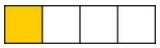
Both ratios and proportions can be simplified if the parts share common factors.

In the bar

- *the ratio of red : yellow is 4 : 8 which simplifies to 1 : 2*
- *the proportion of red is* $\frac{4}{12}$*, which simplifies to* $\frac{1}{3}$
- *the proportion of yellow is* $\frac{8}{12}$*, which simplifies to* $\frac{2}{3}$*.*

Support

Write as many different ways as possible to describe this bar mathematically.



Students' own answers. Possible answers include

- *the ratio of yellow to white squares is 1 : 3*
- *the ratio of white squares to yellow squares is 3 : 1*
- *the proportion of the squares that are yellow is* $\frac{1}{4}$ *or 0.25 or 25%*
- *the proportion of the squares that are white is* $\frac{3}{4}$ *or 0.75 or 75%*
- *the proportion of squares that are white is greater than the proportion that are yellow.*

Depth

In this lesson, you were asked questions about ratios and proportions.

Are ratio and proportion the same thing or different? Explain.

They are different. A ratio compares a part with a part. A proportion compares a part with a whole.

Purposeful practice Reflect and reason

Purposeful practice 1

Look at your answers to **Q1a** and **Q2a**; **Q1b** and **Q2b**; **Q1c** and **Q2c**, and so on.

How can you use the proportions of red and blue counters to find the ratio of red to blue counters in a game? And how can you use the ratio to find the proportions?

When the proportion is written as a fraction, the ratio is numerator of red counters : numerator of blue counters.

To find the proportions, add the two numbers in the ratio to give the denominator of the proportion written as a fraction. The ratio part for red gives the numerator of the proportion of red written as a fraction; the ratio part for blue gives the numerator of the proportion of blue written as a fraction.

Purposeful practice 2

Here are Fred's answers to **Q1**. What mistakes has Fred made?

11 am $\frac{15}{35} = \frac{3}{7}$ *2 pm* $\frac{24}{36} = \frac{2}{3}$ *5 pm* $\frac{54}{66} = \frac{9}{11}$ *8 pm* $\frac{26}{78} = \frac{13}{39}$

Fred has not used the total number of tickets as the denominator, i.e. he has worked out $\frac{number\ of\ adult\ tickets}{number\ of\ childrens\ tickets}$ *or vice versa, rather than* $\frac{number\ of\ adult\ tickets}{total\ number\ of\ tickets}$

Objectives

Core objectives

- Use percentages to describe proportions
- Use percentages to compare simple proportions
- Understand and use the relationship between percentages, ratio and proportion

Support objectives

- Use percentages to describe proportions
- Use percentages to compare simple proportions
- Understand the relationship between ratio and proportion

Depth key concepts

- Understand that a ratio is simply another way of comparing parts – and how this relates to comparing parts written in percentage form
- Understand how to decide when it is better/more efficient to use ratios or proportion to make comparisons

Key points

- You can compare **proportions** using **percentages**.

Common errors and misconceptions

- When changing a ratio to a percentage, sometimes students do not write the total as the denominator of the initial fraction. Address this using concrete materials such as coloured counters to represent the objects involved.

Question guidance

Core

Q7–Q10 These questions have been carefully constructed to give students practice in writing proportions as percentages and using these to make comparisons.

Q9 Students need to remember the formula for the area of a rectangle. In this question they are comparing the fraction shaded green in each shape, not the ratio of green to yellow.

Q10 Write each recipe as a ratio in its simplest form. Alternatively, both recipes could be written as fractions, simplified, and then changed to equivalent fractions or percentages.

Q11b, **Q12b** and **Q13b** Add together the answers from parts **a** and **b**; the total should be 100%.

Q11–Q16 These questions have been carefully constructed to give students practice in converting increasingly complex ratios to percentages.

Q17 Suggest that students change the ratio into percentages first, and then compare the medium-risk part with 70%.

Support

General Make sure students have mastered the fundamental method of converting fractions to percentages demonstrated in **Q1** and practised in **Q2** before attempting **Q3** and subsequent questions.

Proportions as percentages Q1 Students learn the multiplicative method to convert fractions to percentages.

Proportions as percentages Q3 Discuss with student how proportion can be expressed as a percentage or a fraction.

Ratios and proportions as percentages These questions are structured to emphasise the difference between ratio and proportion.

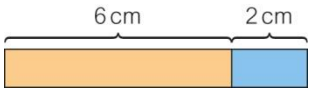
Purposeful practice

Purposeful practice 1 and Purposeful practice 2 These questions give students practice in writing proportions as fractions with different denominators which they then convert to percentages.

Reflect

Core

Use what you have learned in this unit to write five sentences that describe the ratio, proportion, fraction and percentage of the different colours in this bar.



Which word was easiest to write a sentence for? Why?

Which word was hardest to write a sentence for? Why?

This task encourages students to recognise that they have learned many different ways to describe situations using ratio and proportion in this unit. The bar is 8 cm long: 6 cm is orange and 2 cm is blue. Students may write sentences such as:

- *the proportion of the bar that is orange is greater than the proportion of the bar that is blue*
- *the proportion of the bar that is orange is 75%*
- *the ratio of blue to orange is 2 : 6 or 1 : 3*
- $\frac{3}{4}$ *of the bar is orange*

Students are asked to consider which words they found easiest and most difficult to write sentences for. This encourages students to identify what they find easy or difficult, and where they need help.

Support

Is it easier to write proportion as a percentage or a fraction?

Does your answer depend on the total number of objects?

Students' own answers, which may depend on their confidence in working with fractions.

They may say when the total number of objects is a common multiple of 10 (e.g. 10, 20, 50, 100), then it is easier to write percentages, but if the total is a 'less easy' number (e.g. 16, 24, 45), then fractions may be easier.

They may suggest that if they need to compare proportions, then percentages are easier to work with than fractions with different denominators.

Depth

Look back at the questions you have answered.

When is it better to use ratios to make comparisons?

When is it better to use proportions?

You can use ratio to compare the quantities of two different categories, such as the ratio of orange juice to apple juice in a mixed-juice drink. You can use proportion to find the quantity of one category out of the total, such as the proportion of matches lost out of the total matches played.

Purposeful practice Reflect and reason

Purposeful practice 1

Why was it easier to answer **Q2** with the proportions in **Q1** written as percentages rather than fractions?

It is easier to compare percentages than to compare fractions with different denominators.

Purposeful practice 2

How can you check your answers? (What should parts **a** and **b** add up to each time?)

Which class is most likely to be the most successful at sports day? Explain your answer.

*All parts **a** and **b** should add to 100%.*

7D because this class has the highest percentage of students who play sport.

Check, Strengthen and Extend

Core Check up

Q4 This question is designed to test that students can use the unitary method.

Q5–Q7 These questions are designed to check whether students can write ratios in their simplest form.

Q7, Q8 These questions test whether students can scale up given ratios.

Q9 This question is designed to test whether students can work out a total when given a ratio and the value of one part

Q10–Q12 These questions are designed to test whether students can share a total in a given ratio

Q13–Q15 These questions check that students can extract information from a question in order to write a proportion and write it in the required form.

Q17, Q18 These questions are designed to test whether students can rewrite information given in ratio form as a proportion.

Challenge Q1 Work out the total number of squares in the rectangle, then share this number in the ratios 1 : 2 for part **a** and 5 : 3 for part **b**. This gives the area of each section. Work out different ways that the bigger section can be drawn, then check the area of the smaller section is also correct.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

These lessons used bar models to help you solve ratio and proportion problems.

Did the bar models help you or not? Explain.

This task encourages students to think about the purpose of bar modelling, and to reflect on whether they found it useful in their own mathematical problem solving, and how.

Extend

In these lessons, you were asked questions about ratios and proportions.

Are ratio and proportion the same thing or different? Explain.

In which other subjects might understanding ratio and proportion be useful to you?

This task encourages students to consider ratio and proportion carefully. They are different things: ratios compare one part to another part; proportions compare one part to the whole. Being able to find ratios and proportions is useful to other subjects. For example, body proportion in a work of art, ratios of solutions in science, or ratios of men to women in populations in geography.

Depth

Extend

Financial ratios and proportions are used a lot by businesses to show how well they are performing.

Use what you have learned in this unit to give some examples of the types of ratio and proportion you think may be used by a business to demonstrate its performance.

Students' own answers, e.g. profit as a proportion of total annual income; ratio of customers as women : men; revenue at Christmas as a proportion of total annual revenue; ratio of sales as urban : rural.

End of Unit 7 tests: Ratio and proportion

Pearson published resources

Support/Core

- Year 7 End of Unit 7 Support/Core test
- Year 7 End of Unit 7 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 7 Core/Depth test
- Year 7 End of Unit 7 Core/Depth mark scheme

Revision

End of Term tests: Units 5 to 7

Pearson published resources

Support/Core

- Year 7 Spring End of Term Support/Core test
- Year 7 Spring End of Term Support/Core mark scheme

Core/Depth

- Year 7 Spring End of Term Core/Depth test
- Year 7 Spring End of Term Core/Depth mark scheme

Unit opener: Lines and angles

Pearson published resources

Objectives

Core objectives

- Use a protractor to measure and draw angles
- Recognise acute, obtuse and reflex angles

Support objectives

- Use a protractor to measure and draw angles

Depth key concepts

- When do you need to measure, and when can you estimate angles?
- Understand the possible types of angles on a straight line, round a point, and in shapes

Key points

- **Angle** measures a turn. Angles are measured in **degrees** (°).
- An **acute** angle is smaller than 90°. An **obtuse** angle is between 90° and 180°.
- A **reflex** angle is between 180° and 360°.

Common errors and misconceptions

- Students sometimes do not see angle as a measure of turn. Address this by demonstrating how an angle size depends on the amount of turn using electronic or other materials. You could also as a student to face the front of the room and then to turn 90° to the left (or right) to emphasise physically the link between a quarter turn and a right angle.
- Students fail to measure angles correctly with a protractor. Demonstrate using large scale or electronic equipment, emphasising the inner and the outer scales.

Question guidance

Core

General Ensure that students have seen that the two scales on a protractor go in opposite directions and should be read differently.

Q9 Ask students to discuss why you subtract the smaller angle from 360°. The angle that Isabel needs can be found by measuring the smaller angle and subtracting the result from 360°. Ensure that students understand why this gives the correct result.

Q10 These questions have been carefully constructed to give students practice in drawing acute and obtuse angles.

Q14 These questions have been carefully constructed to give students practice in drawing reflex angles. Remind them to use the same technique as in **Q9**.

Support

General Ensure that students understand that there are two scales on a protractor and know how and when to use each scale, counting up from 0° each time. Also make sure they understand how to position a protractor over an angle correctly when measuring it. They should always use a sharp pencil and ruler when drawing angles.

Measuring angles Q5 Make sure that students understand why they are measuring the smaller acute angle in order to calculate the size of the reflex angle. Consider making 360° protractors available.

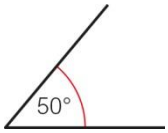
Purposeful practice

These questions give students practice in drawing and measuring different types of angle, recognising them by name and angle size, and understanding the relationships between them.

Reflect

Core

Dylan measures this angle as 130°.

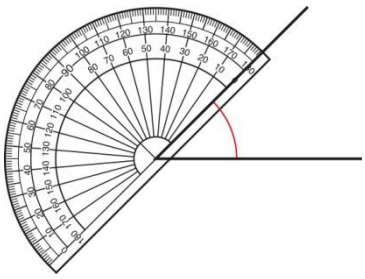


What mistake has he made?

The angle is 50°. Dylan has read the wrong scale on the protractor, using the outer scale (and measuring the angle from a horizontal out to the left) instead of the inner scale.

Support

Sam uses a protractor in this way to measure the angle. What mistake has Sam made?



Sam should have placed the protractor so its baseline was exactly on top of the baseline of the angle, with the cross exactly where the two lines forming the angle meet.

Depth

Explain why it is impossible to have two obtuse angles only

a in a right angle

b on a straight line

c around a point.

a *One obtuse angle is bigger than a right angle, so impossible.*

b *Obtuse angles are bigger than 90°, so two of them together give an angle greater than 180°, so impossible.*

c *Obtuse angles are less than 180°, so two of them together give an angle less than 360°. Another angle is required to make up the 360° around a point.*

Purposeful practice Reflect and reason

Purposeful practice 1

Use 'acute' or 'obtuse' to complete these sentences.

An _____ angle has a reflex angle between 180° and 270°.

An _____ angle has a reflex angle between 270° and 360°.

*An **obtuse** angle has a reflex angle between 180° and 270°.*

*An **acute** angle has a reflex angle between 270° and 360°.*

Purposeful practice 2

How does knowing the type of angle help you use a protractor to draw or measure it?

Knowing an angle is acute means you know it is less than 90°, so use the correct scale of the protractor for an angle less than 90°.

Knowing an angle is obtuse means you know it is between 90° and 180°, so use the scale on the protractor for an angle between 90° and 180°.

Knowing an angle is reflex means you know it is greater than 180°. Subtract the angle from 360° and draw the smaller angle.

Objectives

Core objectives

- Estimate the size of angles
- Describe and label lines, angles and triangles

- Identify angle and side properties of triangles

Support objectives

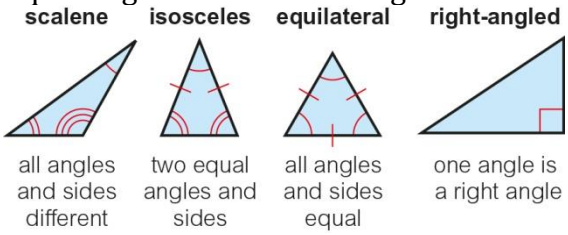
- Name and label lines, angles and triangles
- Estimate the size of angles

Depth key concepts

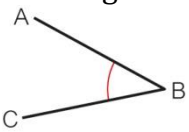
- Understand how to draw a diagram from a written instruction
- Classify triangles using more than one name, e.g. right-angled scalene or right-angled isosceles

Key points

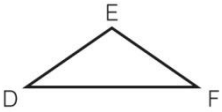
- The number of equal sides and angles can help you identify a **triangle**.
- Equal sides are marked using a dash.
- Equal angles are shown using the same number of arcs.



- This angle is called angle ABC or ${\rm{A\hat BC}}$ or $\angle {\rm{ABC}}$.



- **Perpendicular** lines are at right angles (90°) to each other.
  Parallel lines are always the same distance apart and never meet.
- You can describe a triangle using the letters at its **vertices** (the plural of **vertex**). The vertices are the corners. This is triangle DEF.



Common errors and misconceptions

- Students sometimes do not use a protractor incorrectly. Use a large scale or electronic protractor to demonstrate effective measuring.
- Students sometimes assume that all triangles have 3 lines of symmetry. Use counterexamples of triangles with obtuse angles (non-isosceles) to show why this is not the case.

Question guidance

Core

Q6 *If you add two acute angles, do you get an obtuse angle?* Encourage students to think of examples where this is true and examples where it might not be.

Q9 *Can a right-angled triangle be isosceles?* Encourage students to try and draw one.

Q12 Explain that a roof truss is a structural framework that is usually made out of timber or metal. Suggest using a protractor to check for equal angles. If students do this, ensure the protractor is being used correctly.

Challenge The task is to find as many triangles as possible on 3 × 3 square dotted paper and then on triangle dotted paper. This is an individual activity, but students could compare results. Take this opportunity to discuss whether rotations, etc. count as different triangles.

Support

Naming and describing lines, angles and triangles Q6 This question helps students identify parallel and perpendicular lines. Use it to uncover misconceptions about how things should look (such as a pair of parallel lines being the same length).

Estimating angles Students use 90° (in **Q1**) and 45° (in **Q2**) as benchmarks to estimate the size of acute angles. Encourage them to describe angles as 'a bit less/more than ...' or 'a lot less/more than ...' to help them quantify their estimates in **Q3**.

Purposeful practice

Purposeful practice 1 Students practise labelling and naming angles using different conventions, reaching the understanding that the order of letters doesn't matter as long as the vertex is the middle letter.

Reflect

Core

'Notation' means symbols. Mathematics uses a lot of notation.

For example:

= means 'is equal to' ° means 'degrees' means 'a right angle'

Look back at this lesson on angles, lines and triangles.

Write a list of all the maths notation used.

Why do you think mathematicians use notation?

Students can be put off by maths notation, seeing it as something rather 'alien' that makes maths appear difficult. Therefore, this task asks students to recognise maths notation and its importance to understanding, encouraging them to acknowledge that notation is simply shorthand, in the same way that '?' is shorthand for indicating a question. There is

notation in most subjects, for example, in computing <b> means bold, and languages such as French use accents.

Support

Is it possible to draw a triangle with two parallel sides? Is it possible to draw a triangle with two perpendicular sides?

It is not possible to draw a triangle with parallel sides; parallel sides never meet so they would never meet to form an angle in a triangle.

It is possible to draw a triangle with two perpendicular sides; this is a right-angled triangle.

Depth

a Which pairs of words can apply to a single triangle?

b Is it possible for three of these words to apply to a single triangle? Explain.

scalene isosceles right-angled equilateral

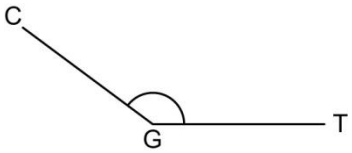
a *Scalene and right-angled; isosceles and right-angled*

b *No; a triangle can only be one of scalene, isosceles or equilateral.*

Purposeful practice Reflect and reason

Purposeful practice 1

When you label an angle like this, is the order of the letters important?



Yes, because the middle letter has to be G. But the order of C and T does not matter, as long as G is in the middle. So ∠ CGT and ∠ TGC are both correct.

Purposeful practice 2

Is an equilateral triangle isosceles?

Can a right-angled triangle be isosceles or equilateral?

Yes, an isosceles triangle has 2 equal sides, so an equilateral triangle is also isosceles.

*A right-angled triangle can be isosceles (see **Q2b**).*

A right-angled triangle cannot be equilateral as all angles in an equilateral triangle are 60°.

Objectives

Core objectives

- Use a ruler and protractor to draw triangles accurately

Support objectives

- Use a ruler and protractor to draw triangles accurately

Depth key concepts

- Understand that you can draw more than one triangle with the same angles, but different side lengths
- Given one side and two angles, understand that you can only draw one triangle

Key points

- To make an accurate drawing, you must use a ruler and protractor.
- 'Sketch' means 'draw a rough diagram'. It does not have to be accurate.

Common errors and misconceptions

- Some students are unable to draw accurately. This can be overcome with practice or, if there are physical needs for some children, with specially adapted equipment.

Question guidance

Core

Q5–Q7 These questions have been carefully constructed to give students practice in accurately drawing triangles when given diagrams of triangles with two sides and the included angle and then with two angles and the included side.

Q6d Encourage students to recognise isosceles triangles when the equal angles are not at the bottom of the triangle. Emphasise that two angles are equal as well as two side lengths.

Q11–Q14 These questions have been carefully constructed to give students practice in accurately drawing triangles when given information about side and angle measurements in words.

Challenge Students should discover that they keep creating equilateral triangles.

Support

These questions are structured carefully to give students confidence in drawing triangles when given specific measurements. Students should start by drawing the horizontal base. Ensure they place the protractor correctly measuring up from 0°. Some students may want to change the orientation of their page.

Purposeful practice

Purposeful practice 1 Students should realise that the sketches are of the same triangle but in different orientations. (This is helpful preparation for work on congruence in Unit 10.)

Purposeful practice 2 Students need to read the measurements carefully to identify the correct triangles. Encourage them to be systematic as they check each sketch.

Reflect

Core

In this lesson, you used two tools to help you draw triangles accurately – a ruler and a protractor.

Which did you find more difficult to use? Why?

Write down a hint or hints, in your own words, to help you use this tool in future.

This task asks students to consider any problems they have with using a ruler or protractor. In particular, many students find protractors difficult to use accurately. Therefore, they will benefit from reflecting on not only what it is that they struggle with, but also on how to write a hint to help them overcome their difficulties in future.

Support

Larry says: 'You should always use a sharp pencil when drawing triangles.'

Why do you think this is important?

A sharp pencil is needed when drawing triangles so that clear, sharp angles will be created. This means that the angles can be measured accurately if needed.

Depth

Liam says, 'You can draw many different triangles all with the same size angles but different length sides.'

Sam says, 'You can also draw many different triangles all with the same length sides but different angles.'

Are they both correct? Explain your answer.

Liam is correct, but Sam is not. If two triangles have the same lengths of sides, then the angles will be the same in the two triangles.

Purposeful practice Reflect and reason

Purposeful practice 1

For each question in Purposeful practice 1

- what is the same about all three triangles?
- what, if anything, is different about all three triangles?

Same: the triangles are identical in size and shape, angles, lengths.

Different: they are all in different positions/orientations/facing different ways.

Purposeful practice 2

Why is it helpful to draw and label a sketch before you draw a triangle accurately?

Drawing and labelling a sketch first helps you work out which angle and side measurements go where.

Objectives

Core objectives

- Use the rules for angles on a straight line, angles around a point and vertically opposite angles
- Solve problems involving angles

Support objectives

- Use the rules for angles on a straight line, angles around a point and vertically opposite angles

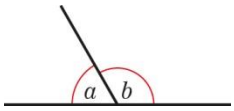
Depth key concepts

- Understand relationship between angles, e.g. there are two right angles on a straight line and four right angles around a point
- Understand that there is no limit to the number of angles around a point

Key points

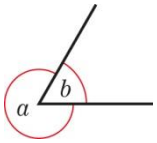
Angles are sometimes labelled with lower case letters inside the angle.

The angles on a straight line add up to 180°.
$a + b = 180°$

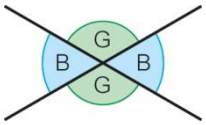


In a diagram, angles labelled with the same letter are the same size.

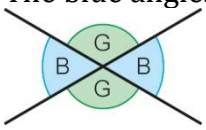
The angles around a point add up to 360°.
$a + b = 360°$



When two lines cross, they make pairs of **vertically opposite angles**.



Vertically opposite angles are equal.
The green angles are equal.
The blue angles are equal.



Common errors and misconceptions

- Students sometimes make simple errors when calculating. Ensure that students have reliable written or mental strategies for subtracting.

Question guidance

Core

Q4 Encourage students to measure accurately, and to correct their measurements when they realise the total should be 180°.

Q5, Q6 These questions have been carefully constructed to give practice in calculating unknown angles on a straight line in increasingly complex angles.

Q6 *Can you make a straight line from two acute angles? Can you have two obtuse angles on a straight line?*

Q8 Ensure that students understand how reflections work (link to mirror and torch in main teaching).

Q9 Make the link to algebra if appropriate.

Q11, Q12 These questions have been carefully constructed to give practice in calculating unknown angles around a point in increasingly complex angles.

Q15 This question has been carefully constructed to give practice in applying different angle facts to calculate missing angles.

Support

General Remind students of the right-angle symbol and that it means 90°.

Angles on a straight line Q4 Scaffolding is provided initially to provide support, but removed for parts **d–f**.

Angles around a point Q2 Again, scaffolding is provided for parts **a** and **b**, but removed for parts **c** and **d**.

Purposeful practice

Purposeful practice 1 Students practise calculating angles on straight line and around a point, using addition/subtraction and division.

Problem-solving practice Ensure that students understand that sketched diagrams are not accurate, so they should not calculate unknown angles by first measuring labelled angles.

Reflect

Core

How do you know whether to subtract from 90°, 180° or 360° when calculating angles?

Students' answers will vary. Possible answers include:

- *subtract from 90° when calculating an angle within a right angle*
- *subtract from 180° when calculating an angle on a straight line or an angle in a triangle when given the other two angles*
- *subtract from 360° when calculating an angle around a point or when calculating a reflex angle.*

Support

Copy and complete each sentence with one of the options in the box.

360° 180° equal

a Angles on a straight line add up to _____

b Vertically opposite angles are _____

c Angles around a point add up to _____

***a** Angles on a straight line add up to **180°**.*

***b** Vertically opposite angles are **equal**.*

***c** Angles around a point add up to **360°**.*

Depth

Steve likes right angles. He says he remembers 180° for a straight line because it is two right angles, and 360° around a point is from four right angles.

Ruth finds it easy to remember 180° for a straight line and then mentally joins two straight lines back to back to make 360° around a point.

Tom finds the 360° around a point easiest to remember, then cuts it in half to make two straight lines (so each is 180°) and half again to make right angles.

Who are you most like? How do you remember the size of a right angle, the angles on a straight line and the angle around a point?

Students' own answers.

Purposeful practice Reflect and reason

Purposeful practice 1

In **Q1** and **Q2** Jack says, ' a is always 90° because the same letter means the angles are the same.' Use another example from Purposeful practice 1 to show why Jack's reasoning is incorrect.

*In **Q1**, $b = 80°$ but in **Q2** $b = 60°$. Jack is incorrect because the same letter only means 'the same value' in the same diagram.*

Purposeful practice 2

In Purposeful practice 1 and Purposeful practice 2,

- in which questions did you use only addition or subtraction to find the angles?
- In which questions did you use only division?

Explain when you can use division to find missing angles.

Only addition or subtraction: Purposeful practice 1 **Q1**, Purposeful practice 2 **Q1** and **Q2**

Only division: Purposeful practice 1 **Q2** and Purposeful practice 2 **Q3**

You can use division when two or more angles have the same letter and so are the same size.

Objectives

Core objectives

- Use the rule for the sum of angles in a triangle
- Calculate interior and exterior angles
- Solve angle problems involving triangles

Support objectives

- Use the rule for the sum of angles in a triangle
- Calculate exterior angles

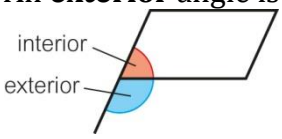
Depth key concepts

- Explore the relationship between exterior and interior angles of triangles
- Use angles in triangles to solve problems involving other shapes made up of triangles

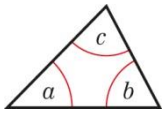
Key points

An **interior** angle is inside a shape.

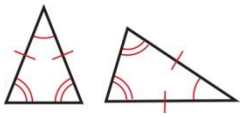
An **exterior** angle is outside the shape on a straight line with the interior angle.



The angles in a triangle add up to 180°.
$a + b + c = 180°$



In an isosceles triangle, the angles at the base of the equal sides are equal.



Common errors and misconceptions

- Sometimes students do not extend a side to form the exterior angle. Demonstrate how to do this, emphasising extending a side in a straight line.
- Students often do not subtract correctly from 180°. Give further mental arithmetic practice, or encourage students to use written methods if required.

Question guidance

Core

Q5 Cut out any triangle, mark the three angles and tear the corners off. Students will be able to see that these can always be arranged in a straight line to form 180°, thus seeing that the angles in a triangle must add up to 180°.

Q6c *Is there a quick way of working out the missing angle in a right-angled triangle?* Students should see that, in a right-angled triangle, the sum of the other two angles is always 90°.

Q6, Q7 These questions have been carefully constructed to give practice in finding unknown angles in triangles, first where diagrams are given before progressing to requiring students to sketch triangles from angle facts supplied in words.

Q9 Students divide 180° by 3 and should get 60°.

Q10, Q11 These questions have been carefully constructed to give practice in applying angle facts in order to find more than one unknown in triangles and in exterior angles.

Challenge There are two possible solutions, depending on whether the 50° is unique or one of the pair of equal angles.

Support

Missing interior angles in a triangle Q2, Q3 Scaffolding is provided for support in **Q2** and initially in **Q3** but removed for parts **b** and **c**.

Purposeful practice

Purposeful practice 1 Students practise calculating unknown angles in right-angled triangles in different orientations.

Purposeful practice 2 Students practise calculating unknown angles in scalene and isosceles triangles in different orientations.

Problem-solving practice Q5, Q6 Link to algebra.

Reflect

Core

In this lesson, you did a lot of subtracting from 180°.

What strategy did you use to mentally subtract from 180°?

Would your strategy still work to subtract 46°? If not, what strategy would you use?

Would your strategy still work to subtract a number from 360°?

If not, what strategy would you use?

This task encourages students to consider the number skills and mental strategies they use, even when learning geometry topics. They may describe a strategy to count back from 180° in tens first and then units. Students are also encouraged to consider the usefulness of their strategy when subtracting different numbers, or when subtracting from 360° (which they are required to do in the next lesson), and whether there may be a better strategy.

Support

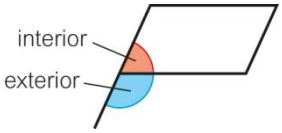
William says: 'Interior angles are on the inside of a shape and exterior angles are on the outside.'

Write a better definition, in your own words, of interior angles.

Then write a definition of exterior angles.

Draw diagrams to help with your explanations.

An interior angle is inside a shape. An exterior angle is outside the shape on a straight line with the interior angle. Interior angle + exterior angle = 180°.

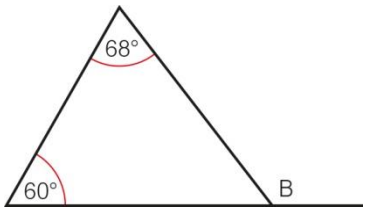


Depth

Look at the diagram.

Explain how you would work out the interior and exterior angle at point B.

Can you see a way to find the exterior angle without first finding the interior one?



You could add 60° and 68° and subtract from 180° to find the interior angle, and subtract that from 180° to find the exterior angle. So the exterior angle will be 60° + 68°, since the two 'subtract from 180°' will 'cancel' each other.

Purposeful practice Reflect and reason

Purposeful practice 1

Is there a quick way to work out the missing angle in a right-angled triangle?

Yes, you can subtract the given acute angle from 90°.

Purposeful practice 2

How many angles do you need to be given to find all the angles in a scalene triangle?

And what about an isosceles triangle?

Scalene: you need to be given two angles to be able to find the third.

Isosceles: you need to know one angle and whether it is 'the top' or 'base' angle, to be able to find the other two angles.

Objectives

Core objectives

- Identify and name types of quadrilaterals
- Use the rule for the sum of angles in a quadrilateral
- Solve angle problems involving quadrilaterals

Support objectives

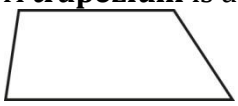
- Identify types of quadrilateral
- Use the rule for the sum of angles in a quadrilateral
- Solve angle problems involving quadrilaterals

Depth key concepts

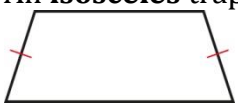
- Know that the order in which you find angles can make solving a problem more or less efficient
- Use angles in quadrilaterals to solve problems involving other shapes made up of quadrilaterals

Key points

A **trapezium** is a quadrilateral with **one** pair of parallel sides.



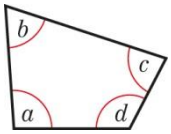
An **isosceles** trapezium is a trapezium with one pair of equal sides.



The **properties** of a shape are facts about its angles and sides.

A **diagonal** is a line joining two opposite vertices.

The angles in a quadrilateral add up to 360°.
$a + b + c + d = 360°$



Common errors and misconceptions

- Students often to not calculate correctly. Give lots of mental arithmetic practice, or encourage students to use written methods if required.
- Students often fail to remember the names of shapes. Give frequent practice and quizzes, especially for quadrilaterals.

Question guidance

Core

General Display or draw accurately a variety of quadrilaterals on the board. Ask students to identify them, and begin to explore the properties of each one.

Q4 This could be a whole-class activity. Students could create their own descriptions.

Q7 Students should become aware that the sum of the angles in a quadrilateral is 360°. Check this is understood correctly before proceeding. Encourage students to create quadrilaterals and check whether they can be cut into two triangles.

Q8 This question has been carefully constructed to give practice in applying angle facts to calculate unknown angles in quadrilaterals.

Challenge Students could do this as a paired activity to encourage discussion. The limiting factor is that two opposite angles are right angles, so a rectangle, a square, a right-angled isosceles triangle and a kite can be made. *Why can't some of the shapes be made?* Assuming that special cases of quadrilaterals are not allowed (if a square is considered as a special case of a rhombus, it can also be a special case of a trapezium or a kite, and by logical extension, any rectangle is also a trapezium) leads to these conclusions: a (non-square) rhombus is impossible because it has two pairs of angles other than right angles; a trapezium is impossible due to the perpendicular sides meaning that it is impossible to have only one pair of parallel sides.

Support

General Establish that students can identify a quadrilateral before moving on.

Properties of quadrilaterals Q2 Extend this question by asking questions such as *Can a parallelogram contain a right angle? Justify your answer.*

Calculating missing angles in a quadrilateral Q1 Check that students remember that the angles in a quadrilateral sum to 360°.

Calculating missing angles in a quadrilateral Q4 This is a slightly different way of finding the missing angle which students may find easier than the method shown in the Core lesson.

Purposeful practice

Purposeful practice These questions list different properties of quadrilaterals, to help understand and memorise these. Students will practise visualising shapes as they answer the questions.

Reflect

Core

Look back at all the lessons you have completed in this unit so far.

Write down at least two new or important mathematics words you have learned. Make sure you spell them correctly.

Beside each word, write a definition in your own words. You can draw diagrams to help.

This task asks students to revisit new or important maths words and terms. It encourages students to check their understanding by writing definitions in their own words (perhaps with accompanying diagrams). Students will suggest their own ideas but likely terminology in this unit includes:

- *types of angle: acute, right, obtuse, reflex, vertically opposite, interior, exterior,*
- *types of triangle: scalene, isosceles, equilateral, right-angled*
- *types of line: perpendicular, parallel*
- *types of quadrilateral: trapezium, isosceles trapezium,*
- *other vocabulary: vertex, vertices, properties, diagonal*

Support

Two of the angles in a quadrilateral are larger than 90°. What do you know about the other two angles? Can there be three angles larger than 90°?

Both the other two angles must be less than 90°. A quadrilateral cannot have three angles greater than 90° – it is an impossible shape.

Depth

In this section, you have used the fact that angles in a quadrilateral always add to 360°.

What additional information do you know about angles in a parallelogram, a rhombus, a kite and a trapezium?

Looking at the angles which are the same – following knowledge about parallel lines.

Purposeful practice Reflect and reason

Purposeful practice

Samira says a rhombus is a special type of parallelogram. Explain why she is correct.

Is a square a rhombus? Explain your answer.

Is a square a rectangle? Explain your answer.

Samira is correct because a parallelogram has two pairs of equal parallel sides and a rhombus also has two pairs of equal parallel sides, and all of them are equal to each other.

A square is a rhombus because it has four equal sides in two parallel pairs.

A square is a rectangle because it has opposite sides equal and all angles are right angles.

Check, Strengthen and Extend

Core Check up

Q1 This question tests students' ability to measure accurately to the nearest degree using a protractor. Watch out for students using the wrong scale.

Q3, Q4 These questions are designed to check that students can draw accurately using a ruler and protractor. Students frequently find this difficult. A suitable level of accuracy is ± 2°.

Q5 This question assesses students' ability to calculate a missing angle using knowledge of angles on a straight line. You may need to remind students that 180° is half a complete turn.

Q7 This question is designed to test students' recall of and ability to use the fact that angles in a triangle add up to 180°.

Q8 This question checks recall that vertically opposite angles are equal. Students tend to call this 'opposite' without using the 'vertical' part of the definition. (Note, however, that both angles could be calculated using angles on a straight line if b is calculated first.)

Q9This question tests knowledge that the angles in a quadrilateral add up to 360°.

Q13 This question is designed to check understanding of the properties of isosceles triangles, quadrilaterals and interior/exterior angles.

Challenge Students should be able to create every type of quadrilateral on the 4 × 4 square.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Write down one thing you find easy and one thing you find difficult when

- drawing angles accurately
- calculating angles
- solving geometric problems

For each thing you find difficult, write a hint in your own words.

This reflective task asks students to consider each topic in these strengthen lessons and list one thing that they found easy and one thing that they found difficult. For example, students may find it easy to draw acute angles, but difficult to draw reflex angles; they

may find it easy to find angles in a triangle, but difficult when there is more than one missing angle in an isosceles triangle. Then students are asked to write themselves a hint to help with those things they found most difficult.

Extend

In these extend lessons, you have answered questions about angles and

i a periscope **ii** the shadow from a tree.

Where else do you think it is useful to have an understanding of angles in maths, in science and in real life?

This task encourages students to seek and be aware of the links between angles and other maths topics, science and real life. Students may recognise that an understanding of angles can help

- *in maths, for example, when finding probabilities with spinners*
- *in science, for example, when exploring slip angles and friction of different surfaces*
- *in real life, for example, when exploring the angle of a solar panel on a rooftop.*

Depth

Extend

List all the things you need to consider when finding a missing angle.

Students' own answers, for example, the number of sides of the shape, whether the shape is regular, the properties of the shape, the values of any other angles, whether any of those angles are the same or equal, etc.

End of Unit 8 tests: Lines and angles

Pearson published resources

Support/Core

- Year 7 End of Unit 8 Support/Core test
- Year 7 End of Unit 8 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 8 Core/Depth test
- Year 7 End of Unit 8 Core/Depth mark scheme

Unit opener: Sequences and graphs

Pearson published resources

Objectives

Core objectives

- Recognise, describe and continue number sequences
- Generate terms of a sequence using a one-step term-to-term rule
- Find missing terms in a sequence

Support objectives

- Recognise, describe and continue number sequences
- Generate terms of a sequence using a one-step term-to-term rule
- Find missing terms in a sequence

Depth key concepts

- Know that the first term and term-to-term rule together define a sequence
- With just one of these, there is (an infinite) number of sequences that could be generated
- Understand that an infinite sequence doesn't necessarily tend to ± infinity, e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$

Key points

- A number **sequence** is a set of numbers that follow a rule.
  Each number in a sequence is called a **term**.
  The **term-to-term rule** tells you how to get from one term to the next in a sequence. It can use adding, subtracting, multiplying and dividing.
- Sequences where the numbers **increase** are **ascending** sequences.
  Sequences where the numbers **decrease** are **descending** sequences.
- A sequence that carries on for ever is **infinite**.
  A sequence with a fixed number of terms or a 'last term' is **finite**.
- When you know the first term and the term-to-term rule, you can work out all the terms in the sequence.

Common errors and misconceptions

- Sometimes students make basic mistakes when adding and subtracting negative numbers and/or decimals. Resolve this by giving them lots of practice calculating with negative numbers and decimals before the lesson.

Question guidance

Core

Q5 This question has been carefully constructed to give students practice in generating sequences when given the first term (positive and negative integers and decimals) and the term-to-term rule (all four operations).

Q7 This question has been carefully constructed with pairs of sequences that share the first two terms in order to demonstrate the need to look at all terms to understand the term-to-term rule.

Q10 Stress that if a sequence has a beginning and an end, it is finite.

Q13c Draw out that this is a poor mathematical model because people do not continue to grow at a constant rate throughout their lives – by the time he is 30 Rasheed will have stopped growing. Encourage students to look at the answers to parts a and b to help them answer part c.

Q18 Students could work in pairs to answer this question. A sequence beginning 1, 4, … could have a number of rules, e.g. + 3, × 2 + 2, × 4. A sequence beginning $\frac{1}{2}$, $\frac{1}{4}$, … could also have a number of rules, e.g. the denominator goes up by 2 each time or the denominator doubles each time. $\frac{1}{27}$ will not be part of the first sequence because it is increasing. $\frac{1}{27}$ may be part of the second sequence, but it would be a very complicated term-to-term rule.

Q19 This question gives students practice in working out the missing terms in increasingly complex sequences.

Support

Number sequence terms and rules These questions reinforce the basic vocabulary of sequence, term and rule, giving students practice in completing ascending (increasing) and descending (decreasing) sequences.

Number sequence problems These questions set sequence problems in the familiar context of money.

Depth

Q7 Define real numbers, 'Real numbers are all the positive and negative numbers, or zero, including fractions.' Discuss why infinity is not a real number. Consider that infinity doesn't have a precise value and address the misconception that it is just the 'biggest number'.

Purposeful practice

Purposeful practice 1 These questions provide practice with term-to-term rules using all four operations, and identifying ascending and descending sequences.

Purposeful practice 2 These questions provide practice with term-to-term rules using negative, fraction and decimal rules and terms.

Reflect

Core

Think carefully about your work on sequences. How would you define a sequence in your own words? Write down your definition. Compare your definition with someone else's in your class.

This reflective task encourages students to think carefully about what a sequence is, and so how they recognise and understand sequences. It also encourages them to acknowledge that other students may have written different definitions and that these may be just as valid.

Support

Eve says: 'Different sequences can have the same first term.'

Write two different sequences with a first term of 3 to show that Eve is correct.

Students' own answers. e.g. 3, 5, 7, 9, 11, ... and 3, 6, 9, 12, 15, ...

Riz asks: 'Does that mean these sequences share the same rule too?'

Write a sentence in response to Riz.

Students' own answers but should indicate that the answers is no, the sequences have different rules, e.g. the term-to-term rule for the first exemplar sequence '+2' but the term-to-term rule for the second sequence is '+3'.

Depth

What is always the same and what can be different about infinite sequences?

All go on forever, but some go on forever towards positive infinity; some towards negative infinity; some may tend to a particular number, e.g. zero.

Purposeful practice Reflect and reason

Purposeful practice 1

Which types of term-to-term rule give ascending sequences? Which types give descending sequences?

How can you use this to help you find the term-to-term rules in **Q3**?

*When n is a whole number, term-to-term rules with $+ n$ or $\times n$ give ascending sequences. Term-to-term rules with $- n$ or $\div n$ give descending sequences. In **Q3**, deciding first whether a sequence is ascending or descending tells you the two possible types of term-to-term rule to consider.*

Purposeful practice 2

Look at your answer to **Q3f**. When does a term-to-term rule where you multiply by a number give a descending sequence?

Does this change your answers to the Reflect and reason questions in Purposeful practice 1?

Multiplying by a number less than 1 leads to a descending sequence.

Revised answer to previous reflect is

Term-to-term rules with $+ n$ or $\times n$ *(where n is a whole number) give ascending sequences.*

Term-to-term rules with – n or ÷ n where n is a whole number, or with × m where 0 < □ < 1 give descending sequences.

Objectives

Core objectives

- Find patterns and rules in sequences
- Describe how a pattern sequence grows
- Write and use number sequences to model real-life problems

Support objectives

- Find patterns and rules in sequences
- Describe how a pattern sequence grows

Depth key concepts

- Understand that the first pattern gives the first term, and what is added each time is the term-to-term rule (for simple pattern sequences, and for more complex ones that 'grow' in two dimensions, such as the Investigation and **Q6**)

Key points

- You can draw the next pattern in a sequence by working out how the pattern grows.

Common errors and misconceptions

- Students often do not relate the 'pattern number' to the term number, instead thinking they are the same, or confusing them. Understanding the relationship is key for developing position-to-term skills. Resolve this by giving students concrete materials such as counters or blocks to make the patterns and arrange them in a sequence. Then point to the 'first', 'second', 'third', etc.

Question guidance

Core

Q5–Q7 Ask students to describe how the pattern grows. Encourage them to relate this to the term-to-term rule.

Q9b Different people may use different methods; it is important that students use a method they are comfortable with. Students could compare methods with a partner to see how others work things out, and then decide which method is more efficient.

Challenge The sequence of rotten potatoes continues 1, 8, 16, 24, 32, …, adding 8 each time (except the first one). Encourage students to view the sequence in different ways. *How could you explain to someone how the sequence grows?* For example: add 2 to each side of the square, or the perimeter of all the odd-length squares. Establish that the model would not be realistic for humans because humans move around whereas the potatoes are stationary, humans aren't packed neatly in a box, some humans might be more likely to contract the virus than others (e.g. elderly people / babies).

Support

Pattern sequences and rules These questions progress slowly, requiring students to extend patterns only on the right-hand side (**Q1** and **Q2**) before involving patterns where they need to extend in different directions (**Q3**).

Using tables for pattern sequences These questions give students practice in completing tables linking pattern numbers and numbers of shapes in each pattern, which helps prepare them for work in later lessons on tables of values when drawing graphs.

Purposeful practice

Purposeful practice 1 These questions give practice in continuing patterns, finding term-to-term rules and describing how a pattern grows.

Purposeful practice 2 These questions give practice in growing rectangle patterns, and using the pattern to predict the 10th term.

Reflect

Core

Look back over lessons 9.1 and 9.2. Sequences are shown in three different ways:

A Diagrams **B** Tables **C** Lists of numbers

Which of these ways most helped you to understand sequences and answer the questions?

This reflective task asks students to think about how different representations help them to understand, specifically sequences, and then more generally other areas of maths. Some may understand sequences better when given diagrams, others may prefer the structure of a table which connects pattern/term number with number in sequence, while others may prefer to lists in which terms are adjacent to each other.

Support

Kinga says: 'It is possible for a pattern sequence to have a subtract rule.'

Draw an example to show that Kinga is correct.

You could start with this pattern made from counters.

Write the rule for your pattern sequence.

Students' own answers; their sequences could have patterns that decrease by 1, by 2, by 3, etc. each time, illustrating term-to-term rules such as '–1', '–2', '–3', etc.

Depth

In **Q1**, how does the shape that the dots are arranged in relate to the first term and the term-to-term rule?

Describe a similar pattern sequence that begins with dots in an octagonal arrangement.

The number of sides in the shape is the first term, and the term-to-term rule is to add the number of sides.

In an octagon: first term = 8, term-to-term rule = +8.

Purposeful practice Reflect and reason

Purposeful practice 1

How can you describe how a pattern grows?

Write down the numbers of dots/squares/etc. in each pattern. Find the first term and the term-to-term rule. These tell you how many dots/lines/squares in the first pattern and how many are added each time.

Purposeful practice 2

In **Q3**, describe the sequence of the first numbers in the multiplications, and the sequence in the second numbers.

How can you use sequences to find the multiplications in **Q4**?

*In **Q3**, the first number increases by 1 each time and the second number increases by 1 each time.*

*In **Q4**, the first number increases by 2 each time and the second number increases by 1 each time – follow this pattern to write the multiplications.*

Objectives

Core objectives

- Generate and plot coordinates from a rule
- Solve problems and spot patterns in coordinates
- Find the midpoint of a line segment

Support objectives

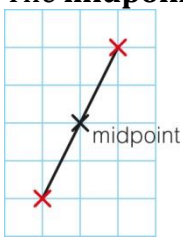
- Read and plot coordinates
- Generate and plot coordinates from a rule
- Find the midpoint of a line segment

Depth key concepts

- Know and understand that the midpoint is (mean of x-coordinates, mean of y-coordinates), just as midpoint of two numbers is the mean of the two numbers

Key points

- The **midpoint** of a line segment is the point exactly in the middle.



Common errors and misconceptions

- Some students confuse x- and y-coordinates (especially when one or the other is 0). Resolve this by giving students lot of practice before the lesson, so they become fluent. Remind them that coordinates are given as (x, y), and that x comes before y in the alphabet; to help remember which is which, they could use the aide-memoire 'in the house and up the stairs'.
- Some students confuse positive and negative coordinates. Address this by encouraging students to relate the coordinate grid to a number line (or a thermometer for the y-axis).

Question guidance

Core

Q6, Q7 These questions prepare students for the next lesson, by seeing a pattern in coordinates and the rule that generates them.

Q9 *Look at your table. How can you find the midpoint of a line segment by just looking at the beginning and end coordinates?*

Q11 Students practise finding the midpoint of a line segment with positive and negative coordinates. Ensure they use pairs of coordinates correctly when calculating the midpoints.

Support

Reading and plotting coordinates These questions give practice in reading unlabelled divisions on number lines, and reading and plotting coordinates in the first quadrant before progressing to reading divisions on negative number lines, and reading and plotting coordinates an all four quadrants.

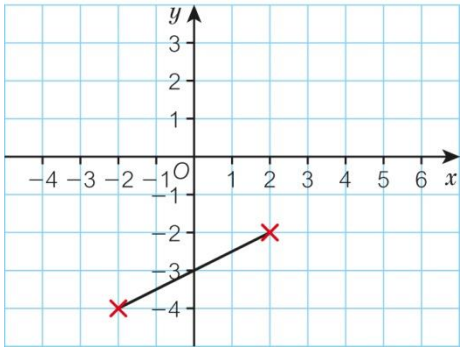
Purposeful practice

Purposeful practice 2 These questions provide structured practice in finding the midpoint of a line segment leading to understanding/stating the rule.

Reflect

Core

Patty, Sally and Dave are talking about this line.



Patty says 'One end is at the point (2, –2).'

Sally says 'The midpoint is at (–3, 0).'

Dave says 'The other end is at the point (2, 2).'

a Who is right and who is wrong?

b Write a hint on reading coordinates, in your own words, to help the students who are wrong. Check your hint. Will it stop them making their mistakes?

a *This reflective task encourages students to recognise common mistakes, including stating the x- and y-coordinates the wrong way around (Sally), and forgetting the negative sign for points below the x-axis (Dave). Only Patty is correct.*

b *Students' own answers. Their hint should reinforce the correct way to read and write coordinate points, e.g. a coordinate is written as (x, y) where x is the x-value and y is the y-value; any coordinate below the x-axis has a negative y-value and any coordinate on the left-hand side of the y-axis has a negative x-value.*

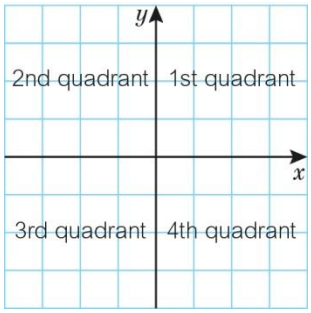
Support

Write the coordinates of a point in the

a 1st quadrant **b** 2nd quadrant

c 3rd quadrant **d** 4th quadrant

e Without plotting the point, how do you know that the point (–3, –4) is in the 3rd quadrant?



Students' own answers but should reflect the following

a *both x and y must be positive* ***b*** *x is negative, y is positive*

c *both x and y must be negative* ***d*** *x is positive, y is negative*

e *Both x and y are negative so the point must be in the 3rd quadrant.*

Depth

Ben says he can use his knowledge of averages to find midpoints of lines.

What average can Ben use?

Describe how this average can be used to find the midpoint of a line segment from (3, 8) to (5, 16).

Take the mean of the x-coordinates and the mean of the y-coordinates to give you the x- and y- coordinates of the midpoint.

$\left(\frac{1}{2}(3+5), \frac{1}{2}(8+16)\right) = (4, 12)$

Purposeful practice Reflect and reason

Purposeful practice 1

What is the same about the sequences of y-values in **Q1** and **Q2**? What is different?

What is the same about the graphs in **Q1** and **Q2**? What is different? How is the difference in the two sequences shown in the two graphs?

Sequences

- *Same: both sequences have term-to-term rule + 1.*
- *Different: **Q1** has first term 2, **Q2** has first term –2.*

Graphs

- *Same: both sets of coordinates form a straight line.*
- *Different: **Q1** the straight line starts at (0, 2), **Q2** the straight line starts at (0, –2).*

- *The different first terms give the different 'starting points' (y-intercepts) for the graphs.*

Purposeful practice 2

How can you find the coordinates of the midpoint first by looking at the coordinates of the beginning and endpoints?

Each x-coordinate of the midpoint is halfway between the two x-coordinates of the endpoints and similarly for the y-coordinates.

Objectives

Core objectives

- Describe and continue special sequences
- Use the term-to-term rule to work out more terms in a sequence
- Recognise an arithmetic sequence and a geometric sequence

Support objectives

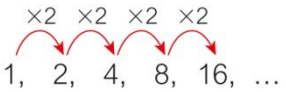
- Use the term-to-term rule to work out more terms in a sequence
- Recognise an arithmetic sequence and a geometric sequence

Depth key concepts

- Understand that when you plot an arithmetic sequence, it will always give a straight line
- Relate this to 'going up or down in equal size steps' – and this is why we sometimes call them linear sequences

Key points

- An **arithmetic sequence** goes up or down in equal steps.
  For example, the sequence 14, 11, 8, 5, 2, … goes down in steps of 3.
- You can describe an arithmetic sequence using the first term and the **common difference** (the difference between terms).
  For the sequence 14, 11, 8, 5, 2, … , the first term is 14 and the common difference is –3.
- In the **Fibonacci sequence**, the next number is found by adding the two previous numbers together. The sequence starts with 0 and 1, to give 0, 1, 1, 2, 3, 5, 8, …
- In a **geometric sequence** the term-to-term rule is 'multiply or divide by a number'.
  For example:



Each term is multiplied by 2.

Common errors and misconceptions

- Students sometimes think that any sequence that follows a rule is arithmetic e.g. 2, 4, 8, 16, … Resolve this by asking students to compare lots of different types of sequences – in addition to examples of arithmetic sequences, give them examples of geometric sequences (e.g. 1, 2, 4, 8, ... or 200, 100, 50, 25, ...), Fibonacci- type sequences (e.g. 0, 1, 1, 2, 3, 5, 8, ...), square numbers (1, 4, 9, 16, 25, ...), triangle numbers (1, 3, 6, 10, 15, ...). Those which increase or decrease in steps of different size are not arithmetic sequences.

Question guidance

Core

Q4 This question is carefully constructed to give practice in identifying arithmetic sequences from a set of increasingly complex sequences.

Q5 Make the link between the shape of the patterns and the name 'square numbers'. The sequence is not arithmetic because there is no common difference between terms.

Q7 Students identify arithmetic sequences and then work out the first term and common difference, which are skills they need when finding the nth term (covered in lesson 9.6).

Q11a Students could use two-step function machines to generate the terms of the sequences.

Q12c Encourage students to use a quicker method to answer this than writing all the terms in between. They could do this mathematically or they could draw diagrams of the first and the last pattern, and decide how many beams there are between them.

Q15 This question is carefully constructed to give practice in identifying whether a sequence is arithmetic, geometric or neither.

Challenge *How did you solve parts* ***a*** *and* ***b****?*

Support

Arithmetic sequences These questions introduce the term arithmetic sequence and give practice in working with arithmetic sequences by writing the first term and finding the common difference.

Geometric sequences These questions give practice in distinguishing geometric sequences from other sequences, and in continuing geometric sequences when given the first term and term-to-term rule.

Sequences with two-step rules This question gives structured practice in using function machines to generate the terms of sequences with 2-step term-to-term rules.

Purposeful practice

Purposeful practice 1 Students identify arithmetic sequences and their term-to-term rules, and use given rules and first terms to generate arithmetic sequences.

Purposeful practice 2 Students use number patterns to find the 10th term of different sequences without finding all terms in between.

Reflect

Core

Hassan says this riddle about a sequence:

'My first term is 3. My third term is 27. I am geometric. What is in between?'

Write down every step you take to work out Hassan's sequence.

You might begin, 'I write down all the numbers I know, leaving gaps for any missing numbers: 3, ___, 27, ...'

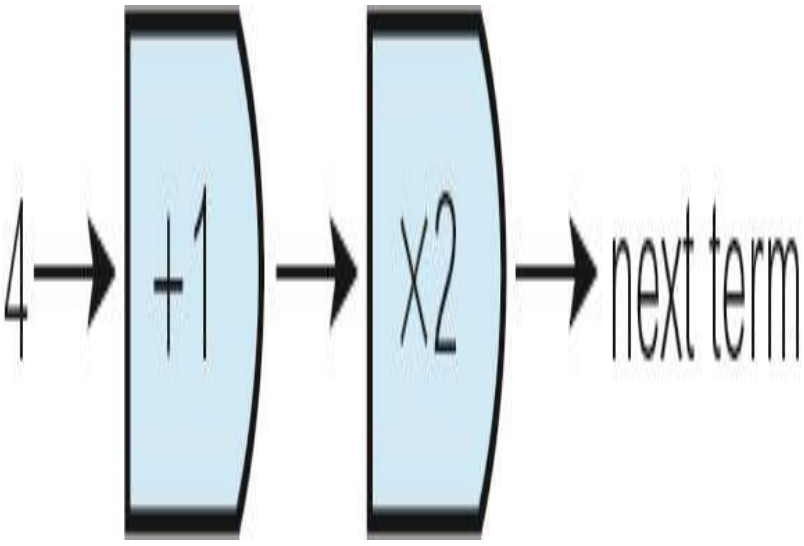
Students' own answers, for example

- *I write down all the numbers I know, leaving gaps for any missing numbers: 3, ___, 27, ...*
- *the sequence is geometric so I know that I need to multiply 3 by a number(?) to get the 2nd term (T). ? is the number I need to multiply T by to get the 3rd term (27). 3 × ? = T and T × ? = 27*
- *using inverse operations, this means 27 ÷ ? = T*
- *3 × ? = T = 27 ÷ ?*
- *so ? = 3, giving the sequence 3, **9**, 27*

Support

The first term of a sequence is 4 and the term-to-term rule is 'add 1, then multiply by 2'.

Draw a function machine and write the first three terms of this sequence.



First three terms are: 4, 10, 22

Joe says that this is the same as a sequence with the term-to-term rule 'multiply by 2, then add 1'.

Show that Joe is incorrect.

The first three terms of Joe's second sequence are: 4, 9, 19. The sequences are different so Joe is incorrect.

Depth

Write three facts you know about arithmetic sequences.

Compare your facts with others in your class.

Students' own answers. Answers may include

- *in an arithmetic sequence the terms increase or decrease in steps of equal size*
- *the graph of an arithmetic sequence is linear*
- *you can describe an arithmetic sequence using the first term and the common difference.*

Purposeful practice Reflect and reason

Purposeful practice 1

How can you tell from its term-to-term rule if a sequence is arithmetic?

Term-to-term rules that only involve addition or subtraction give arithmetic sequences.

Purposeful practice 2

For **Q1–Q5**, Hayley says, 'The 10th term is always first term + 9 × common difference.'

Use examples from **Q1–Q5** to show she is correct.

*For example in **Q1**, the first term is 1, the common difference is 3. 10th term = 1 + 9* × 3.

Objectives

Core objectives

- Recognise, name and plot graphs parallel to the axes
- Recognise, name and plot the graphs of $y = x$ and $y = -x$
- Plot straight-line graphs using a table of values
- Draw graphs to represent relationships

Support objectives

- Recognise, name and plot graphs parallel to the axes
- Recognise, name and plot the graph of $y = x$
- Plot straight-line graphs using a table of values

Depth key concepts

- Understand that the equation of a straight line is a function that generates a y-value for every x-value, and when you input $x = 1, 2, 3,$ (consecutive terms) into the function, the y-values form an arithmetic sequence

Key points

- The equation $y = 2$ means that the y-coordinate is always 2, whatever the x-coordinate is. The line is **parallel** to the x-axis.
  The equation $x = 3$ means that the x-coordinate is always 3, whatever the y-coordinate is. The line is parallel to the y-axis.
- When you draw a graph, it should go to the edge of the grid. Label your graph by writing the equation next to the line.

Common errors and misconceptions

- Some students think that the graph of $y = a$ is parallel to the y-axis, and that $x = b$ is parallel to the x-axis. They do not realise that the graph of $y = a$ crosses the y-axis and is parallel to the x-axis, and vice versa. Encourage students to look for points on vertical and horizontal lines to check their answers. Establish that every point on the line $y = 3$ has 3 as its y-coordinate.
- Students sometimes think that when $n = 4$, $3n = 34$. Give students lots of practice manipulating algebra, so they become fluent. If necessary, provide concrete materials such as counters or beads to demonstrate that $3n$ means 3 lots of n.
- Students don't always draw graphs to the edges of the grid. Encourage them to do this even when the table of values does not extend that far.

Question guidance

Core

Q5b Students will notice that the y-coordinate is always 3. A clearer way of writing this is $y = 3$.

Q6 Students can check their equations by writing down several coordinates and identifying the pattern, e.g. all the x-coordinates are -3 therefore the equation of the line must be $x = -3$.

Q7 If students are not able to immediately identify the graph suggest that they plot several coordinates first, using that they have already learned. They can then join these points with a straight line.

Q8, Q10, Q11 These questions are carefully constructed to give practice in drawing coordinate grids, plotting points, drawing graphs and interpreting the graphs.

Q9 This reasoning question cements understanding of the relationship between a table of x and y values and the corresponding graph.

Q11c Make sure students draw the graphs on the same axes, or close together, to ensure they notice that the lines in **Q8**, **Q10** and **Q11** are parallel.

Q13 *What is the same and what is different about the graphs of* $y = x$ *and* $y = -x$? Where one has positive x-coordinates, the other has negative x-coordinates, and vice versa.

Challenge This leads into Hooke's law of elasticity. The extension of the spring is directly proportional to the force applied to it, up to its elastic limit (where the force applied causes the spring permanent stress). This is where the graph is no longer linear, and can be used as a discussion point.

Support

Graphs parallel to the axes and $y = x$ Students plot and draw horizontal and vertical lines and learn to recognise that the equations of these lines are of the general form $y = b$ and $x = a$.

Drawing a table of values and plotting straight-line graphs Students use the familiar model of the function machine to generate tables of values for given functions and then to plot points and draw graphs of the functions.

Purposeful practice

Purposeful practice 1 Students practise naming vertical and horizontal lines and realise that the x- and y-axes have equations $y = 0$ and $x = 0$, respectively.

Purposeful practice 2 Students recognise the features of the graphs of $y = x$ and $y = -x$, and complete tables of values and draw graphs of given equations.

Reflect

Core

Which of these did you find easiest? Which did you find hardest? Explain.

A Completing a table of coordinate pairs.

B Drawing a straight-line graph.

C Reading coordinates from a straight-line graph.

This reflective task encourages students to reflect on the aspects of graphs that they find easy and hard, and so where they may require more help or work to improve understanding. In doing so, it urges students to take responsibility for their own learning.

Support

Look back at your graphs.

How can you tell, by looking at your graph, if you have made a mistake in the table of values?

Students' own answers, e.g. if the graph line is not a straight line, then there is probably a mistake in the table of values.

Depth

In this lesson, you have drawn a lot of linear (straight-line) graphs.

A graph of linear function has *only one* y-coordinate for each x-coordinate.

Look back at the linear graphs you have drawn in this lesson.

Give two examples of equations of graphs of linear functions.

Give two examples of equations of graphs that are not linear functions. Explain.

Students' own answers, for example

- *linear:* $y = 2x - 3$; $y = 6 - x$
- *non-linear:* $x = -2$; $x = 4$; *for each of these graphs there is more than one* y*-value for every* x*-value, so these are non-linear functions.*

Purposeful practice Reflect and reason

Purposeful practice 1

Jim says, 'The equation of line H in **Q1** is $y = -3$, because it is parallel to the y-axis.'

Explain why Jim is incorrect.

All the x*-values of points on the line are –3, so the line is* $x = -3$*. Graphs with equation* $x = a$ *number are parallel to the* y*-axis.*

Purposeful practice 2

In **Q1**, what is the same and what is different about the graphs of $y = x$ and $y = -x$?

Same: both $y = x$ *and* $y = -x$ *pass through (0, 0) and divide each quadrant 'in half' with diagonal lines.*

Different: $y = x$ *slopes up from left to right whereas* $y = -x$ *slopes down from left to right.*

In **Q2**, what is the same and what is different about the graphs of $y = 2x$ and $y = 2x + 1$?

Same: both $y = 2x$ *and* $y = 2x + 1$ *cross the* y*-axis at* $y = 1$*; also* $y = 2x$ *and* $y = 2x + 1$ *are parallel and have the same slope (gradient). Students should be able to explain this even though then have not yet met the word 'gradient'.*

Different: for the graph of $y = 2x + 1$, the y*-values are 1 more than the* y*-values for the graph of* $y = 2x$ *for corresponding* x*-values.*

Objectives

Core objectives

- Generate terms of a sequence using a position-to-term rule
- Use linear expressions to describe the nth term of simple sequences

Support objectives

- Generate terms of a sequence using a position-to-term rule
- Use linear expressions to describe the nth term of simple sequences

Depth key concepts

- Understand the connection between: nth term, term-to-term rule or common difference and first term (arithmetic sequences only)

Key points

- Each term in a sequence has a position. The 1st term is in position 1, the 2nd term is in position 2, the 3rd term is in position 3, and so on.
  The **position-to-term** rule tells you how to work out a term in a sequence when you know its position.
- You use algebra to write the position-to-term rule.
  It is called the ***n*th term** because it tells you how to work out the term at position n (any position).

Common errors and misconceptions

- Some students confuse position-to-term rule and term-to-term rule, as the same operation could be used in both, e.g. a position-to-term rule could be $5n$ and a 'similar' term-to-term rule could be ×5. Give students lots of practice with sequences asking them to give both types of rule, to show how they are different.
- Try to use sequences of bigger numbers, so that students are less likely to confuse step size and term, e.g. 23, 33, 43, 53, … or 27, 34, 41, 48, …

Question guidance

Core

Q4 Encourage students to describe things in term of 'shape number' or position, moving away from term-to-term rules. If students are struggling, remind them that the position numbers are the counting numbers: 1st, 2nd, 3rd, ... *What is the relationship between the term-to-term rule and the position-to-term rule?* Draw out that the term-to-term rule tells you what number the position number is multiplied by in the position-to-term rule.

Q5, Q6 These questions are carefully constructed to give students practice in dealing with sequences when given the nth or general term, initially finding the first five terms of a sequence and then using the nth term to find the 10th term.

Q9–Q12 These questions are carefully constructed to give students practice in finding the nth term for a range of sequences.

Q12 Allow students to write the nth term as, e.g. $7 \times n$, and move on to more formal algebraic representations when they are comfortable with the idea of the nth term.

Support

General The jump to algebra can be quite daunting for some students.

Finding terms of a sequence using the position-to-term rule Q1 Check that students understand the vocabulary *term* and *position*.

Finding the nth term These questions provide structured practice in finding the position-to-term rule and making the connecting to nth term.

Purposeful practice

Purposeful practice 2 These questions give structured practice in using the position-to-term rules of related sequences to generate the sequences and compare the common differences, leading to the understanding that the common difference is the coefficient of n in the rule.

Reflect

Core

Think about the term-to-term rule and the position-to-term rule for the 5 times table.

Which rule is '×5' and which is '+5'?

Make sure that you know the difference between the term-to-term rule and the position-to-term rule.

Write a hint, in your own words, to help you to remember which is which.

This reflective task gives students the opportunity to clarify (and so avoid confusion between) the term-to-term and position-to-term rule.

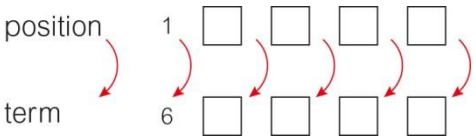
The position-to-term rule is ×5 and the term-to-term rule is +5.

Students will write their own hints to remember which is which, but may include

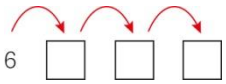
- *position-to-term rule starts with a term's position in a sequence and describes how to get from the position number to the term*
- *term-to-term rule starts with the first term in the sequence, and it describes how to get from one term to the next*

Support

Write the first five numbers in the 6 times-table. Now you have a sequence. Write the position above each number. Is the position-to-term rule '×6' or '+6'?



What is the term-to-term rule?



Make sure you understand the position-to-term and term-to-term rules.

Make some notes to help you remember.

position 1 2 3 4 5

term 6 12 18 24 30

The position-to-term rule is '×6'. The term-to-term rule is '+6'. The position-to-term rule describes how to get from the position number to the term in the sequence. The term-to-term rule describes how to get from one term to the next.

Depth

Which information is enough to allow you to work out *all* the terms in a sequence?

the position-to-term rule the general term the first term the common difference

the 100th term the first term and the term-to-term rule

The first term and the term-to-term rule, the general term or the position-to-term rule.

Purposeful practice Reflect and reason

Purposeful practice 1

What is the nth term for the multiples of 2? What is the nth term for the multiples of 9?

What sequence has nth term $5n$?

2n; 9n,

Multiples of 5

Purposeful practice 2

What do you notice about the numbers in the sequences in **Q2**?

Which of these sequences are related to multiples of 5? Explain your answer.

A 5, 12, 19, 26, 33, ... **B** 8, 13, 18, 23, 28, ...

C 12, 17, 22, 27, 32, ... **D** 5, 9, 15, 19, 25, ...

*In **Q2**, the sequences are related to multiples of 5 – there is a common difference of 5, and the ones digits follow a repeating pattern.*

***B**, **C**; student's own explanation, e.g. the sequences that go up in 5s – because sequences with 5n in the nth term go up in 5s.*

Check, Strengthen and Extend

Core Check up

Q2 This question has been designed to show students that sequences can generate certain sets of numbers.

Q3–Q5 These questions have been designed to check that students can identify term-to-term rules and/or continue sequences when given the first term and term-to-term rule.

Q5e This question part is designed to test whether students can identify sequences as arithmetic, geometric or neither.

Q6 This question tests whether students can work out a term-to-term rule in order to find the missing numbers in a sequence.

Q7, Q8 These questions are designed to test that students can use the nth term to find specified terms in a sequence.

Q9 This question tests students' understanding of position-to-term and nth term, i.e. they are the same rule.

Q13, Q14 These questions check that students can recognise, draw and label graphs parallel to the x- and y-axes.

Q11 This question has been designed to highlight the misconception of confusing nth term and term-to-term rule.

Q15 This question tests students ability to complete a table of values, use this to draw a graph and then find a pair of values from the graph. (Some students may also wish to test their answer by solving the equation, but this is beyond the scope of this unit.)

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Lars says, 'The words *term* and *coordinates* are used a lot in these lessons.

These must be important words for understanding sequences and graphs.'

Write definitions, in your own words, for *term* and *coordinates*.

This reflective task encourages students to notice the most common maths words in a unit, and so the most important words to understand. In this case, the words are 'term' and 'coordinates'. Students are asked to define them in their own words, to reinforce their meaning. Students' definitions will vary but they may write definitions such as

- *A term in a sequence is a number or pattern in the sequence. For example, 1, 3, 5, 7 are the first four terms in the sequence of positive odd numbers.*
- *A coordinate describes a point on a graph; it takes the form (x, y), where x is value on the x-axis and y is the value on the y-axis.*

Extend

Sandra says, 'Sequences and straight-line graphs are all about following patterns.'

Look back at the work you have done in this unit. Write three sentences that describe how what you have learned is all about 'following patterns'.

This reflective task asks students to bring together everything they have learnt in this unit and consider how it has been about following patterns. Therefore, students may write statements such as

- *sequences in diagrams add a predictable number of tiles each time, so follow a pattern*
- *a term-to-term rule is all about applying the same pattern to make a sequence*
- *coordinates of straight-line graphs add the same to the x-value each time, and the same to the y-value each time, so they follow a pattern.*

Depth

Extend

List all the words and phrases you can think of that you can use to describe a sequence of numbers. You should have at least eight words and phrases in your list.

Compare your list with others in your class.

Students' own answers. Possible answers include

arithmetic sequence, ascending, descending, Fibonacci sequence, first term, general term, geometric sequence, nth term, position-to-term rule, term-to-term rule.

End of Unit 9 tests: Sequences and graphs

Pearson published resources

Support/Core

- Year 7 End of Unit 9 Support/Core test
- Year 7 End of Unit 9 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 9 Core/Depth test
- Year 7 End of Unit 9 Core/Depth mark scheme

Unit opener: Transformations

Pearson published resources

Objectives

Core objectives

- Identify congruent shapes
- Use the language of enlargement
- Enlarge shapes using given scale factors
- Work out the scale factor given an object and its image

Support objectives

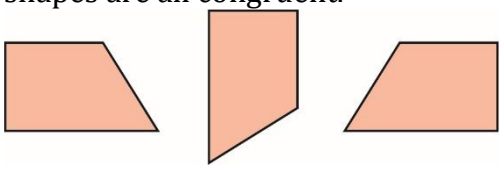
- Identify congruent shapes
- Enlarge shapes using given scale factors
- Work out the scale factor given an object and its image

Depth key concepts

- Understand the language of 'scale factor' – scale relating to scaling up/down and multiplicativity; factor relating to one measure being divisible by another (also about multiplicativity)
- Understand how ratio and enlargement relate to each other (including side lengths and perimeter and area)
- Know that in enlargements, angles in shapes remain unchanged.

Key points

- Shapes are **congruent** if they are the same shape and size. For example, these shapes are all congruent.



- In congruent shapes, **corresponding sides** (matching sides) and **corresponding angles** (matching angles) are equal.
- An **enlargement** transforms a shape. It is a type of **transformation**.
  When enlarging a shape, you multiply all its side lengths by the same number.
  The number you multiply by is called the **scale factor**.
- The original shape is called the **object**.
  The enlarged shape is called the **image**.

Common errors and misconceptions

- Some students confuse congruence and similarity or think they are the same. This unit does not cover similarity, so focus on congruence and give plenty of practice using sets of shapes (some congruent, some not) to reinforce the concept of congruence.
- Sometimes students do not understand enlargement as a multiplicative relationship. Address this by using dynamic geometry software or similar electronic images that model enlargement by a scale factor to demonstrate the effect of multiplying by a scale factor on all sides of a shape.

Question guidance

Core

Q4c There are two possible pairs of congruent rectangles.

Q13 In addition to using the scale factor of 10, students need to divide by the correct power of 10 to give their answers in the specified units.

Q14 This links to ratio, as students need to connect the scale factor of the enlargement to the multiplicative changes in length.

Q17 Ensure that students do not confuse perimeter and area here, as the relationships are different.

Challenge This uses congruence in the context of flags. Some students will not be aware that the blue triangles in the Union Jack are not all congruent. The Welsh flag has two congruent rectangles, although they are different colours.

Support

Congruence Emphasise that congruent shapes must be the same shape and size. This will help students understand that reflections, rotations and translations of a shape give a congruent shape, but enlargements do not. Use **Q2** to check for misconceptions about congruence.

Enlargement Emphasise that when a shape is enlarged, the angles remain the same size, even though the sides change length. Reinforce that the ratio of sides does not change. Suggest students use squared paper when drawing shapes to make it easier for them to count squares.

Enlargement Focus on the Worked example and discuss why we only find the side lengths that are along the grid lines – because the diagonals are hard to measure accurately. Students can check on their enlargement to confirm that the sloping sides are also multiplied by same scale factor.

Purposeful practice

Purposeful practice 2 Students practise finding corresponding sides and corresponding angles in pairs of congruent shapes.

Purposeful practice 3 The shapes being enlarged are increasingly complex, with the shape in Q3 involving diagonal sides and a non-integer scale factor.

Reflect

Core

Look back at this lesson.

List all the new mathematical words you learned. Be careful to spell them correctly.

Write a short definition for each of them. Where possible, use your own words.

Draw your own shape or shapes with each definition, to show what you mean.

This task asks students to identify (and ensure they can spell) any new mathematical vocabulary they have met in this lesson. For example, students may choose the words congruent, object, image, enlargement. Students are encouraged to demonstrate and embed their understanding by not just writing a definition, but also drawing shapes to show what they mean.

Support

Lily writes:

Congruent shapes must be the same shape, the same size and the same way up.

Explain what is wrong with Lily's statement.

Congruent shapes must be the same size (and therefore shape) but they can be in different orientations, so they don't need to be the same way up.

Depth

a Two shapes are congruent.

What is definitely the same about them and what might be different?

b One shape is an enlargement of another.

What is definitely the same about them and what might be different?

a *Corresponding angles are the same, corresponding lengths are the same. They may be in different positions, or at different orientations (rotated), or one might be a reflection of the other, or some combination of these.*

b *Corresponding angles are the same, and the ratio of corresponding side lengths is the same. Corresponding lengths are (usually) different (only the same if the scale factor is 1); they may be in different positions, or at different orientations (rotated), or one might be a reflection of the other, or some combination of these.*

Purposeful practice Reflect and reason

Purposeful practice 1

Which of these statements about congruent shapes are true? They

A have the same angles **B** are the same colour **C** are the same size ***A*** *and* ***C***

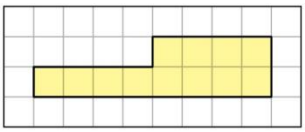
Purposeful practice 2

How did tracing shapes 2 and 4 help you find corresponding sides and angles?

Corresponding sides and corresponding angles are equal.

Purposeful practice 3

Henry says, 'This shape is an enlargement of shape A in **Q1** by scale factor 2.'



Is he correct? Explain your answer.

No, Henry is not correct. He has not multiplied all sides by scale factor 2.

Objectives

Core objectives

- Recognise reflection and rotational symmetry in 2D shapes
- Solve problems using line symmetry
- Identify all the symmetries of 2D shapes
- Identify reflection symmetry in 3D shapes

Support objectives

- Recognise line and rotational symmetry in 2D shapes
- Identify all the symmetries of 2D shapes
- Identify reflection symmetry in 3D shapes

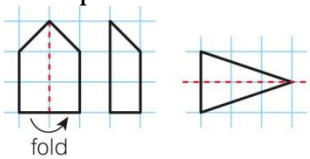
Depth key concepts

- Understand the symmetries of 3D solids and the shapes of their planes of symmetry

- Understand the relationship between rotational and line symmetry in regular polygons

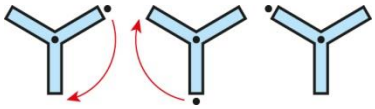
Key points

- A shape has **reflection symmetry** if one half folds exactly on top of the other half.



The dashed line is called a **line of symmetry** or mirror line.

- A shape has **rotational symmetry** if it looks the same more than once in a full turn. This shape looks the same in three positions, so it has rotational symmetry of **order** 3.



A shape that does not look the same when turned has no rotational symmetry.

- If a 3D shape has reflection symmetry, the mirror line is called a **plane of symmetry**.

Common errors and misconceptions

- Some students wrongly mark diagonals as lines of symmetry. Address this by tracing a rectangle (or other shape) and folding along the diagonal, or using dynamic geometry software.
- Some students forget that every shape has rotational symmetry of at least order 1. Make it clear that a shape with rotational symmetry of order 1 does not have rotational symmetry; for a shape to have rotational symmetry it must look the same *more than once in a full turn* and so will have rotational symmetry of order 2 or more. Have a discussion around whether it is possible to have rotational symmetry of order 0.
- Students need to be clear about the difference between line of symmetry (reflection symmetry), rotational symmetry, order of rotational symmetry and plane of symmetry. Model the correct terminology at all times, emphasising dimensions when appropriate.

Question guidance

Core

Q4 *Does a person's face have line symmetry?*

Q5, Q6 These questions have been carefully constructed to give students practice in identifying lines of symmetry in irregular shapes and then drawing them on familiar quadrilaterals.

Q8 From **Q4d**, students should realise that a circle has many lines of symmetry. Encourage them to use the term 'infinite'.

Q9–Q11 These questions have been carefully constructed to give students practice in identifying rotational symmetry in shapes, initially marking the point of rotation on diagrams, before progressing to giving only the names of shapes.

Q9 This introduces rotational symmetry for the first time.

Q10 *Which shapes have no rotational symmetry?* Draw out the idea that shapes without rotational symmetry have rotational symmetry of order 1. Ensure that students understand this.

Q12 Introduces planes of symmetry for the first time.

Q13 Students should be able to visualise a cylinder and a dice.

Q14 Check that students understand that the three 'legs' of the ammonia molecule are equally spaced and of equal length.

Support

Line symmetry Q1 Students may benefit from using a mirror to check if the dashed line is symmetrical. It is important they learn that lines of symmetry are not always horizontal or vertical. In part **c** it is a common error to assume the diagonal is a line of symmetry.

Rotational symmetry Make tracing paper (or fairly see-through paper) available to students.

Rotational symmetry Q2 Use part **b** to show that a shape can have rotational symmetry, but no lines of symmetry.

Symmetry in 3D shapes If possible, provide cuboids that students can look at or even cut (e.g. cheese), to see the different planes that 'cut through' the shape.

Purposeful practice

Problem-solving practice Q2, Q4, Q5 Use these questions to check whether students understand the difference between line of symmetry and rotational symmetry.

Reflect

Core

After this lesson, Evan said, 'A trapezium is *always* symmetrical.'

Robyn said, 'A trapezium is *never* symmetrical.'

Claire said, 'A trapezium is *sometimes* symmetrical.'

Who is correct, Evan, Robyn or Claire? You could draw diagrams to help you.

Use what you have learned in this lesson, and what you know about trapeziums, to explain.

This task ensures students have a good understanding of the meaning of the term 'symmetry'. It also tackles a common misconception that a trapezium always has one line

of symmetry. Students may choose to draw one symmetrical trapezium (showing the line of symmetry), and one non-symmetrical trapezium to explain why Claire is correct.

Support

a What objects in the classroom have no lines or planes of symmetry?

b What objects in the classroom have exactly one line or plane of symmetry?

Students' own answers, depending on the objects in their classroom.

Depth

a Does a shape that has lines of symmetry always have rotational symmetry of order greater than 1?

b Does a shape which has rotational symmetry of order greater than 1 always have lines of symmetry?

a *A shape which has one line of symmetry does not need to have rotational symmetry (e.g. a kite), but once a shape has more than one line of symmetry passing through a single point, it will have rotational symmetry.*

b *A shape which has rotational symmetry does not need to have lines of symmetry, as shown in* ***Q8****.*

Purposeful practice Reflect and reason

Purposeful practice 1

Leo draws lines of symmetry on this rectangle and parallelogram.

Are his lines of symmetry on these shapes correct? Explain how you know.

No, Leo's lines of symmetry are not correct. One part of the shape does not exactly fold on to the other part of the shape if the rectangle is folded along Leo's diagonal dotted lines, or any of the dotted lines on the parallelogram.

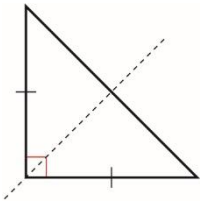
Purposeful practice 2

Is it possible to draw a right-angled triangle with one line of symmetry?

Is it possible to draw a right-angled triangle with rotational symmetry of order 3?

Draw examples to explain your answers.

Example of a right-angled triangle with one line of symmetry:



It is not possible to draw a right-angled triangle with rotational symmetry of order 3 since all three sides must be the same length for this to be possible and so all angles must be 60°.

Purposeful practice 3

How can lines of symmetry help when finding missing sides and angles in an isosceles triangle?

An isosceles triangle has one line of symmetry that cuts it exactly in half. The sides and angles in each half of the isosceles triangle are the same.

Objectives

Core objectives

- Recognise and carry out reflections in a mirror line
- Reflect a shape on a coordinate grid
- Describe a reflection on a coordinate grid

Support objectives

- Recognise and carry out reflections in a mirror line
- Reflect a shape on a coordinate grid
- Find the mirror line for a reflection on a coordinate grid

Depth key concepts

- Identify patterns/rules in coordinates of vertices when a shape is reflected in different straight lines on a coordinate grid

Key points

- A **reflection** is a type of transformation.
  You reflect shapes in a mirror line.
  All points on the image are the same distance from the mirror line as the points on the object, but on the opposite side.
- **Lines of reflection** (or mirror lines) on coordinate grids can be described by their **equations.**

- When you reflect a shape in the line $y = x$ or $y = -x$, count the distance from each vertex of the shape to the line, then count the same distance again on the other side of the line.

Common errors and misconceptions

- Some students make errors in reflections of non-vertical and horizontal lines. These can be modelled using dynamic geometry software.
- Common misconceptions around reflecting in the line $x = y$ can be addressed by using tracing paper.

Question guidance

Core

Q5 These questions will expose common misconceptions about the distance between the mirror line and the image and the orientation of the image. Address these issues by modelling using tracing paper or dynamic geometry software. *Is your body symmetrical?* Humans are largely symmetrical externally, but not precisely so. Images created by using mirror imaging software make the human face look androgynous. Internally, humans are very asymmetric, e.g. the heart is on the left side.

Q6 *When a shape is reflected in a mirror line, what can you say about the perimeter of the shape and its reflection? What about the area of both?*

Q7 *How do you reflect a point on the mirror line?*

Q8–Q12 These questions give structured practice in reflecting shapes in lines of reflection and identifying the equations of horizontal and vertical lines of reflection before progressing to using diagonal lines of reflection.

Q11 Students identify which shapes have been reflected in which lines. Look out for students misnaming $x = -1$ as $x = 1$.

Q12 Place a mirror on the lines $y = x$ and $y = -x$ and angle it in different ways to demonstrate that the distance between the object and the mirror and the image and the mirror remains the same.

Challenge This activity investigates how the coordinates are related when reflecting through $y = x$ and $y = -x$, i.e. (0, 3) becomes (3, 0) in $y = x$ and (–3, 0) in $y = -x$.

Support

Reflections in horizontal and vertical mirror lines Q1 Students should take care to check that the distance from the reflection line to the reflected shape is the same as the original.

Reflections in horizontal and vertical mirror lines Q2 Showing incorrect options allows students to realise what the words ‘same distance from the mirror line’ mean.

Reflections on a coordinate grid Q1 This question provides purposeful practice in reflecting shapes in the same line with slight variation between each shape and the mirror line.

Purposeful practice

Purposeful practice 2 Q2 Check that students reflect in the correct lines, especially $y = -x$ and $x = -1$ (the latter is not drawn on the coordinate grid given).

Reflect

Core

a Look back at **Q5**.

Write down the steps you took to draw the reflected images.

You might begin with:

Step 1: I found the mirror line.

b Look back at **Q12**.

Write down the steps you took to draw the reflected images.

c Which steps were the same or different for **Q5** and **Q12**?

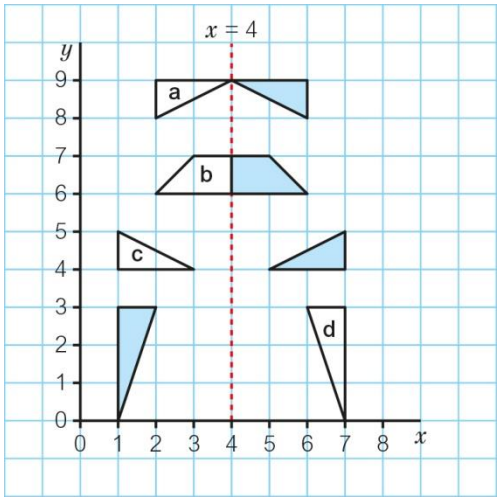
*This task encourages students to become aware of the steps they take when solving problems that require reflecting in lines parallel to the x- and y-axes (**Q5**), and reflecting in the lines y = x and y = –x (**Q12**). For both questions students may include a step that involves counting distances from the mirror line. However, for **Q5**, these distances are horizontal or vertical whereas for **Q12**, they are diagonal.*

Support

Tristan says: 'The mirror line is always halfway between the original and its image.'

Use examples from this lesson to show he is correct.

*Students' examples will vary, but they could refer to Reflections on a coordinate grid **Q1** or **Q2** or Reflections in diagonal lines **Q1**. For example*



Depth

a What happens to vertical and horizontal lines when they are reflected in a line x = something (for example x = 2) or y = something?

b What happens to vertical and horizontal lines when they are reflected in the line $y = x$?

a *Vertical and horizontal lines remain vertical and horizontal respectively when reflected in $x = c$ or $y = c$.*

b *Vertical lines become horizontal and horizontal lines become vertical when reflected in $y = x$.*

Purposeful practice Reflect and reason

Purposeful practice 1

Why is the dotted line that shows where a shape is reflected called the mirror line?

It is called a mirror line because it acts like a mirror. Place a mirror on the dotted line and you can see the reflected image of a shape.

Purposeful practice 2

After answering **Q2**, Antony writes

F is a reflection of C in the line $x = 3$ and E is a reflection of F in the line $x = 1$

Is Antony correct? Explain your answers for each statement.

No, Antony is not correct.

F is not a reflection of C. F is congruent to C. F is moved (translated) 2 square units to the right of C.

E is not a reflection of F in the line $x = 1$. F reflected in the y-axis and moved (translated) 2 square units down would give shape E.

Students could also refer to the Key point 'All points on the object are the same distance from the mirror line as the corresponding points on the image, but on the opposite side.'

Objectives

Core objectives

- Describe and carry out rotations on a coordinate grid

Support objectives

- Draw and describe rotations

Depth key concepts

- Identify patterns/rules in coordinates of vertices when a shape is rotated by different angles and in different directions on a coordinate grid

Key points

- A **rotation** is a type of transformation.
  You rotate a shape by turning it around a point, called the **centre of rotation**.
  To describe a rotation you also need to give the **angle** and direction (**clockwise** or **anticlockwise**).

Common errors and misconceptions

- Some students neglect to rotate every part of the shape. Address this by showing appropriate models or using dynamic geometry software.
- Sometimes students do not maintain the distance to the centre of rotation. Resolve this by using dynamic geometry software to model correctly preserving the distance, but students can also use tracing paper.

Question guidance

Core

Q3, Q5–Q9 Students are given structured practice in describing and carrying out rotations of shapes in increasingly complex questions.

Q4 Establish that 180° is half a turn, so it does not matter whether the rotation is clockwise or anticlockwise.

Q6 Ensure that students write complete and correct sentences for their answers, i.e. they must specify the angle, rotation and direction, and centre of rotation.

Q8 Check that students are using coordinates correctly, especially negative coordinates.

Q9 Insist on complete answers that include direction, angle and the centre of rotation.

Support

General Students who are struggling with describing rotations may find it more useful to draw rotations first, as this practical experience may enable them to better visualise rotations.

Describing rotations Q3 This question slowly builds up the elements needed to fully describe a rotation.

Purposeful practice

Provide tracing paper to help students rotate shapes and find centres of rotation.

Reflect

Core

In the last lesson you learned about reflection. In this lesson you learned about rotation. Look carefully at some of the shapes you reflected and rotated in these lessons. Can a reflection of a shape and a rotation of a shape give the same result?

This task aims to bring together the knowledge students have gained in lessons 10.2, 10.3 and 10.4. It encourages students to consider not only what it means to reflect and/or rotate, but also the nature of shapes they are working with. A reflection of a symmetrical shape (e.g. a letter T) in the □-axis can give the same result as a rotation of 180° about (0, 0). However, a reflection of a shape with no symmetry (e.g. a letter R) cannot give the same result as a rotation.

Support

Use your answers to **Q1** to help you complete these statements with 'vertical' or 'horizontal'.

Rotating a vertical line 90° gives a _____ line

Rotating a horizontal line 180° gives a _____ line

Rotating a _____ line 90° gives a vertical line.

Depth

When an object is rotated by 180°, what happens to the image if the centre of rotation is moved one unit upwards or one unit to the right?

One unit upwards means the image moves two units upwards. One unit to the right means the image moves two units to the right.

Purposeful practice Reflect and reason

Purposeful practice 1

Predict the coordinates of the triangle in **Q2** rotated

90° anticlockwise about (0, 0)

180° about (0, 0)

Check your prediction by carrying out the rotations.

90° anticlockwise about (0, 0): (–5, 3), (–3, 3), (–4, 5)

180° about (0, 0): (–3, –3), (–3, –5), (–5, –4)

Purposeful practice 2

Danny says, 'A rotation of any shape 180° about (0, 0) is the same as reflecting the shape in the x-axis.'

Is Danny correct? Draw a grid, a shape and a rotation to explain your answer.

No, Danny is not correct. Students should show this by drawing a shape on a grid and rotating it 180°.

Objectives

Core objectives

- Translate 2D shapes
- Transform 2D shapes by combinations of rotations, reflections and translations

Support objectives

- Translate 2D shapes
- Transform 2D shapes by combinations of transformations

Depth key concepts

- Know that in translation, rotation and reflection the image is congruent to the object
- Understand that combined transformations can be equivalent to a single transformation

Key points

- A **translation** is a type of transformation. A translation of a 2D shape is a slide across a fl at surface. To describe a translation you need to give the movement left or right, followed by the movement up or down.
  A translation does not change the size or shape of an object.
- When a shape is transformed by a translation, rotation or reflection, the image has exactly the same side lengths and angles as the object.
  This means that the original shape and the transformed shape are congruent.

Common errors and misconceptions

- Translation errors usually involve incorrect use of negative numbers or travelling vertically first. Remind students of the conventions used with coordinates, i.e. (x, y), horizontal before vertical.
- Some students think that shapes are only congruent if they have been translated, and not if they have been rotated or reflected. Address this by reminding them of the key aspects of congruence – same shape, size and angles – but orientation does not matter.

Question guidance

Core

Q3 *When a shape is translated what can you say about the perimeter and area of both shapes? Are both shapes congruent?*

Q3–Q7 These questions are carefully constructed to give practice in describing translations and in translating shapes.

Q4 Make sure students go across before going up or down.

Q8, Q11 These questions are carefully constructed to give practice in combining transformations. Check students' answers carefully, as minor errors are common when combining transformations.

Q10 Charlie is correct, demonstrating that translations can be added to form a single translation.

Q12 Students use their knowledge of the different types of transformation to assess the truth or otherwise of statements. They need to read the statements carefully to make sure they are looking at the correct pairs of shapes and considering the correct transformation.

Support

Describing translations Q1, Q2 These questions build students' understanding of translating in one direction and using left/right, up/down to describe. A common error is to not count the distance between the 'same' points on both shapes.

Describing translations Q3 You could extend this question by giving a translation (e.g. 1 square right, 3 squares down) and asking students to work out which shapes match the translation.

Drawing translations Provide tracing paper for those who need it.

Combined transformations Q1 Draw students' attention to the two separate translations (2 right, 3 down and 4 left, 4 up) and how these can be added together to give the single translation (2 left, 1 up).

Purposeful practice

Problem-solving practice Q3 This question illustrates the relationship between a translation and its 'opposite' translation, i.e. they both have the same numbers but involve opposite directions.

Reflect

Core

Write these transformations in order, from the one you find easiest to the one you find hardest. Look back at some of the transformation questions in this lesson to help you.

A Reflection in a horizontal line

B Reflection in a vertical line

C Rotation of 90° (either clockwise or anticlockwise)

D Rotation of 180°

E Translation

Write a hint, in your own words, for the transformation you found the hardest.

This task asks students to consider which transformations they found easiest and hardest. Then students are asked to write themselves a hint to help with the transformation they found hardest. Encourage them to write a hint that directly addresses whatever it is they found so hard.

Support

a Write down the different types of transformation you have met in this unit.

b Draw an example to explain each one.

a *Transformations are: enlargement, reflection, rotation, translation.*

b Drawn examples will vary, but should illustrate the named transformation correctly.

Depth

Look at all the objects and their images in this lesson. What can you say about the object and image in translations, reflections, rotations or combinations of these?

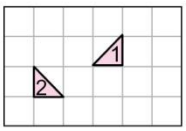
Students' own answers, for example, the object and image in these transformations are always congruent shapes.

Purposeful practice Reflect and reason

Purposeful practice 1

Faisal says, 'I have translated triangle 1 to triangle 2 by moving it 2 squares right and 1 square down.'

What two mistakes has he made?



The translation is left not right. The shape is translated and then reflected. (Alternatively, the shape may have been rotated.)

Purposeful practice 2

Alice says, 'To answer the questions in this exercise, I began by looking for shapes that were reflections, ignoring their position. This helped me to identify the transformations described by **Q1** and **Q2**, but not **Q4** and **Q5**.'

Why was this the case?

Explain your strategy for answering this question.

Descriptions 1 and 2 involve a reflection and a translation, so Alice could easily find the answer to these with her strategy. Descriptions 4 and 5 involve a reflection and a rotation, and so Alice needed also to consider orientation of the shape as well as position.

Students describe the strategies they used.

Check, Strengthen and Extend

Core Check up

Q1 This question is designed to test students' understanding of congruency, including whether they think that colour is involved.

Q2e This question tests the misconception of diagonal symmetry in a rectangle.

Q4 This question checks students' understanding of the order of rotational symmetry, and whether they realise that a shape with no rotational symmetry still has rotational symmetry of order 1.

Q9 This question tests students' understanding of scale factors and their relationship to ratio.

Q11 This question is designed to test whether students can describe a rotation fully by identifying missing information.

Q12, Q13 These questions check whether students can perform combined transformations accurately.

Challenge Nigel is correct but Paul is not. Any shape that has two lines of symmetry must look the same when rotated through 180° and so will have rotational symmetry of order 2. The opposite is not necessarily true. One example is the letter S: it has one line of symmetry but order of rotational symmetry 2.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

In these lessons you have answered questions about these transformations:

reflection rotation translation

Write a definition for each of them, using one of these descriptions:

flips over changes position turns around

For each definition, draw a sketch to show what the definition means.

How did your definition help you choose the shapes?

This task asks students to think about reflection, rotation and translation, and to choose words and shapes to describe them. Students' definitions will vary, but make sure they have correctly matched the given descriptions to the transformations, i.e. reflection and flips over, rotation and turns around, translation and changes position.

Then students are asked how the definition helped them choose the shapes. This emphasises to students the importance of understanding mathematical definitions to help decide what to do.

Extend

Look back at the questions you answered in this Extend lesson.

Beside each one, write the type of transformation you used.

List all the other mathematics topics you used to answer these questions.

This task encourages students to seek and be aware of the links between concepts in maths. For example, students may notice that for rotation they used angles, for enlargement they used multiplication, and for reflection they used coordinates.

Depth

Extend

Where have you seen repeating patterns around you?

What sorts of symmetry do they have?

What different types of patterns can you find?

Examples include tiled floors or walls, brickwork, curtain patterns, and so on.

End of Unit 10 tests: Transformations

Pearson published resources

Support/Core

- Year 7 End of Unit 10 Support/Core test
- Year 7 End of Unit 10 Support/Core mark scheme

Core/Depth

- Year 7 End of Unit 10 Core/Depth test
- Year 7 End of Unit 10 Core/Depth mark scheme

Revision

End of Term tests: Units 8 to 10

Pearson published resources

Support/Core

- Year 7 Summer End of Term Support/Core test
- Year 7 Summer End of Term Support/Core mark scheme

Core/Depth

- Year 7 Summer End of Term Core/Depth test
- Year 7 Summer End of Term Core/Depth mark scheme

Revision

Revision

End of Year tests: Units 1 to 10

Pearson published resources

Support/Core

- Year 7 End of Year Support/Core test
- Year 7 End of Year Support/Core mark scheme

Core/Depth

- Year 7 End of Year Core/Depth test
- Year 7 End of Year Core/Depth mark scheme

Unit opener: Number

Pearson published resources

Objectives

Core objectives

- Use written methods to add and subtract more than two numbers (including decimals)
- Use mental calculation for multiplication
- Estimate answers to calculations

Support objectives

- Use written methods to add and subtract more than two numbers (including decimals)
- Use mental calculation for multiplication
- Estimate answers to calculations

Depth key concepts

- Understand, choose and use a range of strategies for mental calculations by developing an understanding of relationships between numbers

Key points

- **Doubling and halving** is a mental multiplication method.
- Another mental multiplication method involves **rounding and adjusting**.

Common errors and misconceptions

- Students may make errors when dividing decimals e.g. 0.16 ÷ 2 = 0.8. Use long division to demonstrate keeping the decimal point/place value.
- When estimating, students think they must round all of the numbers in a calculation, e.g. 24 ÷ 9.8. Discuss with students whether they need to round all of the numbers.
- When estimating, students may replace numbers with poor estimates, e.g. rounding 43 ÷ 15.1 to 40 ÷ 15 instead of 45 ÷ 15. Emphasise the need to round numbers to the closest easy number.
- When estimating, students may round to numbers that are too difficult to calculate with quickly, e.g. rounding 146 ÷ 5.1 to 145 ÷ 5 instead of 150 ÷ 5. Remind students of the reason for estimating a calculation, i.e. to provide a quick approximation. Some students may be concerned about not getting the 'right' answer when estimating.

Question guidance

Core

Q5, **Q6** These questions have been carefully constructed to show students how to use doubling and halving in calculations. Explain that doubling and halving only works when multiplying numbers. Remind students that doubling and halving must result in simpler calculations.

Q11 This question has been carefully constructed to show students how to use rounding and adjusting in calculations.

Q14, **Q15** Check that students are estimating answers first.

Q18 Ask students to work in decimals first then centimetres.

General *Do you need to round all of the numbers when estimating a calculation?* Not always, e.g. in **Q16c** it is not necessary to round the last number. Explain that the

estimate may be further from the true value as more numbers are rounded. Students should round only enough numbers necessary for each step of the calculation.

Support

Doubling and halving Q3 Ask students if they can they see how the numbers in part **a** were changed to make the numbers in part **b**.

Column addition and subtraction Q3 Discuss with students how they decided which column addition was correct. This will help to draw out their understanding of place value.

Purposeful practice

Purposeful practice 1 and 2 Encourage students to make decisions about which numbers to double and halve, and how best to round numbers for adjusting in calculations.

Reflect

Core

Use two different methods to work out

501 × 18

Which method was easier? Explain.

Student's answers will vary but could include

500 × 18 + 1 × 18 = 9000 + 18 = 9018

or

501 × 20 − 501 × 2 = 10 020 − 1002 = 9018

Other answers may involve rounding to 500 × 18 or 501 × 20 and then doubling and halving to 1000 × 9 or 1002 × 10.

Support

In this lesson, you were asked to estimate.

With a partner, discuss when you might use estimating in your everyday life

a to add two amounts

b to subtract one amount from another.

a *You might use estimating to add amounts of money when you are shopping, or to work out approximate distances.*

b *You might use estimating to work out how much change you should get when shopping.*

Depth

Explain why 35 × 8 gives the same answer as 7 × 4 × 10.

Use this idea to describe how you could make a difficult multiplication such as 18 × 16 easier to work out.

35 × 8 = (7 × 5) × (2 × 2 × 2)

You could write this as 7 × (2 × 2) × (2 × 5), which is 7 × 4 × 10.

If a multiplication is difficult to work out in its current form, the factors of the numbers we are multiplying can be multiplied in any order to make the multiplication easier.

Purposeful practice Reflect and reason

Purposeful practice 1

Which decimal did you double and which did you halve in **Q4a**? Explain why.

Halve 2.2, double 1.5, as this results in the easy calculation 1.1 × 3. Otherwise if double 2.2 and half 1.5, then the calculation is not easy (4.4 × 0.75).

Purposeful practice 2

How do you decide whether to add or subtract when adjusting?

If the number you are multiplying has been rounded down to a multiple of 10, then you add to adjust; if it has been rounded up, then you subtract to adjust.

Purposeful practice 3

Alice works out **Q1g** like this.

$$\begin{array}{r} 836 \\ -724 \\ \hline 112 \\ \hline \end{array} \qquad \begin{array}{r} 7295 \\ +\ 112 \\ \hline \\ \hline \end{array}$$

Does Alice still get the correct answer? Explain why.

Do you think Alice's method is easier or harder than working out 7295 + 836 and then subtracting 724? Explain why.

Yes, because it doesn't matter which order you add and subtract in.

Easier, because there is no exchanging (carrying/borrowing) required in the calculations.

Objectives

Core objectives

- Know and use divisibility rules

- Use a written method to divide decimal numbers by integers

Support objectives

- Know and use divisibility rules
- Use a written method to divide decimal numbers by integers

Depth key concepts

- Understand why divisibility rules work
- Understand the relationships between divisibility rules and relate to factors and multiples

Key points

- An **integer** is a whole number. It can be positive, negative or zero.
- When you divide an integer by a second integer, if there is no remainder then the first integer is said to be **divisible** by the second integer.
  e.g. 64 ÷ 4 = 16, so 64 is divisible by 4.
- A number is divisible by
  - 3, if the **sum** of its digits is divisible by **3**
  - 4, if the last **two digits** are divisible by 4
  - 6, if the number is divisible by **2 and 3**
  - 8, if the last **three digits** are divisible by 8
  - 9, if the **sum** of its digits is divisible by **9**

Common errors and misconceptions

- Students may not understand the effect of adding a 0 to the end of a decimal number when dividing.
- Students may make errors with the alignment of digits in written division.

Question guidance

Core

Q5d, Q8d, Q9 These questions have been carefully constructed to show students how to use divisibility rules for 3, 4 and 8 using progressively larger numbers.

Q12 Students may need to be reminded that they can add zeros to the end of the decimal numbers to complete the divisions.

Q13, **Q14** Students may need reminding or telling that a **deposit** is a sum of money that is part of a full price. It is usually paid to show that you agree to buy something. An **instalment** is one of several sums of money, paid over an agreed amount of time, until the full payment has been made.

Q15 Remind students about estimating answers. Students should be able to estimate 2500 ÷ 25 = 100 to show that Alice is wrong.

Support

Divisibility Q6 Show that all numbers divisible by 10 are also divisible by 2 and 5 by making an array of counters 2 × 5 to demonstrate that all groups of 10 counters are divisible by both 2 and 5.

Purposeful practice

Purposeful practice 1 demonstrates the relationship between factors and divisibility.

Purposeful practice 2 provides practice in adding zeros on the end of a decimal number to complete a division calculation.

Reflect

Core

When might it be helpful to:

a use a divisibility rule, rather than divide?

b divide, rather than use a divisibility rule?

a *When the number being divided is big (3 or 4 digits or more).*

b *When the number is small and easy to divide; when there is no easy divisibility rule.*

Support

Look at the numbers on the tiles.

Which number is the odd one out?

Explain your answer to a partner. Use the word 'divisible'.

40 is the odd one out. It is the only one divisible by 10.The others are divisible by 5.

Depth

Explain how knowing that a number is in both the 6 times table and the 9 times table also lets you know that it is in the 18 times table.

If the number is in the 6 times table, it is divisible by 3 and 2. If it is in the 9 times table, it is divisible by 3 two times. A number in the 18 times table is divisible by 2 and also by 3 two times.

Purposeful practice Reflect and reason

Purposeful practice 1

Explain why a number is divisible by 6 if it is divisible by 2 and 3. Give examples.

Explain why a number is divisible by 12 if it is divisible by 3 and 4. Give examples.

A number is divisible by 6 if it is divisible by 2 and 3 because 2 × 3 = 6. Examples: 204 and 432 are divisible by 2 and 3 and 6.

A number is divisible by 12 if it is divisible by 3 and 4 3 because 3 × 4 = 12. Examples: 276 and 552 are divisible by 3 and 4 and 12.

Purposeful practice 2

Look at your working for **Q2**. When do you put a zero at the end of a decimal when doing written division?

When there is a remainder.

Objectives

Core objectives

- Add, subtract, multiply and divide positive and negative numbers, including larger numbers and decimals

Support objectives

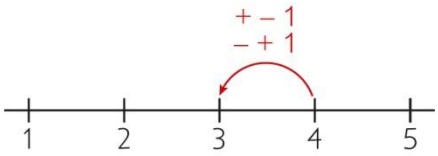
- Add, subtract, multiply and divide positive and negative numbers, including larger numbers

Depth key concepts

- Extend the rules for calculations with negative numbers to very large numbers and decimal numbers
- Understand what happens to the sign in an answer when multiplying more than two positive/negative numbers

Key points

- 4 + −1 is 4 add negative 1. You are adding a negative number. 4 − +1 is 4 subtract positive 1. You are subtracting a positive number. Both give the same answer



Common errors and misconceptions

- Students may think that −2 always means subtract 2. Use the number line to demonstrate the two uses of the – sign. –2 is the name of a number on the number line. To subtract 2 from 5, write 5 − 2. To subtract 2 from −2 write −2 − 2. To multiply 3 by −2 write 3 × −2 or 3 × (−2) to make it absolutely clear that there is no subtraction.
- When calculating −3 + 2 − 4 students may think that the first − has an effect on the other signs. Use the number line to demonstrate otherwise.
- Students may not feel satisfied with the derivation of rules for calculating with negative numbers. See individual questions for more concrete explanations, where possible.
- Students sometimes misuse rules such as 'two negatives make a positive', e.g. −3 − 7 = 21 Explain that the rule only works when multiplying, dividing and for two adjacent + or − signs.
  There is no × sign in the calculation −3 − 7 so the rule cannot be used.
  Calculate −3 − 7 using the number line.

Question guidance

Core

Q2b Ask students *What does the – sign mean*? (negative; −2 is negative 2). Point out that −2 cannot mean 'subtract 2' because there is no number before it, as in say 5 − 2.

Q4a Explain that in the left column of the first table the numbers being added to 3 are decreasing from 3 to –5 with the effect of decreasing the answer. When the numbers being added become negative, it is like adding more and more ice to water, which reduces its temperature.

Repeat for the second table. When the numbers being subtracted become negative, it is like removing more and more ice from the water, which increases its temperature.

Q5 This question has been carefully constructed to show students the resulting patterns with alternating combinations of + and − signs.

Q7 *To find the difference between two numbers, subtract the smaller number from the greater number. What is the difference between 5 and 8, 5 and –8, –5 and –8?*

Q10 Explain that a bank balance is the amount of money in a bank account. A negative bank balance (or overdraft) is an amount owed to the bank. When you put money into a bank account, this is a deposit. When you take money out, this is a withdrawal.

Q11 Explain that hydrogen is a gas, and that all gases can be frozen. Before they freeze, they first become a liquid.

Q10a Say *Mrs Prestwick owed the bank £128 on the 1st. On the 2nd she deposited £20. How much did she owe the bank on the 2nd?* (–£128 + £20 = –£108)

Q12a Check that students have continued the pattern of number correctly before moving to part **b**.

Q13d Remind students that 5 × (−9) is the same as 5 × −9. Say *You can write −9 using brackets (−9) so that you don't confuse it with 'subtract 9'*.

Support

Adding and subtracting with negative numbers Q10 Make sure students see that – – is the same as +.

Purposeful practice

Purposeful practice 1 Remind students that the addition and subtraction operations on the same digits will give different answers if the digits are positive or negative.

Purposeful practice 2 Check that students are clear that multiplying and subtracting negative numbers are not the same.

Reflect

Core

Look back at what you have learned about negative numbers in this lesson.

What is different and what is the same about positive and negative numbers?

This task asks students to reflect on their understanding of negative integers in the context of comparison with positive integers. It helps students by giving some examples. Students may also draw on their pre-existing knowledge of negative numbers. However, they are likely to mostly look again at the rules for negative numbers learnt in this lesson.

Support

Look back at the work you have done in this lesson.

Write one thing that is the same and one thing that is different about working with positive numbers and working with negative numbers.

Same: When you multiply two negative numbers, you always get a positive number. This is the same as when you multiply two positive numbers.

Different: As you move further away from zero, negative numbers get smaller, but positive numbers get greater.

Depth

More than two numbers are multiplied together.

Some are positive and some are negative.

How can you tell whether the answer will be positive or negative?

For example, if you multiply three numbers together and two are negative and one is positive, will the answer be positive or negative?

The answer will always be positive if there is an even number of negative numbers and will always be negative if there is an odd number of negative numbers. The answer to the example will be positive.

Purposeful practice Reflect and reason

Purposeful practice 1

When there are two signs between numbers, does it matter which sign is first?

No, it does not matter, because + − and − + are both the same as subtracting, and − − are the same, and so their order is irrelevant.

Purposeful practice 2

Anna says, '−2 × −2 and 2 − −2 have the same answer.' Is she correct? Explain.

Billy says, '−3 × −3 and 3 − −3 also have the same answer.' Is he correct? Explain.

Anna is correct; −2 × −2 = 4, 2 − −2 = 4

Billy is incorrect; −3 × −3 = 9, 3 − −3 = 6

Objectives

Core objectives

- Calculate using squares, square roots, cubes and cube roots
- Say which integers a square root lies between

Support objectives

- Calculate using squares, square roots, cubes and cube roots
- Give integers that a square root lies between

Depth key concepts

- Know when the negative square root is an appropriate solution to a problem

Key points

- $2^3 = 2 \times 2 \times 2$

  2^3 is '2 **cubed'** or '2 **to the power** 3'

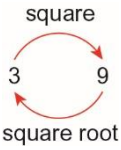
- Finding the **cube root** is the **inverse** of finding the cube of a number. 3 cubed is 27, so the cube root of 27 is 3.

  The cube root of 27 is written $\sqrt[3]{27}$

- $3^2 = 9$ and $(-3)^2 = 9$. The **positive square root** of 9 is 3. The **negative square root** of 9 is −3.
- The $\sqrt{}$ symbol is used for the principal square root, which is always a positive number. For example, $\sqrt{9} = 3$
- A **counter example** is an example which proves that the statement is wrong.

Common errors and misconceptions

- Students confuse powers and multiplications e.g. 2^3 with 2×3. Show that 2×3 is three 2s added together $2 + 2 + 2$ whereas 2^3 is three 2s multiplied together $2 \times 2 \times 2$. You can use cubes to address this in a concrete way: demonstrate 2^3 by building a cube from 8 smaller cubes.
- Students think that -3^2 means the same as $(-3)^2$. Write $3^2 = 9$ and so $-3^2 = -9$. Remind students that −3 can also be written (−3). The square of −3 is -3×-3 which can be written $(-3) \times (-3)$ or $(-3)^2$.
- Students calculate 2×3^2 as 6^2. Remind students of BIDMAS, so that the square must be worked out first, so the calculation can be written $2 \times 3 \times 3$ or 2×9; students meet $(2 \times 3)^2$ in Lesson 1.5.
- Students think that the square root is positive because that's what you get on a calculator. Remind students that the square root of a number is another number that, multiplied by itself, gives the original number, so all numbers have a positive square root and a negative square root.
- Students think that the square root of a number is half of the number and cube root is a third of the number. Use a function machine diagram to demonstrate squares, square roots, cubes and cube roots, e.g.
  - square of $3 = 3^2 = 9$
  - square root of 9 = $\sqrt {\rm{9}}$ = 3



Question guidance

Core

Q3e This question explores the BIDMAS priority of operations (square, multiply, add)

Q7a *Which is the closest square number below/above 6?*

Q8 Students should round the answer to the nearest cm (whole number) or mm (1 d.p.)

Q12 Check that students are able to use their calculators to find cubes and cube roots.

Q16 Students can also use their calculators to show that -7^2 is different from $(-7)^2$.

Support

Squares and square roots Q2b-e, **Q3** Encourage students to work out the squares first and then do the next part of the calculation separately.

Cubes and cube roots Q2, **Q3** Make sure students know how to use their calculators for these questions.

Purposeful practice

Purposeful practice 1 involves squares of negative numbers and positive and negative square roots of numbers.

Purposeful practice 2 Q1 uses cubes of positive and negative numbers to highlight the difference between squares and cubes.

Reflect

Core

The $\sqrt{}$ part of the root symbol began as an old-fashioned letter r in the 16th Century. You could remember r for root!

List all the mathematics notation used in this lesson, and ways you might remember it.

Make sure you know what all the notation in this lesson means.

Sometimes students find notation daunting, especially if it is unfamiliar to them. It is important to make students comfortable with new mathematics notation. Writing down ways to remember notation, and what it does, helps students to revisit and develop a familiarity with it.

Support

Is this statement true or false?

Some numbers are both square numbers and cube numbers.

If the statement is true, give some examples.

If the statement is false, try to explain why.

It is true. 1 is 1^2 and is also 1^3. 64 is 8^2 and is also 4^3.

Depth

When does it not make sense to use the negative solution to the square root of a positive number?

When taking the square root of a measure, e.g. area of a square space to find the side length.

Purposeful practice Reflect and reason

Purposeful practice 1

Katrina says, '$(-20)^2$ is 40, and so $\sqrt{40}$ is 20 or −20.'

What mistake has she made?

Katrina has worked out −20 × −2 instead of −20 × −20 (= 400).

Purposeful practice 2

Miguel writes

$\sqrt[{\rm{3}}]{{{\rm{1000}}}}$ is 10 or −10

Explain why he is incorrect.

$\sqrt[{\rm{3}}]{{{\rm{1000}}}}{\rm{ = 10}}$ *because 10 × 10 × 10 = 1000, −10 × −10 × −10 = −1000*

so $\sqrt[{\rm{3}}]{{{\rm{1000}}}}{\rm{ = 10}}$ *and cannot be −10.*

Purposeful practice 3

Why do **Q1**, **Q5** and **Q9** (and also **Q2**, **Q6** and **Q10**) have the same answer?

Because 2^2 = $\sqrt {16} = \sqrt[{\rm{3}}]{{{\rm{64}}}}$ and it doesn't matter which order you add or multiply numbers.

Objectives

Core objectives

- Calculate combinations of squares, square roots, cubes, cube roots and brackets

Support objectives

- Calculate combinations of squares, square roots, cubes, cube roots and brackets

Depth key concepts

- Understand how to write complex calculations with a given answer

Key points

- For $\frac{calculation\ 1}{calculation\ 2}$ work out (calculation 1) ÷ (calculation 2) using the priority of operations, e.g. $\frac{5+7}{8-5} = (5 + 7) \div (8 - 5)$

- For $\sqrt{addition\ or\ subtraction\ calculation}$ work out the calculation and then the square root, e.g. $\sqrt{40-4}=\sqrt{36}$

Common errors and misconceptions

- Students might not realise that BIDMAS may need to be applied inside a bracket first, e.g. in **Q3c**, apparently breaking the order of operations. Explain that the 'answer' inside the brackets must be found in order for the calculation to continue.
- Students may not realise that the vinculum (horizontal line) on the top of the root sign is treated as a bracket. Compare $\sqrt {{\rm{16}}} {\rm{ + 9}}$ with $\sqrt {{\rm{16 + 9}}}$, and explain the latter is equivalent to $\sqrt {{\rm{(16 + 9)}}}$. Students can use their calculators to show that the answers are different.
- Students may calculate $\sqrt {{\rm{16 + 9}}}$ as $\sqrt {{\rm{16}}} {\rm{ + }}\sqrt {\rm{9}}$. Work out the answers to each calculation to demonstrate they are different.
- Students may not realise that the numerator or denominator of a fraction can be written inside a bracket. Demonstrate this by using a simple example such as $\frac{12}{2+4}$ by writing $\frac{12}{(2+4)}$ = 12 ÷ (2 + 4). Make sure students know how to use their calculators for these calculations.

Question guidance

Core

Q4 Ask students to write down a rule, i.e. the square of a product is the product of the squares.

Q7 49 is, so 49 × 100 and 49 × 10 000 will be too.

Q8 Ask students to work in centimetres.

Q10, **Q12** These questions have been carefully constructed to give students division calculations of increasing complexity.

Q16a Say *40 + 3^2 is inside a bracket, so you must work it out before finding the square root*

Support

Powers, roots and brackets Q3 Encourage students to do these questions step-by-step.

Powers, roots and brackets Q4c Students may be put off by the apparent complexity of this question. Encourage students to break it down into individual steps.

Roots Q2c Make sure students break the division into the correct components.

Purposeful practice

Purposeful practice 1 gives practice in numerical calculations involving squares and brackets.

Purposeful practice 2 gives practice in numerical calculations involving square roots and arithmetic operations.

Purposeful practice 3 gives practice in applying the correct order of operations in complex calculations.

Reflect

Core

Look back at **Q12g**. Write down all the steps you took to work out the answer.

You may begin with, 'Step 1: I worked out 2^3.'

Now look back at each step. How did you decide what to do first, second, third ...?

Compare your decisions with others in your class.

This task encourages students to consider all the steps they took when answering a calculation, and to be aware of how they made decisions about the order of steps. For example, some students may have worked out any powers or roots first, perhaps so that they are then dealing with straightforward integers and operations. Other students may have worked out 2^3 first, and then the calculation on the top, so that then they have their dividend. By asking students to compare steps and decisions with others they should begin to acknowledge other approaches and reasoning.

Support

Hannah said the hardest part of this lesson was remembering the correct priority of operations.

Joseph said the hardest part was remembering the squares, square roots, cubes and cube roots.

Discuss with a partner. Who do you agree with? How can you improve on your weakest skill?

Students' answers will vary. Use the answers to revisit topics if there is confusion about the priorities of operations, or if more work is needed on knowing squares and cubes and their roots.

Depth

You were asked to write a calculation in **Q11** including a cube root and a square that gave a specific integer answer.

What part of the calculation did you write first? Explain why.

Students' own answers, e.g. I calculated the cube root first because there were limited numbers it could be.

Purposeful practice Reflect and reason

Purposeful practice 1

Use your answers to **Q1** and **Q2** to help you copy and complete the missing operation

(+, −, ×, ÷) in this statement:

(number □ number)² = number² □ number²

Give all the possible answers (there is more than one).

(number × number)² = number² × number ²

(number ÷ number)² = number² ÷ number ²

Purposeful practice 2

Use your answers to **Q1** and **Q2** to help you copy and complete the missing operation (+, −, ×, ÷) in this statement:

$\sqrt {number} {\rm{ }}\ □\ {\rm{ }}\sqrt {number} = \sqrt {number\ □\ number}$

There is more than one correct answer. Give all possible operations.

$\sqrt {number} {\rm{ }} \times {\rm{ }}\sqrt {number} = \sqrt {number \times number}$

$\sqrt {number} {\rm{ }} \div {\rm{ }}\sqrt {number} = \sqrt {number \div number}$

Purposeful practice 3

Ali works out $\frac{{{{{\rm{(2 + 3)}}}^{\rm{2}}}}}{{\sqrt {{\rm{9 + 16}}} }}$ as

$\frac{{{{{\rm{2}}}^{\rm{2}}}}{\rm{ + 3}}}}{{\sqrt {\rm{9}} {\rm{ + }}\sqrt {{\rm{16}}} }}{\rm{ = }}\frac{{\rm{7}}}{{{\rm{3 + 4}}}}{\rm{ = }}\frac{{\rm{7}}}{{\rm{7}}}{\rm{ = 1}}}$

What two mistakes has Ali made? What is the correct answer to the calculation?

For the numerator Ali only squares 2 and not (2 + 3).

For the denominator, Ali assumes $\sqrt{9} + \sqrt{16}$ *gives the same answer as* $\sqrt{9 + 16}$.

Correct answer is: $\frac{(2+3)^2}{\sqrt{9+16}} = \frac{25}{\sqrt{25}} = \frac{25}{5} = 5$

Objectives

Core objectives

- Use index form
- Write a number as the product of its prime factors

- Use prime factor decomposition to find the highest common factor (HCF) and lowest common multiple (LCM)

Support objectives

- Use index form
- Write a number as the product of its prime factors
- Use prime factor decomposition to find the highest common factor (HCF) and lowest common multiple (LCM)

Depth key concepts

- Understand that prime numbers are the building blocks for the natural numbers – i.e. that all numbers can be written as a product of prime factors
- Understand when to use HCF and LCM to find the answer to a word problem

Key points

- $2^4 = 2 \times 2 \times 2 \times 2$
  2^4 is '2 to the power 4'. The small number is called the **index** or **power** and tells you how many 2s to multiply together.
  The power key on your calculator may look like
  y^x or $x^{\square}$
- The result of multiplying numbers or letters together is called their **product**. For example, the product of 3 and 10 is $3 \times 10 = 30$.
- **Prime factors** are factors that are prime numbers. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36. The prime factors are 2 and 3.
- All positive integers can be written as a product of prime factors. This is called **prime factor decomposition**. The product is often written in index form (numbers with powers).
- You can use prime factor decomposition to find the highest common factor (HCF) or lowest common multiple (LCM) of two or more numbers.

Common errors and misconceptions

- Students may not completely decompose a number into its prime factors. Remind students of the prime numbers and encourage them to check their factors to make sure they complete the decomposition. They may need reminding about prime numbers from 11 upwards.
- Students may count too many or too few prime factors of a decomposition when finding the LCM, or simply multiply the two numbers together to find the LCM. Show that the decomposed LCM of 12 and 18 ($36 = 2 \times 2 \times 3 \times 3$) contains the primes of each decomposed number ($12 = 2 \times 2 \times 3$ and $18 = 2 \times 3 \times 3$) and no more than primes than are necessary. Demonstrate that simply multiplying the two numbers together gives a number bigger than the LCM.

Question guidance

Core

Q7 This question has been carefully constructed to show students how to find prime factors of increasingly large numbers. Remind students that 1 is not a prime number.

Q8c No. Show that you can multiply the prime factors 2, 3, 3 together in two ways giving two different trees.

Q9 This question has been carefully constructed to give students practice in prime factor decomposition using increasingly large numbers.

Q10 Make sure that students continue the factor tree until only prime numbers are left. Show students how to find the first factor pair by using tests of divisibility: even numbers end in 2, multiples of 10 end in 0, multiples of 5 end in 5, the sum of the digits of multiples of 3 are divisible by 3. Numbers for which there is no test of divisibility should be divided by the remaining prime numbers, e.g. for 91, try dividing by 7, 11, 13 etc.

Q11 Highlight the prime factors of 24 (**2** × **2** × **2** × **3**) in the prime decomposition of 360, i.e. 360 = **2** × **2** × **2** × **3** × 3 × 5

Q17, **Q18** These don't require prime decomposition but encourage students to use the method for pairs of large numbers as it is quicker. In **Q18**, get students started by asking *'Could the vouchers be £2 vouchers?* (yes because 2 is a factor of 72 and 96) *Could the vouchers be worth more? What is the highest amount (HCF)?'*

Support

Prime factors Q4 *Does it matter which factor pair you start with on the factor tree?*

Purposeful practice

Purposeful practice 1 Q1 gives practice in expressing numbers as products of prime factors.

Purposeful practice 2 gives practice in working out factor trees.

Reflect

Core

'Compose' means to make or create something. What do you think 'decompose' means?

Write (in your own words) the meaning of 'prime factor decomposition'.

This reflective task encourages students to consider the individual words in 'prime factor decomposition', so that they may decipher meaning for themselves, rather than view them as mathematical terminology that requires memorising.

Support

Discuss with a partner: How does it help to draw prime factor trees to find the product of prime factors? How does it help to draw Venn diagrams to find the HCF and the LCM?

Students' answers will vary, but students should understand the relationships between their prime factors trees and the products of prime factors, and should be able to use Venn diagrams to understand the differences between HCF and LCM.

Depth

Are there any numbers that cannot be written as a product of prime factors? Explain.

All numbers can be written as a product of one or more prime factors.

Purposeful practice Reflect and reason

Purposeful practice 1

Jack writes the product $3 \times 3 \times 5 \times 5 \times 3$ as

$3^2 \times 5^2 \times 3$

Does Jack's answer give the same answer as the original product?

How can you further simplify Jack's answer?

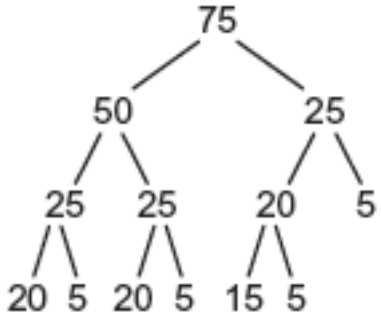
Yes, it gives the same answer.

Simplified answer is $3^3 \times 5^2$

Purposeful practice 2

Here is the start of Sally's factor tree for prime factor decomposition of the number 75.

What mistake has Sally made?



Sally has used numbers that add instead of numbers that multiply for her factor tree, i.e. she has partitioned 75 into 50 + 25 instead of 3 × 25.

Purposeful practice 3

What is the same and what is different about using prime number decomposition for finding the HCF and the LCM of numbers?

Same: Both require finding the product of prime factors for both numbers.

*Different: HCF then requires all the common factors that appear in **both** lists, whereas LCM requires all the factors that appear in **either** list.*

Check, Strengthen and Extend

Core Check up

Calculating with positive and negative numbers

Q1 This question tests the choices that students make when doubling and halving.

Q2 Students mistake the divisibility rule for 3 with the rules for other divisors e.g. they think the last digit needs to be a multiple of 3.

Q4, **Q5** When estimating, students think they must round all of the numbers in a calculation.

Q4, **Q5** Use this question to encourage students to make good estimates when estimating in calculations.

Q4, **Q5** When estimating, students round to numbers that are too difficult to calculate with quickly.

Q6a Students think that −2 always means subtract 2.

Q6 Students misuse rules such as 'two negatives make a positive'.

Powers and roots

Q7a Students confuse e.g. 4^3 with 4×3.

Q7b, **f** Students think that the cube root is a third of the number.

Q7c Students think that -4^2 means the same as $(-4)^2$

Q8 Students think that the square root is always positive because that's what you get on a calculator.

Q10 Students may forget that the vinculum (horizontal line) attached to a root sign is a bracket.

Q10 Students calculate $\sqrt {{\rm{25 }} - {\rm{ 16}}}$ as $\sqrt {{\rm{25}}} {\rm{ }} - {\rm{ }}\sqrt {{\rm{16}}}$

Q10 Students may not realise that the numerator or denominator of a fraction can be written inside a bracket.

Factors and multiples

Q13 Students may not completely decompose a number into its prime factors.

Q14 Students may count too many or too few prime factors of a decomposition when finding the LCM, or simply multiply the two numbers together to find the LCM.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Write five different ways you used your multiplication and division skills in these Strengthen lessons. Your first two might be:

1 When doubling, I multiplied by 2.

2 When dividing a negative number by a negative number, I divided the numbers and then wrote an answer which was positive.

This reflective task aims to demonstrate to students that so much of maths requires multiplication and division. When students acknowledge this, then tasks may seem easier to them. Students may add taking the cube root and writing numbers as products of prime factors to their list as tasks that require multiplication skills.

Extend

Which of the questions in this Extend lesson made you think the hardest? Why?

What could you do so that questions like this don't make you think so hard in the future?

*This task asks students to consider which question made them think the hardest, and why that might be. Students may choose the wordiest question as the one that made them think hardest because it required a lot of interpretation. Perhaps in future, as they read long questions, they might choose to jot down the most important points. Alternatively, they may choose a question involving a formula (such as **Q8** or the Challenge questions), because they find the use of letters off-putting. Perhaps, this indicates a need to become more familiar and comfortable with algebra.*

Depth

Extend

Imagine you are working out calculations without using a calculator.

What steps can you take to try to ensure your answers are correct?

The correct priority of operations must be applied. You should estimate the answer to check if the final answer is sensible.

End of Unit 1 tests: Number

Pearson published resources

Support/Core

- Year 8 End of Unit 1 Support/Core test
- Year 8 End of Unit 1 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 1 Core/Depth test
- Year 8 End of Unit 1 Core/Depth mark scheme

Unit opener: Area and volume

Pearson published resources

Objectives

Core objectives

- Derive and use the formula for the area of a triangle
- Calculate the area of compound shapes made from rectangles and triangles

Support objectives

- Derive and use the formula for the area of a triangle
- Calculate the area of compound shapes made from rectangles and triangles

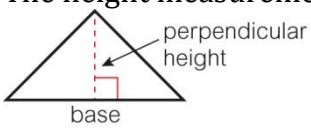
Depth key concepts

- When calculating the area of triangle, it doesn't matter which measurements you choose for the base and height, as long as they are perpendicular to each other
- Every triangle's area is half of the area of a rectangle of the same base and height
- Understand that there are many triangles with the same area (but only one square with a given area)

Key points

- Area of a triangle = $\frac{1}{2}$ × base length × perpendicular height which can be written as $A = \frac{1}{2}bh$

The height measurement must be perpendicular (at 90°) to the base.



Common errors and misconceptions

- Students may forget to use the $\frac{1}{2}$ in the formula for the area of a triangle.
  Demonstrate why the $\frac{1}{2}$ is needed by drawing the rectangle that contains the triangle to show that the area of the rectangle needs to be halved to find the area of the triangle.

Question guidance

Core

Q2 Make sure students write down the correct units for each of their answers, i.e. cm in part **a** and cm^2 in part **b**.

Q6 This question has been carefully constructed to show students how to calculate the area of a triangle with increasingly complex combinations of dimensions and shapes.

Q7 Encourage students to take this one step at a time. Start by working out the total number of red, white and blue flags needed for one length of bunting, then 5 lengths of bunting. Work out the area of one triangle, then finally the total area of material of each colour.

Q9 Tell students to make a copy of the window and divide it into a rectangle and triangle. Work out the individual areas, and then add them to find the total area. Multiply this area by the cost per square metre, to find the total cost.

Support

Area of a triangle Q2 *What do you notice about your answers for **Q1** and **Q2**?* Ask students to explain why they are the same.

Area of a triangle Q5 Students can decide whether they want to draw the rectangles or not. Ask those who didn't draw the rectangles how they worked out the areas.

Area of a triangle Q6 Students could copy the diagrams onto large squared paper, cut out the triangle and show that the two white bits fit exactly on to the triangle each time.

Purposeful practice

Purposeful practice 1 encourages students to use the correct formula, using the correct lengths.

Purposeful practice 2 shows that you can halve at any stage of the calculation.

Reflect

Core

After this lesson George says, 'Area measures the size of a shape.'

Polly says, 'Area is the amount of space inside a shape.'

Hasid says, 'Area is the number of squares a space covers.'

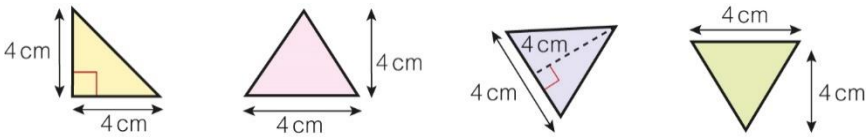
Which definition do you like most? Why?

Use what George, Polly and Hasid say, and your own understanding of area, to write the best possible definition. Compare your definition with others in your class.

It is very easy for students to learn about, and correctly answer questions on, concepts without ever giving much thought to what they actually mean. This reflective task asks students to carefully consider different definitions of area and what they understand the term to mean. Formally, area may be defined as a measure of a surface.

Support

Explain why all of these triangles have the same area.



To use the area formula, does the base have to be on the bottom of the triangle?

The triangles all have the same area because the base and perpendicular height are the same in each case. The base doesn't have to be at the bottom of the triangle, but the height must be perpendicular (at right angles) to the base.

Depth

You are told the area of a shape.

How many possible squares can you draw with this area?

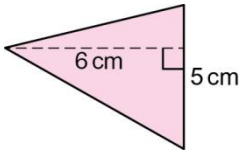
How many triangles can you draw with this area?

You can only draw one square with given area. You can draw more than one triangle, using all the factor pairs of half the area for the base and height, and with different values for the other two side lengths.

Purposeful practice Reflect and reason

Purposeful practice 1

Josie says the area of this triangle is 30 cm^2.



What mistake has she made?

Josie has forgotten to halve the multiplication. She has worked out □□ *and not* $\frac{1}{2}$ □□.

Purposeful practice 2

Explain why all three methods in **Q1** work for finding the area.

Why is it useful to have three methods?

How do you decide which to use?

The three methods are equivalent because they all involve $\frac{1}{2} \times$ □ × □ *and you can multiply in any order:*

A is $\left(\frac{1}{2} \times b\right) \times h$

B is $b \times \left(\frac{1}{2} \times h\right)$

C is $b \times h \times \frac{1}{2}$

It is useful to have three methods because some calculations are easier than others. To decide which to use, students may suggest halving the even numbers (if there is one) first. Where halving either □ *or* □ *leads to decimal × decimal, it may be easier to work out* □□ *first, then halve.*

Objectives

Core objectives

- Derive and use the formula for the area of a parallelogram
- Use the formula for the area of a trapezium

Support objectives

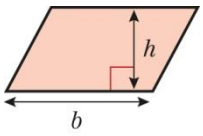
- Derive and use the formula for the area of a parallelogram
- Use the formula for the area of a trapezium

Depth key concepts

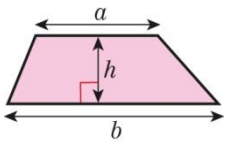
- When calculating area of parallelogram or trapezium it doesn't matter which measurements you choose for the base (or top and base in trapezium) and height, as long as they are perpendicular to each other
- Generalise understanding that all areas are product of perpendicular lengths

Key points

- **area of parallelogram** = base length × perpendicular height, □ = □□

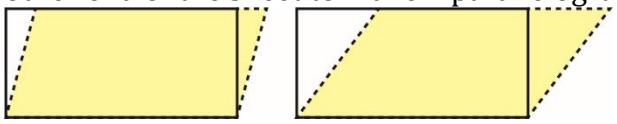


- **area of trapezium**, $A = \frac{1}{2}(a + b)h$



Common errors and misconceptions

- Students may use the slant height instead of the perpendicular height when calculating the area of a parallelogram. Demonstrate why the perpendicular height is used with an activity
  - Start with 2 sheets of A4 paper.
    - Cut a different triangle off the end of each sheet and stick it to the other end of the sheet to make 2 parallelograms. e.g.



Point out that these two parallelograms have the same area as they were made from two rectangles with the same area. The different slant heights do not affect the area of the parallelograms.

- Students may use the wrong order of operations when using the formula to calculate the area of a trapezium. Break the calculation into steps, remembering the first step must be to **Add the Parallel Sides** (**APS**). They can then divide by 2 and multiply by the height in whichever order they prefer.

Question guidance

Core

Q4e Remind students that a formula should have an equals sign, so needs to start A = …

Q6d Make sure students understand the difference between the slant height and the perpendicular height. 5 cm in part **a**, 32 mm in part **b** and 2.7 cm in part **c** are the slant heights and are not used to find the area.

Q7 Make sure students write the correct units in their answers, i.e. cm^2 in part **a** and mm^2 in part **b**.

Q8 Students need to work out the area of the trapezium first, and then multiply by the cost per square metre. They will probably need to use a calculator. Check they write their answer of 170.1 using correct money notation i.e. £170.10

Support

Area of a parallelogram Q1 Ask students why parts **e** and **f** have the same answer.

Area of a trapezium Q4 Some students may need reminding here to look out for different shapes.

Purposeful practice

Purposeful practice 1 encourages students to use the correct formula, using the correct lengths.

Reflect

Core

Q9b asked you to draw a shape with a particular area.

Write all the steps you took to work out the lengths of the shape.

Choose another shape. Will the steps be the same or different? Explain.

*This task asks students to reflect on the last part of **Q9** in this lesson, and, in doing so, make explicit their reasoning and problem-solving strategies. Many students will have chosen to work with a parallelogram, because its formula simply requires multiplying together two numbers (base and perpendicular height). However, what if they had chosen a triangle or trapezium? What steps would they have taken to work back from the area to find the necessary lengths then?*

Support

Look back at **Area of a trapezium Q4**. How did you decide which formula to use for each shape?

You need to be able to identify correctly the different shapes: triangle, parallelogram and trapezium and know the formula for each.

Depth

In each question you multiplied two lengths to calculate the area.

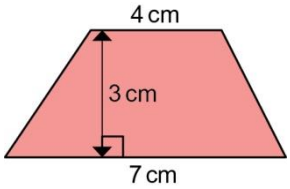
What is the relationship between the two lengths you multiplied each time?

The two lengths are always perpendicular.

Purposeful practice Reflect and reason

Purposeful practice 1

Jonty works out the area of this trapezium.



Area = $\frac{{\rm{1}}}{{\rm{2}}}{\rm{ }} \times {\rm{ 4 + 7 }} \times {\rm{ 3}}$

= 2 + 21

= 23 cm^2

Explain what Jonty has done wrong.

Jonty hasn't put the brackets in the formula. He should have worked out

$\frac{1}{2} \times (4 + 7) \times 3 = \frac{1}{2} \times 11 \times 3$ *= 16.5 cm^2*

Purposeful practice 2

When you use base and height in an area formula, does the base have to be horizontal and the height vertical? Use examples from this page to explain.

No, base does not have to be horizontal and height does not have to be vertical. But the base and height have to be perpendicular to each other. Students should give examples.

Objectives

Core objectives

- Calculate the volume of cubes and cuboids
- Calculate the volume of 3D solids made from cuboids
- Solve volume problems

Support objectives

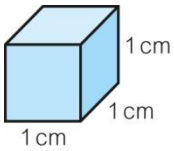
- Calculate the volume of cubes and cuboids
- Calculate the volume of 3D solids made from cuboids
- Solve volume problems

Depth key concepts

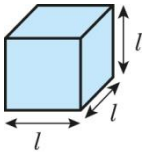
- Understand why volume is measured in cube units.
- Understand that composite volumes can be calculated by 'subtracting' a shape, as well as by splitting into two different shapes.

Key points

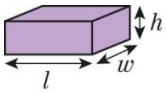
- The volume of a 3D solid is the amount of 3D space it takes up.
  The units of volume are cubic units (e.g. mm^3, cm^3 or m^3).
  A cube with sides of length 1 cm has a volume of 1 cm^3.



- **volume of a cube** = side length × side length × side length
  $V = l \times l \times l$
  $V = l^3$



- **volume of a cuboid** = length × width × height
  $V = lwh$



Common errors and misconceptions

- Students may add the length, width and height rather than multiply them. Demonstrate the multiplication by making cuboids out of centimetre cubes. Count the number of cubes in each layer and show this is the same as length × width. Then multiply by the number of layers to get the total number of cubes i.e. the volume.

Question guidance

Core

Q15 Encourage students to draw the two separate cuboids and write the length, width and height on each one. Then they can go on to do the volume calculations.

Q16 *In how many different ways can you work out the volume of a water tank that is three quarters full?*

Q17 Students need to work out how many dice will fit along the length of the box, the width of the box and the height of the box. By multiplying these numbers together they will work out how many dice will fit in the box.

Support

Volume of cubes Q1c What do students notice about their answers to parts **b** and **c**?

Volume of cuboids Q3 Ask students how their answers to **Q2** helped them to work out the volume of these cuboids.

Volume of cuboids Q4 You can give students the option here of using the answers to **Q2** to help them if necessary. *How did your answers to **Q2** help you to work out the volume of these cuboids?*

Purposeful practice

Purposeful practice 1 gives practice in calculating volumes of cubes and cuboids, switching between the two formulae.

Purposeful practice 2 Q2 requires students to break the composite shapes into the correct component parts and calculate the individual volumes.

Reflect

Core

Maths is not the only subject where you use volume. You use it in science too.

Describe when you have used volume in science.

In which ways is volume the same or different in science and in this maths lesson?

Do you think volume means the same in all subjects? Explain.

This task asks students to consider their understanding of the term volume. In this maths lesson, students have specifically learned about volume of cubes and cuboids. In science, students may have learnt about volumes of liquids or gases. Students should recognise that they use different units to measure volume of liquids or gases. However, they should understand that in all cases volume is a measure of three-dimensional space taken up, regardless of whether it is a solid, liquid or gas being measured.

Support

A cube is a cuboid with all sides the same length. Imagine a cube with side length 10 cm.

Show how to use the formula for volume of a cuboid to work out the volume of this cube.

Do you need to remember two different formulae for volume of a cube and volume of a cuboid?

Volume of a cuboid = □ × □ × □

For a cube, the length = width = height, so the volume of this cube = 10 × 10 × 10 = 1000 cm^3.

No, you can use the same formula for a cuboid and a cube.

Depth

a Why is area measured in square units?

b Why is volume measured in cube units?

a *Area is measured in square units because length × length = length².*

b *Volume is measured in cube units because length × length × length = length³.*

Purposeful practice Reflect and reason

Purposeful practice 1

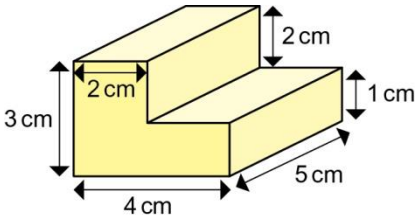
Mia says, 'A cube is a type of cuboid, so you can use the formula □ = □□□ for cubes and cuboids.'

Is Mia correct? Explain why.

Mia is correct. A cuboid is a 3D solid with 6 rectangular faces. A square is a rectangle. So a cube is a 3D solid with 6 rectangular faces: a cuboid. The formula □ = □□□ works for all cuboids. For the special case of a cube, □ = □□□ = □ × □ × □ = □³.

Purposeful practice 2

Here is Sophie's calculation for the volume of this solid.



$V = 3 \times 4 \times 5 = 60 \text{ cm}^3$

Explain what Sophie has done wrong.

Sophie has chosen the 'longest' height, length and width values for the solid and substituted these into the volume of a cuboid formula. But the shape is not a cuboid. To find the volume you need to split it into two cuboids.

Objectives

Core objectives

- Sketch nets of 3D solids
- Draw 3D solids on isometric paper
- Draw plans and elevations of 3D solids

Support objectives

- Sketch nets of 3D solids
- Draw 3D solids on isometric paper

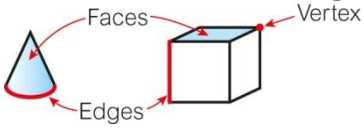
- Draw plans and elevations of 3D solids

Depth key concepts

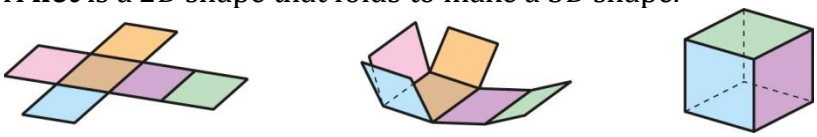
- Understand that different representations of a 3D shape convey different information about the faces and edges of the shape, and move between different representations.

Key points

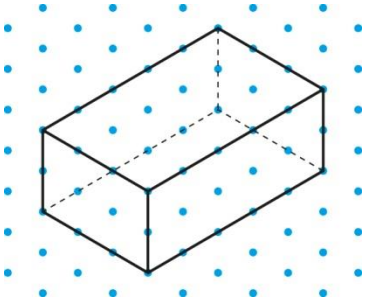
- A 3D solid has **faces**, **edges** and **vertices**. Faces and edges can be flat or curved.



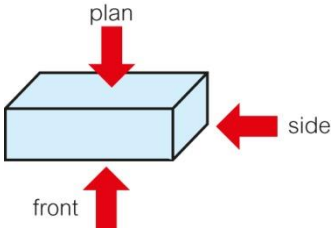
- A **net** is a 2D shape that folds to make a 3D shape.



- You can draw 3D solids on **isometric paper**.
  This cuboid has width 3 cm, length 5 cm and height 2 cm.



- The **plan** is the view from above the object.
  The **front elevation** is the view of the front of the object.
  The **side elevation** is the view of the side of the object.



Common errors and misconceptions

- When making a net, students often draw inaccurate lines and don't fold along a line accurately. Get students to sketch the net first, and then draw the net on squared paper to make sure lines are straight. Score the fold lines on the net before trying to fold.
- Drawing on isometric paper may be confusing for some students. In particular, they may try to draw horizontal lines. Make sure that their isometric paper is oriented correctly, and get them to turn their textbooks slightly so the cuboids are angled in the same way they need to be drawn.

Question guidance

Core

Q4 Check that students understand the terms vertex, parallel sides and perpendicular sides.

Q10 Suggest students make the net for the cube first. Fold it to check it makes a cube. The final step is to make it into a dice by drawing the dots on the faces, making sure that the numbers on the opposite faces add up to 7.

Q12 Emphasise that the question says *draw* and not *sketch*, so drawings need to be accurate. Make sure students write the lengths onto their diagrams and label them plan, front and side elevation.

Q13 If students can think of more than one answer for each part, and then encourage them to write down all their possible answers. Discuss the different answers with the whole class.

Support

Nets Q1 This question should prompt students to realise that a net of a cuboid has 6 faces which are rectangles/squares.

Isometric drawings Q1 Make sure students' isometric paper is oriented correctly.

Plans and elevations Q1 It may help to have physical models of similar cuboids to help students visualise the plan and elevations.

Purposeful practice

Purposeful practice 1 gives practice in using the shapes of faces to identify nets.

Reflect

Core

In this lesson you have learned about nets.

Sketch all the nets that a tea bag manufacturer might use.

Beside each one, explain

a what shape the net will make

b the advantages and disadvantages of using this shape.

This task asks students to consider nets in the context of real life. Students may think of, and draw, the nets for round tea bags, square tea bags, pyramid tea bags, as well as the boxes that tea bags are packaged in. They may use their understanding of tessellation to consider wastage for different nets of tea bags, or different shapes and sizes of boxes.

Support

In this lesson you have drawn 3D solids in different ways – nets, isometric drawings and plans and elevations.

Which type of drawing did you find easiest? Explain why.

Which did you find hardest? Explain why.

Students' answer will vary. Use models of the solids and encourage students to look at the solids from different angles to help them understand how to project from what they see to what they draw.

Depth

In **Q3**, you were asked to draw the side elevations of a curved object.

This diagram shows the side elevation of a 3D solid.

a Is there enough information for you to identify the shape? Explain why or why not.

b What other representation of this shape would give you more information?

a *There is not enough information. From the side view you cannot tell the shape of the top face (it could be a circle, semicircle, square, rectangle or triangle) or the shapes of the other sides. shapes.*

b *A net would show you all faces.*

Purposeful practice Reflect and reason

Purposeful practice 1

How can you use the shapes of faces of a 3D solid to help you identify its net?

Students' own answers along the lines of identifying the same shapes in the net and on the solid.

Objectives

Core objectives

- Calculate the surface area of cubes and cuboids

Support objectives

- Calculate the surface area of cubes and cuboids

Depth key concepts

- Know that two cuboids can have the same volume but different surface area, but all cubes with the same volume have the same surface area.

Key points

- The **surface area** of a 3D solid is the total area of all its faces. You can draw a net to help you find the surface area.

Common errors and misconceptions

- Students may find the volume instead of surface area. Remind students to read the question carefully. If they're asked to find area, they need to think of the net of the box, and think of the area of cardboard that is needed to make the box.
- Students may not find the area of all six faces of the cube/cuboid. Make a sketch of the net of the cube/cuboid and write the area of each face on the net so that none are forgotten. Remind students that they need to find three pairs of faces.

Question guidance

Core

Q4 *Did you sketch a net? How else could you work it out?*

Q7c Remind students that letters are used to represent the lengths of sides and the letters can be treated in the same way as numbers in formulae. Show a letter or letters to represent surface area and put this equal to the answer to part **b**, e.g. □□ = $6x^2$.

Q11 Suggest students draw a sketch of the building with the length, width and height correctly labelled. They can then identify the glass faces and work out the area of those faces only.

Support

Surface area of cubes Q3 Discuss ways of calculating the surface area of the cube without drawing the net, using what students know about the area of the square and the number of squares on the cube.

Surface area of cuboids Q1 Encourage students to identify the matching pairs of faces.

Purposeful practice

Purposeful practice 1 encourages students to identify the number of identical faces first, to save time on calculation.

Purposeful practice 2 Q2 gets students calculating surface area and working backwards to calculate side length from surface area of the cube.

Reflect

Core

This lesson showed you two methods for finding the surface area of a cube or cuboid.

Method 1: draw then add

Draw a net, write the area of each face on the net, add them together (**Q9**).

Method 2: visualise then calculate

Visualise pairs of opposite faces, calculate 2 × area of face for each pair, add them together (as shown in the worked example).

Which method did you prefer? Why? What are the advantages and disadvantages of your preferred method?

Students can sometimes work through lessons without acknowledging that they have learned different approaches to the same kinds of question. This task encourages students to recognise two different approaches to working out the surface area of a cube or cuboid.

They are asked to comment on their preferred approach, its advantages and disadvantages. Students may mention that the advantage of drawing and adding (also a disadvantage to visualising and calculating) is that it ensures every face is included. However, this method may take longer.

Support

Look at the nets you drew for cuboids in this lesson.

For each one, how many faces are identical shapes?

How can this help you calculate the surface area?

For a cuboid, there will be three pairs of identical faces. Identifying the identical faces can help speed up calculations, because you only need to work out three areas.

Depth

a Is it possible for two cubes to have the same volume but different surface areas?

b What about cuboids? Explain.

a *It is not possible for two cubes to have the same volume but different surface areas because the edge lengths must be the same in order that the cubes have the same volume.*

b *Cuboids can have the same volume but different surface areas as the edge lengths do not need to be the same.*

Purposeful practice Reflect and reason

Purposeful practice 1

How does identifying identical faces help you calculate the surface area of a 3D solid?

Saves time and calculations. For example, a cube has 6 identical faces, so if you calculate the area of 1 face, you can multiply by 6 for the surface area of the cube.

Purposeful practice 2

How have you used square numbers and square roots in these problems?

Squares to calculate area of one face.

Square roots to calculate side length from area.

Objectives

Core objectives

- Solve problems in everyday contexts involving measures
- Convert between different measures for area, volume and capacity
- Use tonnes and hectares
- Know rough metric equivalents of imperial measures

Support objectives

- Solve problems in everyday contexts involving measures
- Convert between different measures for area, volume and capacity
- Know rough metric equivalents of imperial measures

Depth key concepts

- Know the relationship between km, metres (cm) and mm, litres and ml, kg and g, and understand how all units in the metric system are multiples/divisors of a 'base' unit (metres, grams, litres, etc) - extend to tonnes, hectares etc
- 1 cm = 10 mm, so 1 cm^2 = 10^2 mm^2 and 1 m = 100 cm, so 1 m^2 = 100^2 cm^2

Key points

- $\times 1000$ ($1\,ml = 1\,cm^3$ / $1\,litre = 1000\,cm^3$) $\times 1000$
- The **capacity** of an object is the volume it can hold.
- Mass: 1 **tonne** (t) = 1000 kg
- You need to convert lengths into the same units before calculating areas or volumes.
- 1 cm^2 = 100 mm^2 1 m^2 = 10 000 cm^2
- Area: 1 **hectare** (ha) = 10 000 m^2

- You need to know these conversions between metric and imperial units.
  1 **foot** (**ft**) ≈ 30 cm 1 **mile** ≈ 1.6 km 1 kg ≈ 2.2 **pounds** (**lb**)
  1 litre ≈ 1.75 **pints** 1 **gallon** ≈ 4.5 litres

Common errors and misconceptions

- Students may get confused about multiplying or dividing when converting between units. Suggest to students that they draw a diagram showing the units they are converting. Write a **B** (for bigger) under the bigger unit and an **S** (for smaller) under the smaller unit.
  - To convert from the bigger unit to the smaller one you multiply by the conversion factor because there will be more of the smaller unit.
  - To convert from the smaller unit to the bigger one you divide by the conversion factor because there will be fewer of them e.g.
    1 litre = 1000 cm^3
    B **S**
    B × 1000 → **S**
    B ← **S** ÷ 1000

Question guidance

Core

Q4 Suggest students take this one step at a time to work out the total amount of medicine Sara needs over the 4 days. Then they can convert the 0.15 litres into ml and compare with their answer.

Q10 *How did you decide which units to use?*

Q11 Suggest students draw two squares identical in size. Give the first a side length of 1 cm and the second a side length of 10 mm. They can then work out the area of both squares and use their answers to complete the conversion.

Q14 The plot of land is a trapezium, so students will need to remember that the formula for the area of a trapezium is $A = \frac{1}{2}(a + b)h$.

Q16 Students need to use the conversion 1 ft ≈ 30 cm. They can either convert 5 ft to cm then m, or 1.4 m to cm then ft. Discuss the advantages and disadvantages of each method.

Q17 Students can either change the miles to km or the km to miles. Discuss which is easier given the conversion 1 mile ≈ 1.6 km.

Support

Converting between ml, litres and cm^3 Q3 This question can be also answered without converting units by recognising that 250ml is one quarter of 1 litre, so 1 litre will fill 4 glasses (and 2 litres will fill 8 glasses).

Converting between metric and imperial measures Q3, **Q6**, **Q10** Encourage students to work out whether their conversion should be a bigger or smaller number than the number they start with.

Purposeful practice

Purposeful practice 1 shows two ways of getting answer in given square units

- convert **lengths before** calculation, or
- convert **area after** calculation.

Reflect

Core

You need to remember the approximate conversions between some metric and imperial units.

Dee says, 'For feet, I remember that my ruler is 1 foot long and I know that is about 30 cm.'

Look at the other metric and imperial conversions in this lesson.

Write down a way to help you remember each of them.

Students are not often given the time to consider how they may best memorise facts. This reflective task gives students examples of how others memorise metric and imperial measure conversions, and asks students to come up with their own ways of remembering.

Support

Look back at the conversions in this lesson.

Copy each statement and write < or >.

1 pint ☐ 1 litre

1 gallon ☐ 1 litre

1 kg ☐ 1 pound

How does knowing which unit is larger help you to check if your conversions are sensible?

Knowing which unit is larger helps you see if your conversion should give a bigger or a smaller number.

Depth

a What unit would you use to measure the weight of

i a hamster

ii a blue whale

iii a sheep?

b What unit would you use to measure the area of

i a garden

ii a playing field

iii a laptop screen?

c Explain how you decided which units to use.

a i *g* ***ii*** *tonnes* ***iii*** *kg*

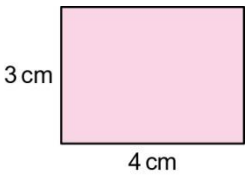
b i *m^2* ***ii*** *m^2 or ha* ***iii*** *cm^2*

c *Students' own answers, e.g. a hamster is small, so you would use grams to measure the weight, etc..*

Purposeful practice Reflect and reason

Purposeful practice 1

Rory works out the area of this rectangle.



$3 \times 4 = 12$ cm^2

12 cm^2 = 120 mm^2

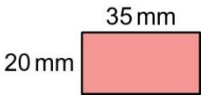
What has Rory done wrong?

Rory multiplied by 10 instead of 100. 12 cm^2 = 12 × 100 = 1200 mm^2

Purposeful practice 2

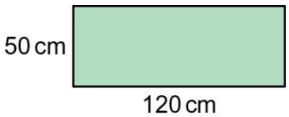
Explain how you would work out the area of each shape in the units given.

a



Area = ☐ cm^2

b



Area = ☐ cm^2

Students' own description of method, e.g. convert length first, then multiply or multiply then convert area with reason why they chose this method. For example:

a *2 × 3.5 is an easier multiplication*

b *500 mm × 1200 mm is 'harder', so prefer 50 × 120 = 6000 and then × 100 to convert to* mm^2

Check, Strengthen and Extend

Core Check up

Q3 This question tests that students use the perpendicular height and not the slant height.

Q6 This question aims to check that students can work out the surface area of a cube and cuboid without drawing a net, so they must remember to include all 6 faces.

Q11 In this question, students need to know the metric to imperial conversions, and be able to use them correctly.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

In this unit you have had to do lots of different things to find the answers to questions.

Write these in order, from the one you found easiest to the one you found hardest.

A Knowing which is the perpendicular height in a shape.

B Using a formula to find the area of a shape.

C Working out what the net of a 3D solid will look like.

D Finding the surface area of a cuboid.

E Knowing when to multiply and when to divide when converting measures.

Write a hint, in your own words, for the one you found the hardest.

This task asks students to identify the things they found easier and more difficult to do.

For example, they may have struggled with finding the perpendicular height (**A**), using a formula (**B**), or visualising a net (**C**).

Students are asked to write themselves a hint for the thing they found most difficult. For example, for using a formula, their hint may be to draw the shape and label the sides, then write the formula in the middle of the shape, before substituting in any numbers.

Extend

Look back at **Q5**. It asked you for a 'counter example'. What did this counter example show about Caroline's statement?

In what sort of situation might you need to prove that a statement is untrue? Could you use a counter example? Explain.

This task highlights the words 'counter example', which is likely to be a new concept for many students. Counter examples are a useful precursor to thinking about mathematical proof, as they demonstrate when something is not (always) true. Counter examples are useful in science too, e.g. when the results of an experiment or observation do not fit the theory (e.g. ladybirds have 6 spots, until a ladybird with 2 spots is observed).

A teacher may use counter examples to sort out a dispute between students, e.g. That is my peg, I always hang my coat there. Counter example: No, you didn't last Friday.

Depth

Extend

To describe a 2D shape, you can name the shape, or the shapes it is made from, draw a diagram, and calculate its perimeter and area.

How can you describe a 3D solid?

Name the solid, describe the solids it is made from, calculate its volume, surface area, mass, draw a net, plan, elevations, isometric drawing.

End of Unit 2 tests: Area and volume

Pearson published resources

Support/Core

- Year 8 End of Unit 2 Support/Core test
- Year 8 End of Unit 2 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 2 Core/Depth test
- Year 8 End of Unit 2 Core/Depth mark scheme

Unit opener: Statistics, graphs and charts

Pearson published resources

Objectives

Core objectives

- Interpret pie charts

- Draw pie charts

Support objectives

- Interpret pie charts
- Draw pie charts

Depth key concepts

- Understand that pie charts show the proportions of data

Key points

- A **pie chart** is a circle divided into slices called **sectors**. Each sector represents a set of data.

Common errors and misconceptions

- Students may have difficulty in drawing angles accurately. Warm up **Q3** gives practice in drawing angles. Demonstrate the method using a protractor, emphasising the need to line up the zero line with the base line of the angle. Make sure students know how to read the correct scale (start at 0 and count up, to make sure you have the correct one), and make sure they use a sharp pencil.
- Some students may draw all the angles from the first radius, rather than from edge of previous sector. Encourage students to label each sector with what it shows, as they complete it, to help reinforce that the sector is now finished and they should start a new one for the next data item.

Question guidance

Core

Q5–Q8 Students can work with fractions and percentages only, and don't need to use angles.

Q9a Make sure students calculate the total correctly. Errors here will make the rest of the question impossible.

Q12 Students may draw angles inaccurately, or draw each angle from the original radius, rather than from the edge of the previous sector.

Support

Interpreting pie charts Q1 Students need to be competent at identifying fractions of a circle.

Interpreting pie charts Q2 If necessary remind students how to find a fraction of a quantity.

General Some students find using a protractor tricky. You may wish to spend some time revising this skill.

Purposeful practice

Purposeful practice 1 gives practice in interpreting pie charts using fractions.

Reflect

Core

Rosie says that fractions help you to interpret pie charts (as in **Q5**). What other areas of mathematics help you to interpret pie charts?

What maths skills do you need to draw pie charts?

This task aims to emphasize the mathematics concepts and skills necessary for interpreting and drawing pie charts. It encourages students to reflect on their interpretation and drawing of pie charts, and to make links between different areas of mathematics topics.

Students should recognise, for example, that an understanding of percentages is useful for interpreting pie charts, and that accurate angle measuring is a useful skill for drawing pie charts.

Support

What did you find easiest when drawing pie charts? What did you find most difficult?

What skill do you think you need to practise?

Students' answers will vary, but may include difficulties with calculating angles, or calculating fractions from given angles. Remind students that there are 360° in a circle and that it helps to remember 180° in a semicircle (50%), 90° in a quarter circle (25%).

Depth

Francesca is comparing two pie charts.

She says, 'The pie chart shows that more students take history at GCSE in School A than in School B.'

Explain the problem with Francesca's statement.

Francesca can only know the numbers of students taking history if she knows the numbers of students in each school. Without this information, she can only compare the proportions of students taking history.

Purposeful practice Reflect and reason

Purposeful practice 1

In **Q3** and **Q4**, do you need to work out the number of students who chose each subject to work out the total number of students?

No, for example, if you know $\frac{1}{2}$ then multiply by 2 to find the total, if you know $\frac{1}{4}$ then multiply by 4 to find the total and if you know $\frac{1}{8}$ then multiply by 8 to find the total.

Purposeful practice 2

Did the table in **Q1** help you work out the angles in **Q2**? Explain.

Students' own answers, e.g. from the table 25% = 90°; from the table 10% = 36° so double to find 20% and halve to find 5%.

Objectives

Core objectives

- Calculate the mean from a frequency table
- Use two-way tables
- Use tables for grouped data

Support objectives

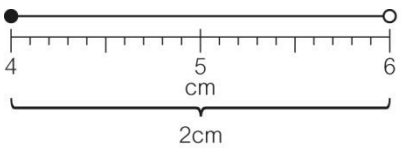
- Calculate the mean from a frequency table
- Use two-way tables
- Use tables for grouped data

Depth key concepts

- Understand that a table presents data from lists or that could be represented in other types of diagram. Move between tables and other representations
- Understand that the method for calculating mean from a frequency table is the same as the method for calculating the mean from a list, but is more efficient
- Understand which average is most appropriate to represent a set of data for particular contexts

Key points

- A **two-way table** divides data into groups in rows across the table and in columns down the table. You can calculate the totals across and down.
- The class $4 \leq l < 6$ includes all values of length l from $l = 4$ cm up to, but not including, 6 cm.



The width of this class is 2 cm. ≤ means 'less than or equal to'.

Common errors and misconceptions

Students may put boundary values in wrong classes. Emphasise that the < sign means that x cannot be equal to that value.

When calculating the mean using data in tables, students may divide by the number of rows in the table, not by the total frequency. Encourage students to get in the habit of adding an extra column to the table to calculate the totals by multiplying the number by the frequency.

Question guidance

Core

Q4–7 These questions have been carefully constructed to give students practice in calculating means using tables with increasingly complex ranges of values and frequencies.

Q11c Tim's aim is to make the biggest possible profit so he should cut the least profitable item.

Q12e You can only estimate the range, using the largest possible and smallest possible lengths, 8 â˜’ 0 = 8 cm. This is not the same as using the actual lengths.

Support

General When calculating averages encourage students to think carefully about sensible answers – is the average representative of the data?

Calculating the mean from a frequency table Q2 If necessary, model this showing 17 lots of 13 and 12 lots of 14.

Two-way tables General Student often struggle with two-way tables. Take some time to make sure students understand the data being shown. Establish that the totals horizontally should sum to the same as the totals vertically.

Grouped data General Ensure that students understand and can use the inequalities symbols before you attempt this section.

Purposeful practice

Purposeful practice 1 gives practice in calculating means from frequency tables including cells with 0 value, increasing number of classes, and 'harder' numbers.

Purposeful practice 2 gives practice in grouping data into equal width groups.

Reflect

Core

Freddie and Claudia are talking about tables.

Freddie says, 'Tables show information in columns and rows.'

What do you think of Freddie's definition of a table? Is it true for all the tables in this lesson?

Where else do you see tables displaying information in everyday life?

This task encourages students to consider what a table is, and its use in everyday life. Some students may find it difficult to read tables in maths, but once they recognise their everyday use (for example, school timetables, bus timetables, sports league tables), then this should make them more comfortable with tables in maths too.

Support

Look at the tables you have used in this lesson. Why are tables a good way of displaying information?

Students' own answers, but should include reference to the organisation of data, making it easier to work out which data and which calculations to use.

Depth

In **Q6**, you were asked to compare data using tables and using diagrams.

a Which did you find more useful and why?

b Why did you choose the diagram you did to represent the data?

a *Students' own answers, for example, 'The diagram was more useful because it was easier to see the shape of the pattern.'*

b *Students' own answers, for example, 'I chose a bar chart so I could see where the most marks were.'*

Purposeful practice Reflect and reason

Purposeful practice 1

Lola says, 'If one of the values is zero, you can ignore that row in the table, because any number × 0 = 0.' Look at **Q2** and **Q3** and explain why Lola is wrong.

*If the zero is in the right-hand column (frequency column) the product number × frequency = 0, and the frequency for the row is also 0 so does not affect the total frequency. If the zero is in the left-hand column (e.g. table in **Q2**, first row), you cannot ignore the row because you need to include the frequency (12) when calculating total frequency.*

Purposeful practice 2

Find the smallest and largest values in the set of leaf lengths. How can these help you work out the classes for **Q2**?

Do you always need to start the first class at zero?

Smallest is 2.1 cm, largest is 13.9 cm. All data is in between 2 cm and 14 cm so you can use these as smallest and largest values for the classes. You don't have to start at zero when there are no values of zero or ≤1.

Objectives

Core objectives

- Draw stem and leaf diagrams for data
- Interpret stem and leaf diagrams

Support objectives

- Draw stem and leaf diagrams for data
- Interpret stem and leaf diagrams

Depth key concepts

- Understand the similarities and differences between stem and leaf diagrams and bar charts, including back to back bar charts and stem and leaf diagrams.

Key points

- In a set of 9 data values, the median is the $\frac{{{\rm{9 + 1}}}}{{\rm{2}}}{\rm{ }}$ = 5th value
  In a set of 10 data values, the median is the $\frac{{{\rm{10 + 1}}}}{{\rm{2}}}{\rm{ }}$ = $\frac{{{\rm{11}}}}{{\rm{2}}}{\rm{ }}$ = 5.5th value
  In a set of 9 data values, the median is the $\frac{{n{\rm{ + 1}}}}{{\rm{2}}}$ the value
- A **stem and leaf diagram** shows numerical data split into a 'stem' and 'leaves'. The key shows you how to read the values.

Common errors and misconceptions

- Students may not order the leaves. Emphasise that drawing the diagram is a two-step process
  - Step 1 is to decide on the stem, write in the leaves as you cross them off the data list
  - Step 2 is to copy the data with the leaves in order.
- Students may write only the leaf as the mode or median and forget the stem. *Is your answer sensible? Is it one of the data values? For example, if the student says the modal parking time is 2 minutes, ask Did anyone park for only 2 minutes?*

Question guidance

Core

Q5, **Q6** These questions have been carefully constructed to give students practice in creating stem and leaf diagrams using larger data sets and data values.

Q7f Because the data in a stem and leaf diagram is already ordered.

Q9 Students should find median and mode for each diagram and compare them, to see if either has increased by £1.

Q9, **Q11** These questions have been carefully constructed to give students practice in interpreting stem and leaf diagrams showing larger data sets, including back-to-back stem and leaf diagrams.

Support

Find the median of a set of data General Students often find the median term by counting from both ends of the data and hence make errors. The section focuses on finding the median term.

Find the median of a set of data Q3 Discuss with students what to do when there is an even number of pieces of data. A common misconception is that $\frac{1}{2}$ of an even number can be calculated and this value will be the median term. Model this by putting an even number of students in a row and asking who is in the middle, repeat for an odd number of students.

Purposeful practice

Purposeful practice 1 gives practice in finding median for different numbers of pieces of data.

Purposeful practice 2 gives practice in finding the mode, median and range from stem and leaf diagrams with different keys/same numbers.

Reflect

Core

Hannah, Sam and Tilly discuss how they use worked examples.

Sam says, 'I read the question, then the answer, then all the note boxes telling me what to do.'

Hannah says, 'I only read the note boxes and bits of the answer when I get stuck.'

Tilly says, 'I read the question, then the first part of the answer and its notes.

Then I read the next bit of the answer and its notes, and so on.'

Describe how you read the worked example for this lesson.

Try reading it again in different ways (like Sam, Hannah and Tilly).

Which way do you think is best? Why?

This task aims to encourage students to be independent learners, and make best use of the worked examples in lessons. It urges them to think about how they currently read worked examples and whether they may employ alternative and better strategies.

Support

A scientist calculates $\frac{{{\rm{20 + 1}}}}{{\rm{2}}}{\rm{ = 10}}{\rm{.5}}$

The scientist says: 'The median length of the worms is 10.5 cm.'

Explain what the scientist has done wrong.

The scientist has identified which term is the median term, but this is not the median value.

Depth

Compare a bar chart and a stem and leaf diagram. Which is better for:

- finding the modal class
- finding the median
- finding the mode
- showing the shape of the data distribution?

It is easier to find the modal class from a bar chart. It is easier to find the median and mode from a stem and leaf diagram. They both show the shape of the data distribution clearly.

Purposeful practice Reflect and reason

Purposeful practice 1

For the set of numbers 12, 15, 18, 20, 20, 21

Jo writes

$n{\rm{ = 6, }}\frac{{n{\rm{ + 1}}}}{{\rm{2}}}{\rm{ = }}\frac{{\rm{7}}}{{\rm{2}}}{\rm{ = 3}}{\rm{.5}}$ so median = 3.5

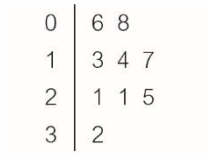
Explain what Jo has done wrong.

Jo has given the position of the median as the median. The median is the 3.5th value, which is halfway between the 3rd and 4th values, so the median is halfway between 18 and 20 = 19.

Purposeful practice 2

Three students find the mode, median and range of this data.

	Kate	Leo	Jonah
Median	7	21	2.1
Mode	1	17	1.7
Range	2 − 6 = −4	26	2.6



Who is correct? Explain what the others have done wrong.

Jonah is correct. Kate has only used the 'leaves' i.e. mode is 1, not 21, range is 2 â˜ 6, not 32 â˜ 6, median is 7 not 17. Leo has not used the key. He has the correct digits but not correct values.

Objectives

Core objectives

- Compare two sets of data using statistics or the shape of the graph
- Construct line graphs
- Choose the most appropriate average to use

Support objectives

- Compare two sets of data using statistics or the shape of the graph
- Construct line graphs
- Choose the most appropriate average to use

Depth key concepts

- Understand how to make comparisons between data.

Key points

- A small range shows that the data items are close together. This means the data is more **consistent**.
- **Statistics** are values that represent a set of data. Mean, median, mode and range are statistics.
- An extreme value that doesn't fit the pattern of the other values is called an **outlier**.

Common errors and misconceptions

- Students may not know where to start in a 'compare the data' question. Emphasise there should be two sentences in their answer
  - one comparing an **average**
  - one comparing the **ranges**.

Suggest suitable sentence openings: *'The mean/median/mode for …' and 'The range for …'*

- Students may not be able to decide which is the most appropriate average to use in an 'open' question. Suggest a strategy to work out all the averages, and then decide which is closer to most of the data values.

Question guidance

Core

Q5 A company that makes ice cream would expect to sell more in the summer, i.e. 3rd quarter.

Q8d All the questions can be answered simply by looking at the shape of the graphs. Values can give extra detail, e.g. the greatest and least difference in scores. If students didn't use the shape of the graph, encourage them to do so now.

Q6d The mean is not a real shoe size, so is not very helpful in this situation. The range of sizes given is 3.5 to 8.5, but there were no sales of 3.5 and 8.5, so the actual range is 8 â^' 4 = 4. The manager will need to buy shoes at all sizes in the range.

Q7 Extreme values or anomalies affect the mean as they are included in the total. An extreme value at one end of the data does not affect the median, which only considers the middle value.

Support

Using a graph to compare two sets of data General You may wish to discuss why it is sometimes easier to compare data on a graph than simply numerical data.

Purposeful practice

Purposeful practice 1 Q1 demonstrates that greater range means the data is less consistent and vice versa.

Purposeful practice 2 Q1 demonstrates that a higher mean does not mean that all values are higher.

Reflect

Core

In this lesson, you compared data by comparing averages and the range, and by comparing graphs or pie charts of the data.

Which did you prefer, and why?

Students' answers will vary. Some students will find the visual comparisons with graphs and pie charts easier, but make sure their comparisons are valid (e.g. not getting confused about proportions in pie charts compared with actual data values). Students may not be confident about choosing the right averages for comparisons.

Support

In this lesson you have used different averages in different situations. Why do you think it is important to know which 'average' you are given?

Discuss your answer with a partner.

You have to know which average gives the best representation of the data, and the purpose the average is used for.

Depth

Two students have a mean score of 78 in general knowledge tests.

What other measure do you need, to decide which student would be best to represent the school in a general knowledge quiz? Explain.

Need the range, so we could see which student's score was most consistent.

Purposeful practice Reflect and reason

Purposeful practice 1

What does the range tell you about a set of data?

How consistent, or not, the data values are.

Purposeful practice 2

Two more students, F and G, took all the tests.

The mean for student F is 5.3 and the mean for student G is 7.1.

Jim says, 'This shows G always does better in the tests than F.'

Sally says, 'You can't tell who did better in each test from these statistics.'

Who is correct? Explain why.

Sally. F may do better in some tests than G but still have lower mean.

Objectives

Core objectives

- Draw a scatter graph
- Draw a line of best fit on a scatter graph
- Describe types of correlation

Support objectives

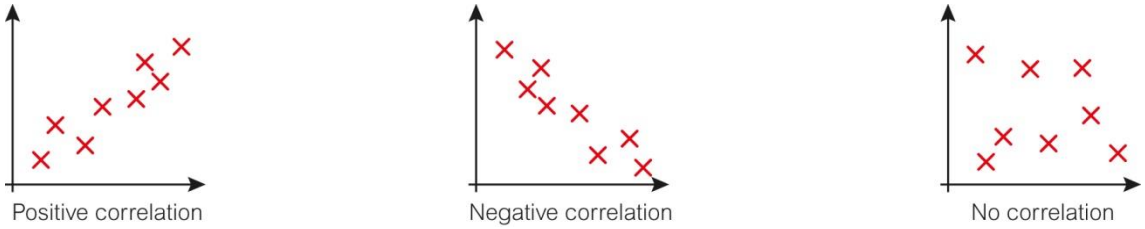
- Draw a scatter graph
- Draw a line of best fit on a scatter graph
- Describe types of correlation

Depth key concepts

- Deepen understanding of correlation by considering examples where there is weak or no correlation, as well as examples where there is correlation that you might not expect (between two seemingly random quantities)

Key points

- A **scatter graph** plots two sets of data on the same graph. The shape of the graph shows if there is a relationship or **correlation** between them.



- A **line of best fit** shows the relationship between two sets of data. Draw a line of best fit so that there are the same number of crosses on each side of the line. There may or may not be crosses on the line.

Common errors and misconceptions

- Students may join the points with straight lines. Emphasise that a scatter graph shows a scatter of points, not a line.
- Students may think the line of best fit has to go through zero. Emphasise that the line has to go through the middle of the data points (i.e. there should be roughly the same number of points on either side of the line), should follow the shape of the data distribution, and doesn't need to go through the first and last points.

Question guidance

Core

Q7 Students could use a spreadsheet to produce this graph. Data is entered as a vertical table, and the spreadsheet program is then used to insert a scatter graph. They can edit or adjust axis labels and scales. Poverty is defined as having less than 60% of median income.

Support

Plotting a scatter graph Q2 The data in the table does not follow simply from smallest to biggest. Emphasise the need to treat the data as pairs of coordinates, read vertically in the table, and that they do not follow a pattern.

Drawing a line of best fit Q3 Lines of best fit should not be forced through the origin.

Purposeful practice

Purposeful practice 1 gives practice in understanding relationships and correlations.

Purposeful practice 2 provides practice in drawing suitable lines of best fit.

Reflect

Core

Think about the different ways to display data. Think back to Year 7 too.

- Which is the easiest to read and understand? Which is the hardest? Why?
- Which is the easiest to draw? Which is the hardest? Why?

This task asks students to consider different ways of displaying data and to say which they find easiest and hardest to read and understand, and why. This helps students to recognise where they may need more practice at interpreting charts and diagrams. Students are also asked which ways of displaying data they find easiest and hardest to draw. Students may find it easiest to read and understand bar charts because they are familiar with them, and perhaps pie charts hardest to draw because they involve a calculation, as well as accurate angle drawing.

Support

All the data sets in this lesson could be represented on scatter graphs.

What type of data can you show in a scatter graph?

You can use data where there is a relationship between one set of values (on one axis) and a second set of values (on the second axis).

Depth

A study was conducted in the USA that found that there was a strong positive correlation between police numbers and crime rates. Does this mean that increasing the amount of police will increase the amount of crime? Explain your answer.

No, it does not. It is likely that the high amount of crime causes more police to be hired. Just because there is correlation does not mean that increasing one variable will increase the other.

Purposeful practice Reflect and reason

Purposeful practice 1

What type of correlation is it when

a as one quantity increases, the other decreases

b as one quantity increases, the other increases?

a Negative correlation

b Positive correlation

Purposeful practice 2

Does a line of best fit have to go through (0, 0)?

Does a line of best fit have to pass through some data points?

Use examples from this page to explain.

*No to both. For example, in **Q1**, in graphs **b** and **c** the line of best fit does not pass through (0, 0).*

*In **Q1c** the line of best fit does not pass through any data points, though the other examples do.*

Objectives

Core objectives

- Interpret graphs and charts
- Explain why a graph or chart could be misleading

Support objectives

- Interpret graphs and charts
- Explain why a graph or chart could be misleading

Depth key concepts

- Understand when a statistical diagram is appropriate or inappropriate to represent a set of data, e.g. when to use a bar chart or stem and leaf diagram and when to use a pie chart.

Key points

- Before you read values from a graph or chart
  - read the title
  - read the axis labels
  - read the scales.

  You cannot draw accurate conclusions from an inaccurate graph.
- Changing the scale can make a graph look very different.

shows the axis has ‘missing values’.

Common errors and misconceptions

- Students may assume that scales go up in equal steps of 'nice numbers' such as 1, 2, 5, 10. Encourage a systematic approach to interpreting graphs.
  - Read the title.
  - Read the axis labels.
  - Read the scales.
  - Then read off any values.
- Students may assume that all scales start at zero. Emphasise that not all graphs that do not start at zero have a crumple zone/discontinuity symbol.

Question guidance

Core

Q4 and Q5 These questions introduce students to the discontinuity symbol and reinforce the importance of reading scales

Q6 Refer students to the first Key point. This should help them identify any missing information from the graph.

Support

Misleading scales Q2 Explain to students what the symbol on the vertical axis means.

Misleading scales Q3 Explain to students that in politics data is often misrepresented or missed out to show a particular trend.

Misleading shapes Q1c Explain to students that this shows how important it is to only use one symbol of a constant size in a pictogram.

Misleading shapes Q2 You may wish to discuss with students why the first graph might be produced.

Purposeful practice

Purposeful practice Q1 Students should realise that they can only answer one question on each pie chart at first, they need the key to answer another question, and the total number to answer the last question. This will help students answer the Reflect and reason. Students should know that they need the title, the key and the total frequency to be able to interpret the pie chart.

Purposeful practice Q2 leads to the same Reflect and reason as Q1. Students should realise they need axis labels to help them interpret a bar chart.

Reflect

Core

List five ways that graphs can mislead you. You could begin with:

‘It is misleading when different symbols are used on the same _____ .’

When answering many different multi-part questions, students sometimes find it difficult to recognise the key points to take away from the lesson. This task asks students to look back at what they have learnt and summarise the key points, i.e. the ways in which graphs may be used to mislead.

Support

Marcus says: ‘If the data on a graph or chart is correct then it cannot be misleading.’ Do you agree? Explain.

Not necessarily. Although the data might be correct, it might still be misleading if scales or values on axes are shown in a certain way, so that trends are not accurately represented.

Depth

When looking at a graph or chart in the news or on social media, what would you recommend people look out for to avoid being misled?

Students’ own answers, but could include reference to the numbers given on the axes (do they start at 0?), time spans, whether pie charts show proportions accurately, whether percentages add up to 100, etc.

Purposeful practice Reflect and reason

Purposeful practice

What information do you need to be able to interpret

a a pie chart

b a bar chart?

a *For a pie chart: title, key, total number represented*

b *For a bar chart: title, scales on axes, axis labels*

Check, Strengthen and Extend

Core Check up

Q1 This question addresses the common error of writing mode and range as the frequency values (mode 9 and range 9 â˜’ 3 = 6), rather than the data values (mode 2, range 4 â˜’ 0 = 4)

Q2 This question addresses the common error of not comparing both the mean and the range, not understanding what the range shows.

Q3 This question addresses the common error of not identifying the ‘easy’ values to fill in first

Q4 This question checks whether students can deal with boundary data values and understand boundary notation, also whether they give units with the modal class and range.

Q5 In this question students have to interpret a simple pie chart. Part **c** involves finding the total number of data values, given the number represented by one segment. Students need to realise that 90° represents $\frac{1}{4}$ of the data.

Q6 This question checks that students can calculate angles accurately from a table, measure each sector accurately from the edge of the one before, give a key.

Q7 In this question students have to interpret a stem and leaf diagram, checking that students use the key, and correctly find the correct value of the median

median = $\frac{n+1}{2}$th value

Q8 In this question students have to identify correlation in a scatter graph and relating it back to the original context

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

For this Strengthen lesson, copy and complete these sentences.

I found questions _____ easiest. They were on _____ (List the topics.)

I found questions _____ most difficult. I still need help with _____ (List the topics.)

This task aims to raise students' awareness of the questions they found easiest and hardest in these strengthen lessons. It also encourages them to analyse what topics these questions covered, so that they may recognise where they may and may not require some additional help.

Extend

Q5 uses data that climate scientists might use. It also uses these maths topics

- line graphs
- mean and range.

List all the other maths topics you have used in these Extend lessons.

How might climate scientists use these maths topics too?

This reflective task asks students to analyse the topics they have covered in the Extend lessons. It then asks them to think how they may be of use to climate scientists, giving them the opportunity to think about the topics in a different context.

Depth

Extend

During this unit, you have examined many different ways of displaying data.

Give some examples of when you might use each one and why you would choose that diagram.

Students' own answers, but should accurately identify the correct use of the different types of data displays.

End of Unit 3 tests: Statistics, graphs and charts

Pearson published resources

Support/Core

- Year 8 End of Unit 3 Support/Core test
- Year 8 End of Unit 3 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 3 Core/Depth test
- Year 8 End of Unit 3 Core/Depth mark scheme

Unit opener: Expressions and equations

Pearson published resources

Objectives

Core objectives

- Understand and simplify algebraic powers
- Write and use expressions involving powers

Support objectives

- Understand and simplify algebraic powers
- Write and use expressions involving powers

Depth key concepts

- Understand that powers of variables are written in the same way as powers of numbers, and that ab^2 means $a \times b^2$ and not $(ab)^2$

- Understand that an algebraic expression is the generalisation of a rule or relationship
- Understand the meaning of 'variable' and that the choice of letter is not important when writing an expression

Key points

- In an algebraic expression, write numbers first and then letters in alphabetical order.

Common errors and misconceptions

- Students may multiply indices instead of adding them. Write each power of $p^2 \times p^3$ as a product (i.e. rewrite p^2 as $p \times p$ and p^3 as $p \times p \times p$), count up the letters, write the product as a power. *How many ☐'s are there?* (5) *How do you get 5 from the indices 2 and 3?* (Add them.)
- Students may not realise that p can be written p^1. Write $2^3 \div 2^2 = 2^{3-2} = 2^1$ and $2^3 \div 2^2 = 8 \div 4 = 2$ showing that $2^1 = 2$. This works for any number, so it also works for a letter because in mathematics a letter is a number. Write $a^1 = a$
- Students may be confused when trying to simplify. Errors may include incorrect simplifications such as $5^2 \times 10^3 = 50^5$, $2c + 3 = 5c$, $3x + 3y = 3xy$. Make sure students understand what can and what cannot be simplified.

Question guidance

Core

Q7 This question has been carefully constructed to give students practice in simplifying algebraic expressions of increasing complexity.

Q9bi Use the expression from **Q9a** and multiply by 3.

Q9bii Replace 3 by n in the answer to part **bi**.

Q9ci *How many squares are there in this array (grid)?* (5×10)

Support

Substituting into expressions Q1d Students may need to be reminded that xy is the same as $x \times y$.

Purposeful practice

Purposeful practice 1 reinforces the difference between na and a^n

Purposeful practice 2 provides practice in writing number and algebra products in index notation

Reflect

Core

Sally says that $4 \times x \times x$ is $4 \times 2x = 8x$.

Explain why Sally is wrong.

This task aims to make explicit what is happening when you multiply (for example, $x \times x$) and when you add (for example, $x + x$), and so addresses a common mistake. Most importantly, it asks students to explain the difference in their own words, and so articulate their understanding. It also offers a useful hint to substitute in a value of x and see what happens.

Support

Kathy asks, 'Are the results of 2^3 and 2×3 the same?'

Answer Kathy's question, then explain your answer.

No, they are not the same. $2^3 = 2 \times 2 \times 2 = 8$. $2 \times 3 = 6$

Depth

For each part

i state whether the pair of expressions are always, sometimes or never equal

ii if they are sometimes equal, state the values at which they are equal.

a $2x^2$ and $(2x)^2$ **b** $3x^2$ and $(3x)^2$ **c** $3x^3$ and $(3x)^3$ **d** ab^2 and $(ab)^2$

a i Sometimes ***ii*** When $x = 0$

b i Sometimes ***ii*** When $x = 0$

c i Sometimes ***ii*** When $x = 0$

d i Sometimes ***ii*** When $a = 0$, $a = 1$ or $b = 0$

Purposeful practice Reflect and reason

Purposeful practice 1

In algebra, how do you write

a 4 lots of a

b a multiplied by itself 4 times?

a $4a$

b a^4

Purposeful practice 2

You can write $2 \times 2 \times 5 \times 2$ in index form as $2^3 \times 5$ or 5×2^3.

Is there more than one way to write $r \times r \times t \times r$ in index form?

No, r^3t is the only answer as letters should be in alphabetical order and there is no $\times$ sign, $r^3 \times t$ or tr^3 do not follow algebraic conventions.

Objectives

Core objectives

- Expand brackets
- Write and simplify algebraic expressions and formulae using brackets and division

Support objectives

- Expand brackets
- Write and simplify algebraic expressions and formulae using brackets and division

Depth key concepts

- Understand when to use brackets when writing an expression, and when the brackets are not needed.
- Understand that in an expression like $\frac{{u{\rm{ }} - {\rm{ 10}}}}{{\rm{3}}}$, you treat the expression in the numerator as if it were written in brackets, when following the order of operations.
- Understand that an algebraic expression is the generalisation of a rule or relationship.
- Understand the meaning of 'variable' and that the choice of letter is not important when writing an expression.

Key points

- Division can be written as a fraction. For example, $a \div 3$ can be written as $\frac{a}{{\rm{3}}}$
- An **expression** uses variables (letters) to stand for numbers.
- A **formula** uses variables and an equals sign (=) to show the relationship between variables.

Common errors and misconceptions

- Students may only multiply the first term when expanding brackets. Demonstrate using numbers, e.g. 2 × (5 + 1) by expanding and using BIDMAS.
- Similarly, students may only change the sign of the first term when multiplying a bracket by a negative quantity. Demonstrate using numbers, e.g. â˜’2 × (5 + 1) by expanding and using BIDMAS and then â˜’2 × (5 â˜’ 1).

Question guidance

Core

Q5d Explain that the numerator or denominator of a fraction are not usually written inside brackets but that it helps students remember that the numerator and denominator are single numbers.

Q13 If you multiply a bracket by a negative number, what happens to the signs of the terms inside the bracket?

Q13–15 These questions have been carefully constructed to give students practice in expanding and simplifying algebraic expressions of increasing complexity.

Support

Expanding brackets Q2 If you multiply a bracket by a negative number, what happens to the signs of the terms inside the bracket?

Purposeful practice

Purposeful practice 1 Q1, **Q2** give practice in expanding brackets with positive and negative terms.

Purposeful practice 2 Q1, **Q2** give practice in multiplying brackets by +(1) and â˜'(1).

Reflect

Core

Write a definition, in your own words, for

- expand
- simplify

Compare your definitions with those written by others in your class.

Can you improve your definitions?

This task addresses an important literacy point, as well as exploring the mathematics. What do we actually mean when we ask students to 'expand' and 'simplify'? In articulating a meaning for themselves, students not only reflect on how the words may be defined, but also on their own understanding.

Support

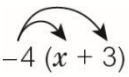
Amir works out

â˜'4(x + 3)

using a grid.

×	x	+3
−4		

Eloise works out â˜’4(x + 3) by drawing arrows.



Work out the answer using Amir's method.

Can you get the same answer using Eloise's method?

Discuss with a partner which method you prefer and why.

×	x	+3
−4	$-4 \times x$ $= -4x$	-4×3 $= -12$

â˜’4x â˜’ 12

Yes, you get the same answer using Eloise's method?

Depth

Look back at the expressions in Q7.

a Would it matter if these expressions used the letters x and y instead of a and b?

b Do the letters help you to understand an expression?

a *No*

b *Students' own answers, for example 'Not necessarily, although in some formulae letters are often chosen to represent a quantity starting with that letter, which makes the formula easier to remember.'*

Purposeful Practice Reflect and reason

Purposeful practice 1

Sadie expands â˜’5(x â˜’ 3) to â˜’5x â˜’ 15. Explain what she has done wrong.

When you multiply brackets by a negative number, what happens to the signs of both terms in the brackets?

Sadie has multiplied â˜’5 × x correctly to get â˜’5x. She has not multiplied â˜’5 × â˜’3 correctly.

The correct expansion is â˜’5x + 15

Purposeful practice 2

Amit says, 'In these expressions, there isn't a term outside the brackets so you can just remove the brackets.'

8 â˜’ (x + 5) 8 + (x + 5)

Simplify the expressions and explain why Amit is wrong.

8 â˜’ (x + 5) = 8 â˜’ x â˜’ 5 = 3 â˜’ x

If you remove the brackets you get 8 â˜ + 5 = 8 â˜ + 5 = 13 â˜

You cannot remove the brackets if there is a negative sign in front, because this negative sign changes the signs of both terms in the bracket.

$8 + (x + 5) = 8 + x + 5 = 13 + x$

You can remove the brackets if there is a positive sign in front of it.

Objectives

Core objectives

- Factorise expressions

Support objectives

- Factorise expressions

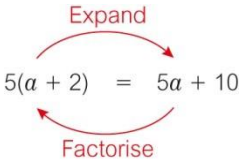
Depth key concepts

- Understand the significance of multiplying by both terms in a bracket - the expression in the bracket is one factor, the term in front of the bracket is another factor - and that factorisation is the inverse of this.

Key points

- **Expanding** removes brackets from an expression.
- **Factorising** inserts brackets into an expression.

To factorise 5a + 10, write the common factor of its terms, 5, outside the brackets. This is called 'taking out the common factor'.



- To factorise an expression completely, write the HCF of its terms outside the bracket.

Common errors and misconceptions

- Students may only partially factorise an expression. For example, $12a + 16b = 2(6a + 8b)$ *Can you factorise the expression inside the brackets?* Write $2(6a + 8b) = 2 \times (2 \times (3a + 4b))$ and simplify. Point out that 4 is the HCF of 12 and 16.

Question guidance

Core

Q6 This question has been carefully constructed to give students practice in factorising algebraic expressions of increasing complexity.

Q8b 5 × 1 month bill = 5 × (*e* â˜’ 7) so the contents of the bracket are 1 month's bill.

What does 7 represent? The reduction in monthly payment, subtracted from the previous payment of £*e*.

Q13b *What is the common factor of the numbers? 2 What is the common factor of the letters? □ What is the highest common factor of the terms? 2□*

Q13, **Q14** These questions have been carefully constructed to show students how finding the HCF of pairs of terms is used when factorising algebraic expressions of increasing complexity.

Support

Factorising – more than one common factor Q4 Complete factorisation uses the highest common factor.

Factorising – common factor includes a letter Q1d The HCF can be a number and a letter e.g. 2*w*.

Purposeful practice

Purposeful practice 1 gives practice in factorisation when one term is a multiple of the other one.

Purposeful practice 2 gives practice in factorisation with more complex combinations of letters and numbers.

Reflect

Core

At the end of Lesson 1.6, you defined 'factor'. (If you didn't do it, or if you cannot find it, then write a definition of 'factor' now.) Use your definition of factor to help you write a definition, in your own words, of 'highest common factor (HCF)'. Use your definition of HCF to help you write a definition, in your own words, of 'factorising'. Be as accurate as possible.

How did your factor and HCF definitions help you to define factorising?

Students may 'panic' if they cannot remember what a mathematical term means. This task aims to demonstrate to students that sometimes they can use definitions of words they know, to help them define words they have forgotten. The task asks students to reflect on what it means to factorise, by considering a definition of 'factor' and then considering a definition of highest common factor.

Support

Expanding and factorising are the inverse (opposite) of one another.

Where else in maths have you learned about two things being the inverse of one another?

Write down as many examples as possible.

Students' answers will vary, but should include multiplying and dividing, and adding and subtracting, and could include squares and square roots.

Depth

Is it possible to use factorisation to check if brackets have been expanded correctly?

Explain your reasoning.

Yes, because factorisation is the inverse of expanding brackets.

Purposeful practice Reflect and reason

Purposeful practice 1

Maisie factorises $6t$ â˜’ 6 to $6(t$ â˜’ $0)$.

Explain why Maisie is wrong. Write the correct factorisation.

Maisie's factorisation does not expand to give the original expression.

Maisie has divided the terms incorrectly. 6 ÷ 6 = 1, not 0.

$6t$ â˜’ $6 = 6(t$ â˜’ $1)$

Purposeful practice 2

a $2(6x$ â˜’ $8)$ **b** $3(2p^2$ â˜’ $p)$ **c** $4(6m$ â˜’ $9m^2)$

The terms in the brackets have a common factor.

***a** factor 2, correct factorisation 4(3□ â˜’ 4)*

***b** factor □, correct factorisation 3□(2□ â˜’ 1)*

***c** factor 3□, correct factorisation 12□(2 â˜’ 3□)*

Objectives

Core objectives

- Find the inverse of a simple function
- Write and solve one-step equations using function machines

Support objectives

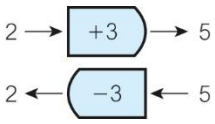
- Find the inverse of a simple function
- Write and solve one-step equations using function machines

Depth key concepts

- Understand the difference between expressions, formulae and equations
- Understand that while you can solve most one step equations 'in your head', you are doing this by identifying and using inverse operations (informally)

Key points

- A **function** is a rule that changes one number into another.
  The function +3 adds 3 to a number.
- The **inverse function** is â^'3 because it reverses the effect of the function +3.



- An **equation** contains an unknown number (a letter) and an '=' sign. '**Solve** an equation' means 'work out the value of the unknown number'. The **solution** is the value of the unknown.

Common errors and misconceptions

- Students may subtract instead of dividing both sides of a one-step equation.
  Write the equation using the sign ($g \times 5 = 20$) and ask students for the inverse function.
- Students may change the signs when turning an equation around.
  Use 3 = 5 â^' 2 as an example.

Question guidance

Core

Q9b *What is the answer when you divide 0 by a number?* 0

Q9d Write the equation as n â^' 2 = 10. *What is the inverse function?* Divide by â^'2 or ÷ â^'2

Q9e *If you divide a negative number by a negative number, is the answer positive or negative?*

Q16a *What is the total angle on a straight line?* 180°

Support

Finding the inverse of functions Q2 Make sure students know the inverse functions of the four arithmetic operations +, â˜', × and ÷

Solving equations using function machines Q5d You may need to remind students that

$\frac{y}{4}$ is the same as $y \div 4$

Purposeful practice

Purposeful practice 1 demonstrates that x (and other letters) take different values in different equations, and is used to show an unknown value.

Reflect

Core

Write the steps you take to solve equations like the ones in this lesson.

You could begin, 'Step 1: Simplify the left-hand side, if needed.'

Beside each step, show whether you found that step OK or difficult.

Ask a friend or your teacher to help you with any difficult steps.

This task encourages students to become aware of the steps they take when solving one-step equations. Students may write

Step 2, draw a function machine

Step 3, find the unknown using the inverse function.

Students should be encouraged to identify any steps they may have found difficult. These difficulties should be addressed before students move on to two-step equations in the next lesson.

Support

In this lesson, you have solved equations by using addition, subtraction, multiplication or division. Write an example of an equation that is solved using each operation. Compare your answers with a partner's and discuss any differences.

Students' answer will vary, but should include suitable equations.

Depth

Look back at the equations and formulae you have used in this lesson.

What are the differences between an equation and a formula?

An equation shows that two mathematical expressions are equal and can be solved to find the value of an unknown variable, while a formula shows a relationship between two or more variables.

Purposeful practice Reflect and reason

Purposeful practice 1

Hamza says, 'The solutions to questions in Purposeful practice 1 include x = â˜'7, x = 0 and x = 2. How can x have more than one value?'

Explain why the value of x is not the same in every equation.

x (or another letter) stands for an unknown number. It does not always stand for the same unknown number (but in some cases it may do).

Purposeful practice 2

Dale says, 'The solution to $\frac{x}{{\rm{5}}}{\rm{ = 10}}$ is x = 2.'

Is he correct? Explain why. What mistake do you think he made?

Dale is not correct. The solution is x = 50. He has divided 10 by 5 instead of multiplying.

Objectives

Core objectives

- Solve and write two-step equations using function machines
- Solve problems using equations

Support objectives

- Solve and write two-step equations using function machines
- Solve problems using equations

Depth key concepts

- Understand that writing and solving an equation is a powerful and efficient method for solving many problems involving an unknown quantity - 'using x for the unknown' is a useful problem solving strategy.
- Know that solutions to equations can be positive and negative integers, and (simple) decimals and fractions

Common errors and misconceptions

- Students multiply or divide before adding or subtracting. Use BIDMAS to show how the calculation
  $5 \times 2 + 1$
  is written using a function machine. Replace 5 with n and write the equation
  $2n + 1 = 11$

using a function machine. To reverse the function machine, work backwards, from right to left.

Question guidance

Core

Q5g Rewrite as $3k + 4$ to solve using function machines.

Q7 Students may realise that the problem can be solved using a one-step equation $12c = 36$.

Ask them to write an equation before carrying out any calculations.

Q10, Q11 These questions have been carefully constructed to give students practice in simplifying terms, expanding brackets and solving equations with expressions of increasing complexity.

Challenge Check that students calculate the correct height ($b + 2$) and base ($7 + 2 = 9$) of the outer rectangle.

Support

Solving two-step equations using function machines Q1biii Encourage students to check their inverse function machine with different inputs and outputs.

Purposeful practice

Purposeful practice 1 reinforces the difference between na and a^n.

Reflect

Core

Choose an equation from this lesson. Do the steps for solving equations that you wrote at the end of Lesson 4.4 work for this equation too? If not, rewrite your steps.

Check that they work for another equation from this lesson.

This task builds on the reflective task in lesson 4.4. It asks students to revisit the steps they wrote down for solving simpler (one-step) equations, and notice if they also apply to more complex (two-step) equations. Students should find that the steps are the same. Students' understanding should be reinforced by writing an example beside every step. This provides a model to students for how they might make notes on, and revise, mathematics.

Support

Write the steps to solve an equation from this lesson. Discuss your steps with a partner.

Step 1 Use the information you are given to write an equation.

Step 2 Substitute the numerical values into the equation.

Step 3 Carry out the calculations in the correct order (using BIDMAS).

Depth

Are the solutions to equations always integers? Are solutions always positive?

Show examples from this lesson to explain.

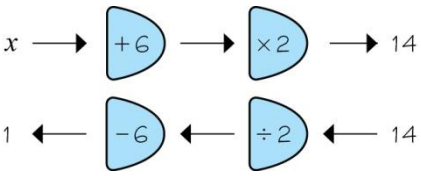
No: they can be fractions, decimals or negative integers. ***Q2c*** *finds a negative value of* □ *and the solution to the equation in* ***Q7*** *is a decimal value less than 1.*

Purposeful practice Reflect and reason

Purposeful practice 1

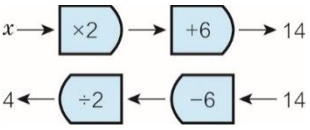
Martin draws these function machines to solve

$2x + 6 = 14$.



Explain what Martin has done wrong.

Martin has not followed the order of operations. The correct function machines are



$x = 4$

Purposeful practice 2

How can drawing function machines help you write an equation?

Drawing function machines helps you work out the order of the operations in the equation.

Objectives

Core objectives

- Solve equations using the balancing method

Support objectives

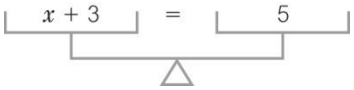
- Solve equations using the balancing method

Depth key concepts

- Understand that algebraic operations follow the same rules as number operations
- Know and use priority of operations to decide on order of inverse operations when using the balancing method.

Key points

- In an equation, the expressions on both sides of the equals sign have the same value.
  You can visualise them on balanced scales.



The scales stay balanced if you do the same operation to both sides.

You can use this **balancing method** to solve equations.

Common errors and misconceptions

- Students may divide before adding or subtracting from both sides of a two-step equation.
  Use function machines to demonstrate the order of working.
- Students may not write each modified equation on a new line, leading to untrue equations,
  e.g. $2n + 1 = 9$ â˜’ $1 = 8 \div 2 = 4$. Cover and uncover parts of their 'equation' to show that it does not make sense. Emphasise vertical presentation.

Question guidance

Core

Q4a Encourage students to write the associated equations, each one on a new line

$m + 8 = 10$

$m + 8$ â˜’ $8 = 10$ â˜’ 8

$m = 2$

Q4e Rewrite â˜’$4g = 12$ as $g \times$ â˜’$4 = 12$ and check that students know that â˜’4 is a negative number rather than a subtraction.

Q10 This question has been carefully constructed to give students practice in solving equations of increasing complexity.

Support

One-step equations Q4e Remind students that $\frac{h}{8}$ is the same as $h \div 8$

Purposeful practice

Purposeful practice 2 gives practice in substituting numerical values into formula, and in using substituted values to solve equations

Reflect

Core

Do the steps for solving equations that you wrote at the end of Lesson 4.5 work for equations with an unknown (a letter) on both sides of the '=' sign? If not, rewrite them.

Check that your steps work for solving equations from Lessons 4.4, 4.5 and 4.6.

This task builds on the reflective tasks in lessons 4.4 and 4.5. It asks students to consider the steps for any equation (one-step, two-step or with unknowns on both sides). Hence it makes explicit to students what they are doing when they solve equations.

Support

You have learned two different ways to solve equations: function machines and the balancing method. Discuss with a partner which method you prefer and why.

Students' answers will vary. Discuss with the students any difficulties they identify and examine reasons for their preferences.

Depth

Which inverse operations will you use, in which order, to solve

3x â˜’ 7 = 12 and 3(x â˜’ 7) = 12?

How do you know in which order to use the inverse operations?

3x â˜’ 7 = 12: *add then divide*

3(x â˜’ 7) = 12: *divide then add*

The inverse operations are in the reverse order to the operations in the equation so, for example, when the equation has multiply then subtract, you add and then divide.

Purposeful practice Reflect and reason

Purposeful practice 1

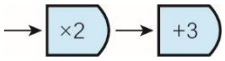
Which is the correct method for solving 3x + 7 = 13?

Ã· 3 then â˜’ 7 â˜’ 7 then Ã· 3

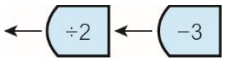
Aurora says, â€˜To solve an equation, use the inverse functions in the reverse order.â€™

Use an equation from Purposeful practice 1 to show why Aurora is correct.

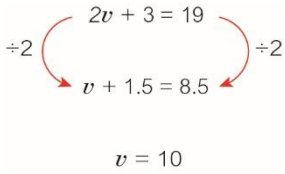
Method B. For example, in 2□ + 3 = 19, the functions are



To solve, use the inverse operations in reverse order



Students may explain without function machines. For example



Substitute to check: 2 × 10 + 3 = 23 ≠ 19

Purposeful practice 2

When does using a formula to find a value involve solving an equation?

When the value you want is not the subject (letter on its own on one side of the equals) of the equation.

Check, Strengthen and Extend

Core Check up

Q6 This question is designed to test if students are over-simplifying when collecting like terms.

Q6a This question is designed to test if students understand that both terms are multiplied when expanding brackets.

Q7b This question is designed to test if students are fully factorising an expression when requested.

Q9 This question is designed to test if students divide both sides of a two-step equation before adding/subtracting, i.e. isolating the unknown term first.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Anna says, 'Algebra is just like arithmetic really, but when you don't know a number, you use a letter.'

Is Anna's explanation a good one? Explain your answer.

Sometimes students do not recognise the connection between arithmetic and algebra, seeing algebra as involving a lot of 'meaningless' letters. This task aims to demonstrate to students the meaning of the variables (letters). This also develops confidence in students, as it encourages them to see that if they are comfortable with arithmetic, then they should be comfortable with algebra too.

Extend

Which do you find easier, working with expressions or equations? Explain why.

This task emphasises the difference between expressions and equations, and asks students to consider what they find easy or difficult about working with each of them. Students may say that they find equations easier to work with because they can easily test the accuracy of their answer by substituting the value of the variable back into the equation.

Depth

Extend

In this unit you have written complicated worded situations algebraically before solving them. In what ways has this helped you to work through a problem or multiple problems in the same scenario?

Students' own answers, for example: 'When looking at changing the numbers, I have been able to see more clearly where each number is used and saved time.'

End of Unit 4 tests: Expressions and equations

Pearson published resources

Support/Core

- Year 8 End of Unit 4 Support/Core test
- Year 8 End of Unit 4 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 4 Core/Depth test
- Year 8 End of Unit 4 Core/Depth mark scheme

Revision

End of Term tests: Units 1 to 4

Pearson published resources

Support/Core

- Year 8 Autumn End of Term Support/Core test
- Year 8 Autumn End of Term Support/Core mark scheme

Core/Depth

- Year 8 Autumn End of Term Core/Depth test
- Year 8 Autumn End of Term Core/Depth mark scheme

Unit opener: Real-life graphs

Pearson published resources

Objectives

Core objectives

- Draw, use and interpret conversion graphs

Support objectives

- Draw, use and interpret conversion graphs

Depth key concepts

- Understand why a conversion graph between currencies or units of length, mass and volume will always be a straight line through the origin

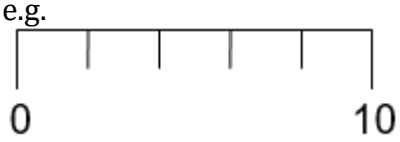
Key points

- A **conversion graph** converts values from one **unit** to another.

Common errors and misconceptions

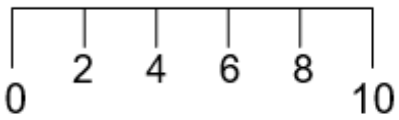
- Misreading the scale on the axes is a common problem. Resolve by drawing a diagram to show the first value on the scale and the number of increments between.

  e.g.

  Work out the value of one increment, and then write the numbers on the scale to check they work.

  i.e.

Question guidance

Core

Q4, **Q5** In these questions, the graph passes through the origin. Discuss why this is the case.

Q4e There are many different ways e.g. convert 10 cm to inches then multiply by 7, convert 35 cm to inches then multiply by 2, convert 30 cm and 40 cm to inches then add etc.

Q6 This question introduces a conversion graph which does not pass through the origin. Emphasise the need to read the scales on both axes carefully.

Challenge Some students might be confused by the two graphs, with one showing NZD on the vertical axis and the other showing NZD on the horizontal axis. Discuss what this means.

Support

Reading scales Q2 Make sure students are interpolating the values correctly.

Interpreting conversion graphs Q1 Make sure students are reading scales correctly and understand that the horizontal and vertical axes can have different scales.

Drawing conversion graphs Q1, **Q2** Discuss with students whether the line should go through the origin.

Purposeful practice

Purposeful practice 1 gives practice in reading from graphs.

Purposeful practice 2 gives practice in drawing graphs.

Reflect

Core

After this lesson, Caroline says, 'In **Q5a i**, I almost read 5 on the dollars axis, instead of on the pounds axis.'

She wrote down: Conversion graphs: be careful to read the correct axis.

Write your own 'be careful to' list for conversion graphs.

This task asks students to recognise where they made (or almost made) errors in working with conversion graphs. Students may also include in their lists: 'read the scale on the axis carefully, so I know what one square represents'; or 'always use units in my answers'.

Support

Would it be easy to draw the graph in **Drawing conversion graphs Q1** with the Maldivian rufiyaa axis going up 0, 1, 2, 3, 4…?

Why do you think each graph in this lesson has different scales on the two axes?

What do you need to check before you read a value from an axis?

You need different scales on the two axes for the different ranges of values. You need to check the scale on the axis, to see what the units are and what each square represents.

Depth

Why will a conversion graph for currencies or units of length, mass and volume always be a straight line that passes through the origin?

Are there any conversion graphs that do not?

They have the same number of extra units for each additional unit so will always make a straight line. They will always pass through the origin because if you have no money in one currency, you also have no money in the other currency.

The conversion graph between Celsius and Kelvin does not pass through the origin because 0 °C is not the same as 0 °K.

Purposeful practice Reflect and reason

Purposeful practice 1

When is it easier to use the graph to convert between pounds and kilograms? And when is it easier to do a calculation?

Students' own answers should include that it is easier to do a calculation when one or both values to convert are not on the graph. Answers may also include that finding 5 kg ≈ 11 lbs when you know 10 kg ≈ 22 lbs is easier than reading a graph.

Purposeful practice 2

How do you decide where to start and end your graph axes?

How do you choose the scale?

Start at the smallest value for both measures (0 in both these questions) and end at the largest or nearest 'round' number after. Choose the scale by what will fit on the graph paper available.

Objectives

Core objectives

- Interpret a distance–time graph
- Draw a simple distance–time graph
- Draw and use graphs to solve distance–time problems

Support objectives

- Interpret a distance–time graph
- Draw a simple distance–time graph

Depth key concepts

- Understand that a distance time graph can represent journeys using different units of distance and time, such as metres per second
- Understand that on a distance time graph showing a journey of 60 miles in 1 hour by a straight line, the car's speed may have varied slightly from minute to minute, but the graph does not show this.

Key points

In a distance–time graph

- the vertical axis represents the distance from the starting point
- the horizontal axis represents the time taken.

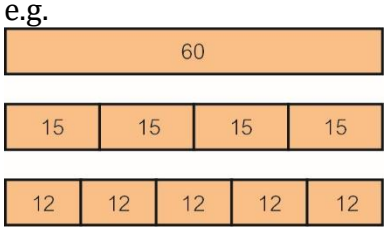
On a distance–time graph the **gradient** (steepness) of the line represents the **speed** of the journey.

The steeper the line, the faster the speed.

Common errors and misconceptions

Students may think that the straight line away from the starting point is uphill, the straight line back is downhill, and the horizontal section is moving along a flat road. Get students to look carefully at the vertical axis e.g. using the graph in **Q3**, ask students *How far is Liam from home at 1:15, 1:30, 1:45? What does this first section of the graph tell you?* Repeat for the other sections of the graph.

Students may misread the horizontal time scale. Resolve this using a bar model. Use the bar to represent 1 hour or 60 minutes. Divide the bar into different numbers of sections and ask students to work out the number of minutes in each section.
e.g.



Question guidance

Core

Q3 Remind students that Liam took 30 minutes to drive to the shopping centre and 45 minutes to drive back. It took him less time to drive there, so on average he must have

been driving faster. Discuss why this may have been e.g. less traffic on the roads, all traffic lights were green, etc.

Q4 This is an opportunity to make sure students understand each part of the graph. *What happened to Peter's distance from home between 60 and 70 minutes?*

Q7b Check students understand that the steeper the line, the greater the speed.

Q9 Remind students to read the label on the vertical axis carefully. It shows the distance from Bath. The red line represents the bus that starts at Bath and the blue line represents the bus that starts at Newport.

Support

Interpreting a distance–time graph Q1a *How do you know when Eva gets to her friend's house?*

Drawing a distance–time graph Q1 Remind students that the gradient and direction of the line changes when Paul gets to his appointment, and when he leaves to go home.

Purposeful practice

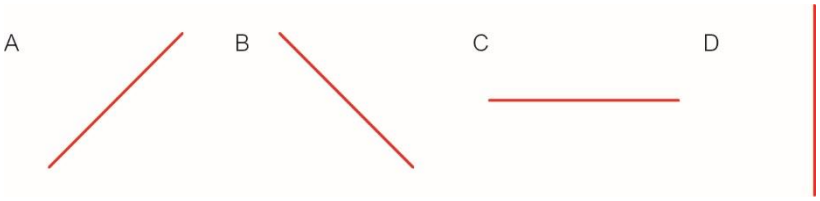
Purposeful practice 1 gives practice in reading distance-time graphs using different scales.

Purposeful practice 2 gives practice in calculating with times and distances, and in plotting times and distances.

Reflect

Core

You have seen lines like lines **A**, **B** and **C** on distance–time graphs.



Describe, in your own words, what each type of line tells you.

What if lines **A** and **B** were steeper? What would each of them tell you then?

Would there ever be a line like line **D** on a distance–time graph? Explain your answer.

This task encourages students to generalise. It asks them to use their experience of answering particular distance-time graph questions in this lesson to recognise what straight line segments on distance-line graphs represent. It also asks students to consider the kind of information that each type of line provides. Students should be able to state that a vertical line is not possible on a distance-time graph as it suggests travel in no time at all.

Support

When you stay in one place, does your distance from home change? Use your answer to explain what a horizontal line on a distance–time graph means.

No, your distance stays the same. The horizontal line shows that the distance is not changing.

Depth

In the investigation, you looked at roller coasters and distance–time graphs.

a What would it mean if a distance–time graph showed a loop?

b Why are distance–time graphs unlikely to have perfectly vertical sections?

a *It would mean that the distance had two different values for the same time, which is not possible.*

b *This would show that the distance had changed without any time passing.*

Purposeful practice Reflect and reason

Purposeful practice 1

The graph lines for Journeys 2 and 3 are the same shape. Why are the journey descriptions not the same?

Because the scales of the graphs are different, so the line represents different distances and times on each graph.

Purposeful practice 2

How can you tell from the graph in **Q2** that car B travels the slowest?

The line representing car B is less steep than the lines for cars A and C.

Objectives

Core objectives

- Draw and interpret line graphs

Support objectives

- Draw and interpret line graphs

Depth key concepts

- On a line graph, intermediate points are only estimates and not actual values. Begin to understand that it is more reliable to predict intermediate values within the data (interpolate) than to assume a trend will continue and predict future values (extrapolate)

Key points

- The shape of a line graph shows whether a quantity is increasing or decreasing.

Common errors and misconceptions

- When drawing a graph, students may not plot points accurately enough. Make sure students have a sharp pencil and use a ruler as a straight edge to help them follow the correct lines.

Question guidance

Core

Q4 Students may find it useful to make a table of values for this graph showing the value of silver each month. They may then find it easier to answer the questions.

Q5 This question can help understand that the different axes can be treated differently. *Why is there a break in one axis but not the other?*

Q5c It is only an estimate because we don't have the actual data for 2017.

Q5d Not really, as the amount she earns may go up or down depending on different factors such as whether she changes jobs, earns a bonus or not etc.

Challenge Encourage students to look at both graphs together. Ask them what kind of weather they would like to have on an outdoor activity holiday, and also what they wouldn't like.

Support

Interpreting line graphs Q3e Make sure students can identify 2005 on the horizontal axis and can read up and across to the vertical axis.

Drawing line graphs Q1 The graph should show a line joining all the data points – not a line of best fit.

Purposeful practice

Purposeful practice 1 gives practice reading different scales.

Purposeful practice 2 gives practice in choosing scales for graphs.

Reflect

Core

In this lesson, **Q3** asked you to 'explain how you know'. Think about how you explained.

Did explaining help you to understand more about graphs? Describe how.

This task asks students to reflect on the way they explained how they answered the question. Encourage students to think about this in this lesson and more generally. Students may mention that discussion allows them to test their own ideas and get new ideas from their classmates. It is also an opportunity to explain things in their own words and learn from each other.

Support

Leo is writing tips on drawing line graphs. He begins: 'Use a sharp pencil.'

Write some tips for yourself on drawing line graphs.

Students' answers will vary but should include:

Choose sensible scales for the horizontal and vertical axis.

Label each axis

Give the graph a title.

Plot each point carefully.

Join the points with a straight line.

Depth

In **Q1** you were asked to plot a line graph.

a Why is your value for 350 g only an estimate?

b Was your estimate for 350 g more or less accurate than your estimate for 1200 g?

Explain your reasons.

a *350 g is between two places we have values for, but we do not know the actual value for 350 g.*

b *350 g is likely to be more accurate than 1200 g because 350 g lies between two values we know, whereas 1000 g lies beyond the graph of values we know.*

Purposeful practice Reflect and reason

Purposeful practice 1

Which two graphs can only represent whole number values? Explain why.

From which two graphs can you read the most accurate values? Explain why.

A and C only represent whole number values because they show numbers of people.

From graphs A and B you can read the most accurate values because the scales are larger. You can read, for example, 10 people in June for August and 23°C in June for B, but for C, 4000 tourists in June may not be accurate; it could be, for example, 4010 or 3999.

Purposeful practice 2

Which of your graphs has a vertical scale starting at 0?

Explain how you decided where to start the scale for each one.

Students' own answers. ***Q1*** *has to start at 0,* ***Q2*** *and* ***Q3*** *do not. Starting them closer to the lowest data value avoids wasted space on the graph and means you can use a larger scale.*

Objectives

Core objectives

- Draw and interpret line graphs and identify trends

Support objectives

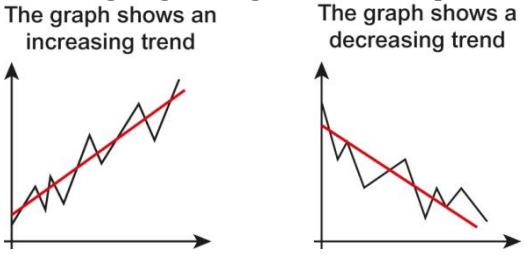
- Draw and interpret line graphs and identify trends

Depth key concepts

- Understand that a graph may show seasonal or other variations, but still show an upward or downward trend

Key points

Line graphs can help you identify **trends** in the data. The trend is the general direction of change, ignoring individual ups and downs.



Common errors and misconceptions

- Students may misread the values from graphs. Encourage students to use a ruler to follow the correct lines across from the vertical axis and up from the horizontal axis on the graph.

Question guidance

Core

Q2c *How do you know when digital sales overtook CD sales?*

Q5 Suggest to students that it might help if they plot the points differently i.e. crosses for holidays abroad and circles for holidays in the UK.

Q6 The accurate readings from the graph are

	Bicycle	Motorbike	Air	Rail	Bus/Coach	Car/Van/Taxi	Total
1970	4	4	2	36	60	297	403
2010	5	5	8	68	43	656	785

Before starting this question it may be worth reminding students how to write one number as a percentage of another.

Support

Describing trends Q2c Encourage students to look at the whole trend from January to July.

Describing trends Q3 Discuss why there is an upward trend during each year.

Comparing trends Q1g *How do you know when the café sold the same number of coffees and sandwiches?*

Plotting graphs to describe trends Q1, **Q2** Ask students how they can plot the graphs to make them easy to understand. (Use different colours.)

Purposeful practice

Purposeful practice 2 gives practice in plotting millions and identifying trends.

Reflect

Core

This lesson asked you to interpret information from some real-life graphs.

Which questions were easiest for you to answer? What made them easier?

Which questions were hardest for you to answer? What made them harder?

This task asks students to identify the questions they found easier and more difficult to answer, and why that may be. They may consider the questions where they had to read off a graph and give a numerical answer, as opposed to the questions where they had to write a sentence to comment on what was happening. They may also consider where there was more than one graph on the same axes.

Support

In this lesson you have learned more about drawing line graphs.

Draw a line graph. Then write in your own words the steps for drawing the graph and say how you can tell if the trend is increasing or decreasing.

Students' answer will vary, but should include

- *choose scales for each axis*

- *plot the points carefully*
- *join the points with a straight line*
- *add labels to each axis*
- *add a title to the graph.*

The trend is increasing if the graph is going up and decreasing if the graph is going down.

Depth

Look back at **Q1**. How is the trend in the first few months different to the trend over the whole time period the graph shows?

The increase in the first few months is much steadier than the overall increase.

Purposeful practice Reflect and reason

Purposeful practice 1

Nadia says, 'To read this scale, I try counting up in steps of 0.1, 0.2, etc. until I find the ones that fit.'

Miles says, 'I see that 5 squares represent 1 000 000 so each small square represents $\frac{1\,000\,000}{5} = 200\,000$

Explain how **you** read a scale.

Students' own answer. If they have another method of their own they should explain it.

Purposeful practice 2

On a graph like this, how can you show clearly which line represents which set of data?

Why is your answer to **Q3** only an estimate?

Use different colours or styles for each line and include a key.

It is an estimate because the scale is small so it is difficult to read the 'year' value accurately and because the actual data for each year may not have followed the straight line plotted here.

Objectives

Core objectives

- Draw and interpret linear and non-linear graphs from a range of sources

Support objectives

- Draw and interpret linear and non-linear graphs from a range of sources

Depth key concepts

- You can use graphs to solve problems, by finding patterns in data, or predicting midpoints or identifying trends, or times when rate of change is slower or faster - easier than from data

Key points

- A **linear graph** is a single straight line.
- A **non-linear graph** is not a single straight line.

Common errors and misconceptions

- Students may use inappropriate scales when drawing their own graphs. Encourage students to plan their graphs before they draw them, working out the values that their graphs need to show on both axes. For example in **Q5**, the horizontal axis (number of hoodies) goes from 0 to 20, and the vertical axis goes from 0 to 250 [(20 × 10) + 50]. Make sure students understand that the different axes can be drawn to different scales. Then draw lightly in pencil on graph paper the different scales they could use. Choose the one that gives the biggest graph with values that are easy to plot and read off.

Question guidance

Core

Q5 This question can be used to make sure students understand the sense of the question. In **Q5d ii**, *Why isn't the answer 12.5 hoodies?* (Can't order half a hoodie.)

Q9 *What happens to the depth of water when you get in a bath?*

Q12 Students may think that the wider the container the steeper the line. Demonstrate what happens with two different width containers. Tell students to keep asking themselves *Will the depth of water rise more quickly in the wider container or the narrower container?*

Challenge Students need to use their graph to work out the surface area of a cube that produces 0.87 ml of melt water. Then they need to divide this answer by 6 to find the area of one face, then square root this answer to find the side length of the cube.

Support

Interpreting real-life graphs Q2e Students may need to be told to take two readings from the vertical axis and subtract one from the other.

Non-linear graphs Q1b *What is represented by the change in the steepness of the graph?*

Purposeful practice

Purposeful practice 1 introduces percentage graphs.

Purposeful practice 2 gives further practice with percentage graphs.

Reflect

Core

Are distance–time graphs linear graphs? Look back at lesson 5.2 to check.

Are conversion graphs linear graphs? Look back at lesson 5.1 to check.

No, distance-time graphs are not linear graphs.

Yes, conversion graphs are linear graphs.

Support

Describe in your own words what a linear graph looks like.

A linear graph is a single straight line.

Depth

In **Q4** you were asked to describe the trend in the percentage of the population aged 0–14 years.

Do you think this would have been as easy to do if you just had the list of percentages?

Explain your answer.

No, because you would have had to compare all the numbers. On the graph you can clearly see the trend without having to compare individual numbers.

Purposeful practice Reflect and reason

Purposeful practice 1

In test 1, Lila got $\frac{8}{20}$ marks.

In test 3, Lila got $\frac{6}{10}$ marks.

Lila says, 'I got a higher mark in test 1. Why does it look lower on the graph?'

Explain why the graph is correct.

The graph is showing the marks as percentages, not the actual marks.

Purposeful practice 2

Lucas says, 'The graph shows there were the same number of 17–25 driving licence holders in 1992 and 1995.'

Thomas says, 'This may not be true.'

Explain what the graph actually shows and why Thomas is correct.

The graph shows there were the same percentages in 1992 and 1995. But because we don't know how many 17–25-year-olds there were in those years, we can't know the actual number of people.

Objectives

Core objectives

- Draw and interpret curved graphs from a range of sources

Support objectives

- Draw and interpret curved graphs from a range of sources

Depth key concepts

- Understand that for some graphs it is more realistic to join data points with a curve than with straight lines, as a curve better represents the data

Key points

- You can **interpret** graphs from real-life situations by reading values and suggesting what they mean.

Common errors and misconceptions

- Students may use inappropriate scales when drawing their own graphs. Get students to write down the minimum and maximum values that need to be plotted on each axis. For example in **Q4**, the temperature goes from 15 to 45, and the number of lumens goes from 2 to 16. Students than need to work out an appropriate scale for each axis based on these values before drawing their graph.

Question guidance

Core

Q7 If students aren't sure what the answer to this part is, ask two students at opposite sides of the room to stand up and pretend to throw a ball to each other and catch it too.

Challenge *When the graph shows a decrease in the percentage growth rate of the yeast, does this mean that the amount of yeast is also decreasing?* No, the amount of yeast is not decreasing, it is still increasing but at a slower rate than before.

Support

Interpreting curved graphs Q3c Students need to find two points on the graph above 80 m, find the corresponding times and subtract one from the other.

Plotting curved graphs Q1b Make sure the curve goes through all the plotted points.

Purposeful practice

Purposeful practice 1 gives practice in interpreting the intercept and the steepness of curved graphs.

Reflect

Core

Look at the curved graphs from this lesson and the graphs from previous lessons.

Write down two things that are the **same** for linear and curved graphs.

Write down two things that are **different** for linear and curved graphs.

Students' answer will vary but could include

Linear and curved graphs are both plotted on horizontal and vertical axes and can be used to identify trends.

A linear graph is a single straight line, but a curved graph is not.

Support

How would you answer a question that says 'estimate the value'? Does it mean take a guess? Discuss with your partner a better explanation of the instruction 'estimate the value'.

'To estimate' means to use the information you have available to find an approximate value.

Depth

Look back at your answers to **Q3**.

a Explain why Graph C is a curve and not a straight line.

b How is this similar to the graph for an object thrown in the air? How is it different?

a *The container does not fill at a constant rate. When the dish is narrow (at the bottom) it fills faster than when it is wider (at the top).*

b *Similar because the curve starts off steeply, and then becomes less steep (as the ball slows down). Different because the curve for the ball curves back downwards as it begins to fall, whereas the curve for the container ends when the container is full.*

Purposeful practice Reflect and reason

Purposeful practice

How can you tell from a curved graph

- whether a quantity is decreasing or increasing

- when it is increasing fastest?

An upward curve shows increasing quantity. A downward curve shows decreasing quantity

The steepest part of the curve shows when it is increasing fastest.

Check, Strengthen and Extend

Core Check up

Q4 This question tests students' ability to read scales correctly.

Q5 This question checks that students can use an appropriate scale when drawing their own graphs.

Q6d This question is designed to test students' understanding that the steeper the line, the faster the rate of growth.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Which do you find harder, reading from graphs or drawing graphs?

What makes it harder?

Write one thing about reading graphs and one thing about drawing graphs you think you need more practice on.

This task asks students to identify what makes some graphs easier to read from than others, and whether they find it easier or harder to read from a graph or draw a graph. This should lead students to identify where they may need additional practice in working with graphs.

Extend

Look again at **Q2**. Why might athletes want to graph their performance?

Look again at **Q4**. Why might doctors want to graph a patient's pain?

This task asks students to consider why graphs may be used by different professionals, and highlights not just the usefulness of graphs but the prevalence of them in all kinds of careers. Students may have seen graphs as graphics to support news readers, and so mention the usefulness of graphs to journalists.

Depth

Extend

What information is easier to see from a graph?

What information is easier to work out from a table?

It is easier to see trends in the data in a graph. A table is more useful for seeing the numbers and calculating averages, for example.

End of Unit 5 tests: Real-life graphs

Pearson published resources

Support/Core

- Year 8 End of Unit 5 Support/Core test
- Year 8 End of Unit 5 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 5 Core/Depth test
- Year 8 End of Unit 5 Core/Depth mark scheme

Unit opener: Decimals and ratio

Pearson published resources

Objectives

Core objectives

- Round decimals to 2 or 3 decimal places
- Round numbers to a given number of significant figures
- Round numbers to an appropriate degree of accuracy
- Order decimals of any size, including positive and negative decimals

Support objectives

- Round decimals to 2 or 3 decimal places
- Round numbers to a given number of significant figures
- Round numbers to an appropriate degree of accuracy
- Order decimals of any size, including positive and negative decimals

Depth key concepts

- Understand when it is more appropriate (and more accurate) to round to decimal places than significant figures (or vice versa)
- Understand the impact of rounding

Key points

A number rounded to **2 decimal places** (2 d.p.) has two digits after the decimal point.
A number rounded to **3 decimal places** (3 d.p.) has three digits after the decimal point.

For rounding to **2 decimal places**, look at the thousandths (the third decimal place).

- 5 thousandths and above, round up.
- 4 thousandths and below, round down.

For rounding to **3 decimal places**, look at the ten thousandths (the fourth decimal place).

- 5 ten thousandths and above, round up.
- 4 ten thousandths and below, round down.

For most calculations, an **appropriate degree of accuracy** is a value you can measure easily and accurately. For example, you can easily and accurately measure 6.1 cm, but not 6.075 cm.

To save writing all the zeros, you can write very large numbers as a decimal number of millions. For example, you can write 2 500 000 as 2.5 million.

You can round numbers to a certain number of significant figures (s.f.). The first significant figure is the one with the highest place value.
It is the first non-zero digit in the number, counting from the left.

When comparing the size of decimals

- first compare the whole number parts
- then, if they are equal, compare tenths
- then, if they are equal, compare hundredths
- continue like this for thousandths, ten thousandths, hundred thousandths ...

Common errors and misconceptions

- Students may be unable to round up or down. Use a number line to model physical proximity.
- Students may forget to round up when 5 is the last digit. Explain this as a convention.
- Watch for students who are uncertain about names for bigger numbers, i.e. confuse 10 000 and 100 000.

- Students may have misconceptions when ordering decimals, such as thinking 3.4 is smaller than 3.13 because 4 is smaller than 13. Use zero placeholders and compare equal number of decimals after the decimal point.
- Make sure students know the difference between ascending and descending.
- Ordering negative numbers can cause confusion. Reiterate that the more negative a number is the smaller it is. Use number lines to demonstrate if required.
- Students often confuse < and >. Remind students that the open end points to the larger number (or that the point of the symbol points to the smaller number).

Question guidance

Core

Q7 Ask students to check if £55 is enough to pay workers the amount they have worked out. (55 ÷ 6 = 9.166... but students need to round down to 9.16 as there isn't enough money to give everyone 9.17). As extension, All leftover pennies go into a charity box. How many leftover pennies will there be in the week?

Q8 Students may need to be reminded how to find the length of one side of a square when they are given the perimeter.

Q9 This question links with Geography. If time is available, get students to check where each city is.

Support

Rounding decimals to 1, 2 or 3 decimal places Q1 Marking numbers on a number line helps student to visualise what they are doing when rounding a number, rather than just following a 'round up' rule.

Rounding numbers to a given number of significant figures Q7 Students will often say that 4350 should be rounded to 4 since zeros do not 'count' as significant figure. Spend time discussing why the zeros are important in terms of place value.

Purposeful practice

Purposeful practice 1 gives practice in rounding to given numbers of decimal places and significant figures.

Purposeful practice 2 gives practice in ordering decimals, including negative numbers.

Reflect

Core

In this lesson you have been doing lots of work with decimals. Imagine someone had never seen a decimal point before.

How would you define it?

How would you describe what it does?

Write a description in your own words.

Compare your description with others in your class.

This task prompts students to consider the decimal point and its purpose. Students should recognise that it separates the whole number part from the fractional part of a number. Students are encouraged to compare their description of the decimal point with others in the class, so that they may learn from each other.

Support

Larry says, 'Rounding a number to 2 d.p. is the same as rounding a number to 2 s.f.'

Discuss with a classmate if Larry is correct. Look back at **Q4** in **Rounding decimals to 1, 2 or 3 decimal places** and **Q4** in **Rounding numbers to a given number of significant figures** to help you.

Larry is not correct. You can round to 2 decimal places and have more than 2 significant figures, and you can round large numbers to 2 significant figures with no decimal places.

Depth

a Give an example where it is not useful to round a number or value.

b Give an example where it is useful to round a number or value.

c Does it matter if you round using decimal places or significant figures? Explain.

a *Student's own answer, for example, measuring medicine doses.*

b *Student's own answer, for example, communicating very large numbers in a news headline.*

c *It doesn't matter if you round using decimal places or significant figures, but you need to be aware of the impact on accuracy.*

Purposeful practice Reflect and reason

Purposeful practice 1

Is it more accurate to round money amounts to 2 d.p. or 3 s.f.? Explain why.

It is more accurate to round to 2 d.p. when there is £10 or more as then significant figure does not round to the nearest penny. It is more accurate to round to 3 s.f. when there is less than £1. Rounding money amounts to 2 d.p. always rounds to nearest penny.

Purposeful practice 2

Repeat **Q2** but insert a new decimal into each list so that they are still in descending order.

Students own answers, for example

2 a *29.6, 29.53, 29.0601*

b *9.20106, 9.2009, 9.20016*

c *–3.4, –3.5, –3.724*

d *0.083, –0.1, –0.803*

v

Objectives

Core objectives

- Multiply any number by 0.1 and 0.01
- Multiply larger numbers
- Multiply decimals with up to and including 2 decimal places

Support objectives

- Multiply any number by 0.1 and 0.01
- Multiply larger numbers
- Multiply decimals with up to and including 2 decimal places

Depth key concepts

- Apply the inverse relationship of multiplication and division to decimal calculations

Common errors and misconceptions

- Students may not naturally estimate before answering. Encourage a sense of the size of the number.
- Students may fail to see the connection between similar calculations. Spider diagrams can help students see how calculations are similar.
- Students may fail to see the connection between inverse and equivalent calculations. Revisit function machines and emphasise that the relationships between the calculations are the same when decimal numbers are used.

Question guidance

Core

Q12 In this question, students are encouraged to see the relationship between the number of digits in the question and the answer.

Q14 Students may not recognise that the multiplication (× 8) only needs to be carried out once.

Q16 You may need to remind students of formulae to find areas of shapes.

Support

Estimating and using multiplication facts Q2 Encourage student to use the language of 10 times smaller/larger. Challenge student to estimate what the answer will be – what size answer are they expecting?

Purposeful practice

Purposeful practice 1 and 2 are carefully constructed to give students practice in column multiplications up to three digits, including decimals, of increasing complexity.

Reflect

Core

Look back at **Q13**. At the end of this question you discovered a mathematical 'rule'.

The rule tells you where to put the decimal point in the answer when multiplying decimals.

Write the 'rule' in your own words.

Does the rule still work for 2.5 × 4.2?

Students should revisit the rule for multiplying decimals, i.e. count the number of digits after the decimal point in the numbers in the calculation to find the number of digits after the decimal point in the answer.

Students are asked to write down the rule and consider why they were encouraged to find it out for themselves, rather than just being given it. Finally, students are invited to think of a strategy for multiplying decimals that they could use if they had forgotten the rule. They may use estimation.

Support

What mistakes do you think it is easy to make when multiplying decimal numbers? How could you avoid them?

Students own answers, but likely to focus on getting the decimal point in the wrong place. Discuss the importance of counting the numbers of digits to the right of the decimal point and encourage students to round and estimate to make sure their answer is sensible.

Depth

Describe the steps you took to answer **Q1c**. Try to use mathematical language in your description, for example 'inverse' and 'place value'.

Students' own answers, for example, 'I used the fact that multiplication was an inverse of division to write a division.

Purposeful practice Reflect and reason

Purposeful practice 1

Caspar says, 'When working out 956 × 784, I multiply each digit in 956 by 4 then 8 then 7.'

Theo says, 'When working out 956 × 784, I multiply each digit in 956 by 4 then 80 then 700.'

Who is correct? How does your working for long multiplication help to explain this?

Theo is correct. You place a zero in the second line of multiplication because you are multiplying by a number of 10s (e.g. 8 tens when multiplying by 784). You place two zeros in the third line of multiplication because you are multiplying by a number of 100s (e.g. 7 hundreds when multiplying by 784.

Purposeful practice 2

How could you use your answer to **Q2a** or **Q2b** to help you work out **Q2c**?

Peter works out **Q3e** and writes 5.72 × 3.6 = 205.92

How could he have used estimation to realise this answer is wrong?

In the calculation the second number in **Q2a** (23) was ÷ 100 in **Q2c**, so the answer is the answer to **Q2a** ÷ 100. Compared to **Q2b**, it was ÷ 10, so the answer was **Q2b** ÷ 10.

5.72 × *3.6 ≈ 6 × 4 = 24*

Purposeful practice 3

How do multiplying by 0.1 and dividing by 10 relate to each other?

How do multiplying by 0.01 and dividing by 100 relate to each other?

What division calculation do you think is the same as multiplying by 0.0001?

Multiplying by 0.1 and dividing by 10 are the same.

Multiplying by 0.01 and dividing by 100 are the same.

Dividing by 10 000

Objectives

Core objectives

- Divide by 0.1 and 0.01
- Multiply and divide by decimals
- Solve problems involving decimals and all four operations

Support objectives

- Divide by 0.1 and 0.01
- Divide by decimals

Depth key concepts

- Understand the relative sizes of answers to related decimal calculations

Key points

- To divide by a decimal, multiply both numbers by a power of 10 (10, 100, …) until you have a whole number to divide by. Then work out the division.

Common errors and misconceptions

- Students may not change both numbers in a decimal division. Encourage students to see this as an equivalent calculation.
- Students may not recognise the relationship between similar calculations. Use a spider diagram if appropriate.
- Students may not line up decimal points when adding or subtracting. Encourage students to work carefully with their written calculations, and remind them to add zero placeholders to help with calculations.

Question guidance

Core

Q6 This question help students to see connections within families of calculations.

Q13 Encourage students to estimate their answer first. They should recognise that 10 × 2.4 = 24, so their answer should be a bit more than 10.

Q14 Encourage students to use column methods of multiplication and check they understand how to deal with decimal points.

Q15 Students may need to be reminded how to calculate the volume of a cuboid.

Support

Dividing by 0.1 and 0.01 Q1 The pictorial representations may help students to visualise the problem.

Dividing decimals Q1 Encourage students to discuss the pattern – *what has happened to each number in the division calculation?*

Purposeful practice

Purposeful practice 2 is carefully constructed to give students practice in multiplying and dividing by 100, 10, 1, 0.1 and 0.01 and to demonstrate the subsequent patterns.

Reflect

Core

What is the same and what is different when dividing an integer by an integer compared with dividing a decimal by a decimal?

This task asks students to think about the steps they take when dividing by integers and by decimals. They should recognise the similarities of the mechanics of the division, and identify the difference that dividing by a decimal gives an answer that is bigger than the decimal being divided.

Support

These are the names of the numbers in a division calculation

dividend ÷ divisor = quotient

Imogen says, 'The quotient is always smaller than the dividend in a division calculation.' Use the questions and your answers in this lesson to decide whether Imogen is correct. Give examples.

Imogen is not correct. If the divisor is less than 1, the quotient will be bigger than the dividend. For example, 6 ÷ 0.5 = 12, so the dividend is 6 and the quotient is 12. Dividing by 0.1 and 0.01 Q1, Q2, Q3 and Dividing decimals Q1 and Q2 are examples in this lesson.

Depth

How do you use your knowledge of multiplying and dividing by powers of 10 when multiplying and dividing decimals? Give examples.

Students' own answers, for example, 'I can make the calculation easier by working without decimals and then dividing and multiplying by powers of 10 to get my answer to the correct place value.'

Purposeful practice Reflect and reason

Purposeful practice 1

Tyler works out 0.4 ÷ 0.2. He writes 0.4 ÷ 0.2 = 0.8

Describe two mistakes Tyler has made.

Tyler has multiplied 4 by 2 instead of dividing. Tyler assumes the answer is of the form 0.? because the two numbers in the calculation are 0.4 and 0.2

The correct answer is 2.

Purposeful practice 2

Copy and complete these statements.

÷ 100 is the same as × ☐ ÷ 10 is the same as × ☐ ÷ 1 is the same as × ☐

÷ 100 is the same as × 0.01

÷ 10 is the same as × 0.1

÷ 1 is the same as × 1

Purposeful practice 3

Q1a and **Q1c** include the word 'times'. Do you always multiply when a question includes 'times'?

Not necessarily. In algebra, a term such as 2 times x is written as 2x, and to solve an equation such as 2x = 4, you need to divide 4 by 2 to find x.

Objectives

Core objectives

- Divide a quantity into three or more parts in a given ratio
- Use ratios involving decimals
- Solve ratio and proportion problems
- Use unit ratios

Support objectives

- Divide a quantity into three parts in a given ratio
- Use ratios involving decimals
- Solve ratio and proportion problems
- Find unit ratios

Depth key concepts

- Understand how to use unit ratios to make comparison
- Deepen understanding of decimal, ratio and proportion calculations by working out problems in real life contexts and relating to previously learnt multiplicative concepts
- Understand the same 'rule' applies to simplifying ratios involving fractions as ratios involving decimals'

Key points

- You can compare ratios by writing them as unit ratios. In a unit ratio, one of the numbers is 1.

Common errors and misconceptions

- Students may not understand that ratios can be simplified like fractions.
- Students may not grasp how to use multiplying by powers of ten to remove decimal points in decimal ratios.
- Students may not realise they should further cancel ratios after multiplying by power of ten.
- Students may not remember the relationship between millilitres and litres.

Question guidance

Core

Q5 This question introduces decimal results from sharing in given ratios. Discuss with students how to work out the appropriate number of decimal places after a calculation.

Q6d This question can be used to show the relationship between ratios and proportions.

Q11 Make sure students break this question down into the appropriate steps e.g. 1 litre = 1000 ml, simplify the ratio by multiplying all parts by 10 (25 : 14 : 1), find the total number of parts (40)

Support

Sharing in a ratio Q1 By using a bar model method students can visualise sharing in a ratio. Remind them that the value in each 'box' must be equal.

Purposeful practice

Purposeful practice 1 gives practice in sharing a given amount by given ratios, of increasing complexity.

Purposeful practice 2 gives practice in expressing ratios and proportions.

Purposeful practice 3 gives practice in working with unit ratios.

Reflect

Core

Look back at **Q6d**.

a Write all the steps you took to work out the answer to part i.

b Write all the steps you took to work out the answers to parts ii and iii.

c Lou says, 'Part i was about ratio. A ratio compares one part to another part. Parts ii and iii were about proportion. A proportion compares one part to the whole thing.'

Is Lou correct?

*This task asks students to spell out the steps they took to answer a ratio question (**Q5a**) and then a proportion question (**Q9**). Students are required to think more deeply about the meaning of the concept of ratio, in relation to comparison of parts, and also proportion, in relation to comparison of one part to a whole. The intention is that students gain greater insight into what ratio and proportion are, and how they are the same and different.*

Support

You found equivalent ratios in all the questions in this lesson. Describe how to find an equivalent ratio.

To find and equivalent ratio, multiply all the parts of a ratio by the same value.

Depth

a When is it useful for a ratio to include a decimal? Give examples.

b When is it useful for a ratio including a decimal to be simplified to give a ratio involving integers? Give examples.

***a** When the simplest form of a ratio is greater than the number you are sharing into a ratio. For example, when partitioning 20 in the ratio 5 : 37, it might be easier to use the ratio 0.12 : 0.88.*

***b** When the sum of the total parts of the integer ratio makes it easier to share the amount. For example, 0.2 : 3 might be easier as 2 : 30 because you only have to divide by 32 instead of 3.2.*

Purposeful practice Reflect and reason

Purposeful practice 1

What did you notice about the total number of parts in the ratios in **Q1** parts **a**–**e** and **Q2** parts **a**–**e**?

*In **Q1** all the ratios involved dividing £64 into 16 parts. In **Q2** all the ratios involved dividing 160 m into 16 parts.*

Purposeful practice 2

What do the fractions in **Q2**, **Q3** and **Q4** add to each time? Explain why.

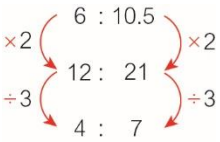
They add to 1 because they show the proportions of all the counters in each board game.

Purposeful practice 3

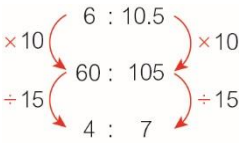
To work out the answer to **Q1b**, Luke says, 'I multiplied both parts by 2, then simplified.'

Ahmed says, 'I multiplied both parts by 10 and then simplified.' Show that they both reach the same answer. Do both of their methods work for **Q1e**? Explain.

Luke



Ahmed



Luke's method does not work for **Q1e**, as doubling does not give a ratio with integer parts.

Ahmed's method does work for **Q1e**.

Check, Strengthen and Extend

Core Check up

Q1 This question tests for correct use of < and >

Q2 This question is designed to test students' ability to write millions as decimals.

Q4 This question is designed to test students' ability to order negative decimals.

Q6 This question uses a known multiplication to find related multiplications.

Q8 This question involves reasoning to find an equivalent calculation to ÷ 0.1.

Q12 This question has the context of money.

Q15 This question involves decimal answers.

Q16 This question has the context of mass of ingredients.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

This Strengthen lesson used lots of diagrams.

Look back at these diagrams.

Which diagrams did you find most useful? Why?

Which diagrams did you find least useful? Why?

This task encourages students to think about the purpose of diagrams, and reflect on whether they found them useful in their own mathematical problem solving, and how. Students may consider number lines, spider diagrams and bar modelling.

Extend

What kinds of jobs might need the maths skills you have used in this Extend lesson?

Look back at the questions to help you. For example, **Q12** was about car gears. Someone working as a Formula One engineer would need to understand car gears.

*This reflective task demonstrates to students how an understanding of decimals and ratio are important in many different jobs. Simply by looking back at the contexts for questions in these Extend lessons, students may identify a need for these mathematics skills by gardeners (**Q7**), financial departments (**Q4**), and nurses (**Q5**).*

Depth

Extend

Would you prefer to complete all your work in fractions or in decimals?

Which do you think is more accurate?

Students' own answers. Students may initially think that it's easier to work with decimals. Encourage students to think about times when fractions are more accurate than decimals, e.g. using $\frac{1}{3}$ may give a more accurate answer than 0.33...

End of Unit 6 tests: Decimals and ratio

Pearson published resources

Support/Core

- Year 8 End of Unit 6 Support/Core test
- Year 8 End of Unit 6 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 6 Core/Depth test
- Year 8 End of Unit 6 Core/Depth mark scheme

Unit opener: Lines and angles

Pearson published resources

Objectives

Core objectives

- Classify quadrilaterals by their geometric properties

- Solve geometric problems using side and angle properties of special quadrilaterals

Support objectives

- Classify quadrilaterals by their geometric properties
- Solve geometric problems using side and angle properties of special quadrilaterals

Depth key concepts

- Understand that the properties of a quadrilateral allow you to name the quadrilateral, and conversely knowing the name of a quadrilateral means you know its side, angle and symmetry properties, and can use them to find missing lengths and angles in quadrilaterals

Key points

- A **diagonal** is a line that joins two opposite vertices of a shape.
- When diagonals **bisect** each other, they cut each other in half.
- The **properties** of a shape are facts about its sides, angles, diagonals and symmetry.

Common errors and misconceptions

- Students may name a shape using its image rather than its properties. Tell students that shapes on examination papers may not be drawn accurately on purpose. The only way to be sure is to look at the properties of the shape or a given description.

Question guidance

Core

Q4, **Q5** Refer students back to the Key points on page 173 if they need help.

Q6 Yes because two of the lines are perpendicular to the same line and so must be parallel.

Q8 Students may need to be prompted to look to both the right and the left of point *B*.

Q11 As a general strategy, students can mark all known angles using a shape's properties before calculating marked angles.

Support

Properties of quadrilaterals Q1 Students often struggle with marking the same length/size or parallel sides.

Properties of quadrilaterals Q2 Asking student to draw the shapes and the markings will help them to remember them and the meanings.

Purposeful practice

Purposeful practice 1 relates lines of symmetry to angles and side lengths.

Purposeful practice 2 gives practice in identifying equal angles in regular quadrilaterals.

Reflect

Core

Frankie says that to identify a shape, he begins by asking himself these questions.

- Are any sides equal in length?
- Are the equal sides next to each other, or opposite?

Write all the questions you ask yourself to identify a shape.

Test your questions on some quadrilaterals. Can you improve your questions?

Compare your questions with other people's. Can you improve your questions?

This task asks students to think carefully about all the questions they need to ask themselves when identifying shapes. Students should consider all properties, to include sides, angles, diagonals and symmetry. Students are asked to test their questions on some quadrilaterals and compare their questions with others in the class, each time refining them. This models the process of creating, testing, and then comparing an idea, to make it as good as possible.

Support

When you describe the properties of a quadrilateral, what do you look out for?

Students' own answers, but should include reference to the lengths of the sides, angles between adjoining sides, parallel sides, right angles, and angles of bisection of diagonals.

Depth

You know that a shape is a kite.

How does this help you to work out missing sides and angles?

If you know it is a kite, you can use all the properties of a kite such as angle sum, equal side lengths and lines of symmetry, and that the diagonals intersect at right angles.

Purposeful practice Reflect and reason

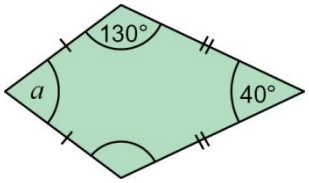
Purposeful practice 1

How can you use symmetry of a shape to identify its equal sides and angles?

Line symmetry identifies equal and opposite sides and angles. When a shape has rotational symmetry, some of its angles must be equal, as the shape fits on to itself at least twice when it rotates through 360°. For a quadrilateral of rotational symmetry of order 4, all angles are equal. For rotational symmetry of order 2, opposite angles are equal.

Purposeful practice 2

Farouk says, 'Opposite angles in a kite are equal, so $a = 40°$



Farouk is wrong. Explain the mistake he has made and work out the size of angle a.

Only one pair of opposite angles are equal in a kite and Farouk has identified the wrong pair. In this diagram, two angles are 130°, so a = 360 – 130 – 130 – 40 = 60°

Objectives

Core objectives

- Identify alternate angles on a diagram
- Understand proofs of angle facts

Support objectives

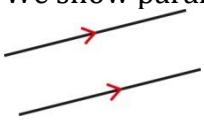
- Identify alternate angles on a diagram
- Understand proof of angle facts

Depth key concepts

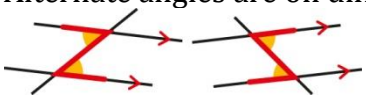
- Understand the difference between demonstration (that a theory works for some values) and proof (where it works for all values)

Key points

- We show parallel lines using arrows.



- When a line crosses two parallel lines it creates a 'Z' shape.
  Inside the Z shape are **alternate angles**.
  Alternate angles are equal.
  Alternate angles are on different (alternate) sides of the diagonal line.



- It is not enough to show that a theory works for a few values. You need to prove that it works for all values. A **proof** uses logical reasoning to show that a theory is true.

Common errors and misconceptions

- Students may fail to realise that alternate angles can be obtuse. Use a diagram to show that, for every pair of acute alternate angles, there is a pair of obtuse alternate angles (giving a stretched and/or backwards Z).
- Students may think that demonstrating a result using a few examples constitutes a proof. Explain that a proof is an explanation that works for all possible values and so must use algebra, with letters representing angles and sides. A geometric proof uses known properties of angles and shapes.

Question guidance

Core

Q3 *Which two alternate angles are acute/obtuse?* (a and d; b and c)

Q7 Split into two separate diagrams to show that p and r are unrelated to q and t.

Q6d Encourage students to identify which angles to calculate first. It may help to cover up parts of the diagram that are not being worked on.

Support

Proof of angle facts Q1 This is an important aspect of the new curriculum. Students need to be able to justify and explain their reasoning.

Purposeful practice

Purposeful practice gives practice in identifying and using alternate angles.

Reflect

Core

Marco says, 'A mathematical proof is an argument that convinces people something is true.'

Read each line of the proofs you worked on in **Q9** and **Q10**.

How does the way they are written help to convince people?

This task encourages students to think about the idea of mathematical proof. Students may read the lines of proof and note that there is a single statement per line, with no extra words. Statements are always true (or false) and supported by a reason. The use of algebra allows you to generalise, rather than using numbers which only offer specific cases.

Support

Did you find it easy to identify alternate angles? How would you describe finding alternate angles to a partner?

Student's own answers, but should include reference to lines crossing parallel lines to create a Z shape.

Depth

Ravi wants to prove that the angles in a triangle add up to 180°. He does this by drawing lots of triangles, measuring the angles and showing that they add up to 180°.

Why has Ravi not proved that the angles in a triangle add up to 180°?

Ravi has only shown that they do for those triangles. To prove it, Ravi would have to look at a general case and show that the result is true even when you don't know any of the numbers.

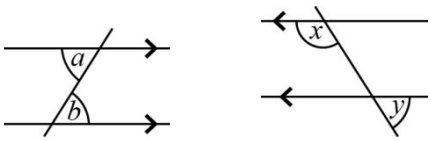
Purposeful practice Reflect and reason

Purposeful practice

Poppy says, 'In this diagram, a and b are alternate angles because

- they are on opposite sides of the line
- one sits on a parallel line, the other 'hangs' from a parallel line.'

Use Poppy's definition to explain why x and y are not alternate angles.



x and y are on alternate sides of the line. But they are both 'hanging' from a parallel line and in alternate angles one should 'sit' and one should 'hang'.

Objectives

Core objectives

- Identify corresponding angles
- Solve problems using properties of angles in parallel and intersecting lines

Support objectives

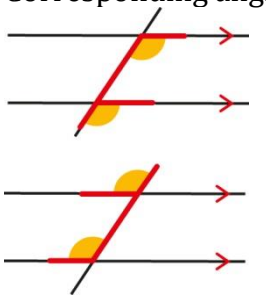
- Identify corresponding angles
- Solve problems using properties of angles in parallel and intersecting lines

Depth key concepts

- Understand that missing angles in parallel lines can be found using angle facts in different combinations, that often there is more than one way of solving the angle problem, and you may need to find angles that are not labelled on the diagram in order to work out the size of the angles you want.
- Understand that angles in parallel lines prove the angle properties of trapezium, rhombus, parallelogram (e.g. opposite angles equal) and extend to co-interior angles in these shapes.

Key points

When a line crosses two parallel lines it creates an 'F' shape.
There are **corresponding angles** on an F shape.
Corresponding angles are equal.
Corresponding angles are on the same (corresponding) side of the diagonal line.



Common errors and misconceptions

- Students may not use the properties of triangles to help solve a problem. Encourage students to identify known triangles when looking at a diagram for the first time. They should look for equal sides and angles, right angles and shape names in the question to identify isosceles, equilateral and right-angled triangles.
- Students may not use the properties of quadrilaterals to help solve a problem. Encourage students to identify known quadrilaterals when looking at a diagram for the first time. They should look for equal sides and angles, parallel lines, right angles, shape names in the question. Refer them to the Key point on p173 of the Core 2 textbook.

Question guidance

Core

Q5, **Q6** Encourage students to see if they can find more than one way to solve these questions.

Q7b, **c** It may help to remind students that angles on a straight line add up to 180°.

Q11 *What shape is the sign*? (rectangle) *What are the angles at each corner of a rectangle*? (90°)

Support

Corresponding angles Q3 Students often struggle to identify corresponding angles. Stress the two parallel lines as the arms of the F.

Corresponding angles Q5 Students often muddle alternate and corresponding angles. Make sure they can identify and understand the difference.

Purposeful practice

Purposeful practice gives practice in identifying and using corresponding angles.

Reflect

Core

After the lesson Otmane wrote this list.

To solve shape and angle problems, look out for

- right angles (marked with a small square)
- parallel lines (marked with arrows)
- alternate angles (alternate sides of any line that crosses parallel lines)

Look back at the questions you answered in this lesson.

Write your own 'Look out for' list.

Often students are simply shown or told angle facts, but they are rarely helped to identify exactly what to look for when solving geometrical problems. This task aims to help them to do this. They are started off with a short list of things to look for, and asked to extend it, making explicit the process they undertake for solving geometrical problems.

Support

List all the angle facts you know. Draw a diagram to illustrate each one.

Student's own answers, but should include reference to angles on a straight line; angles in a triangle; angles in a quadrilateral; alternate and corresponding angles. Some students will also remember opposite angles in a parallelogram; vertically opposite angles.

Depth

What advice would you give to someone who is working on an angles question and cannot see how to work out the missing angle?

Work out the size of any angle, as there are links between different angles and calculating one lets you calculate another, which might help you work out the one you need.

Purposeful practice Reflect and reason

Purposeful practice

For **Q3e**, did you find angle n or angle p first? Work through the question again starting with a different angle to last time.

Is there always only one way to find the angles in parallel lines? Use more examples from this section to explain.

There is sometimes only one way, but usually there is more than one way.

Students' own examples, for example in **Q3d** you can find *p using angles on a straight line when you know m and n, or using alternate angles when you know q.*

Objectives

Core objectives

- Calculate the sum of the interior and exterior angles of a polygon
- Work out the sizes of interior and exterior angles of a polygon

Support objectives

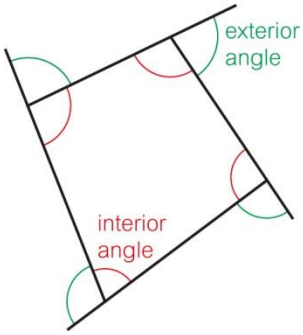
- Calculate the sum of the interior and exterior angles of a polygon
- Work out the sizes of interior and exterior angles of a polygon

Depth key concepts

- All 2D shapes have sum of exterior angles 360 (e.g. because if you walked around the outside of a polygon you would end up back where you started), but only quadrilaterals have sum of interior angles 360. The interior sum has to be a multiple of 180 because n (number of sides) has to be an integer.

Key points

- The **interior** and **exterior angles** of a polygon are shown in the diagram.
  In an **irregular polygon** sides are not all equal lengths, and angles are not all equal.



- Sum of interior angles of an n-sided polygon = $180°(n - 2)$

Common errors and misconceptions

- Students may assume that a polygon is regular. Emphasise that polygons must be assumed to be irregular unless the question states otherwise or the diagram shows all sides equal or all angles equal (or you can show them to be equal).
- Students may draw exterior angles in two different directions. Use a pencil to demonstrate that the exterior angles only add up to 360° if it moves in the same direction.

Question guidance

Core

Q4c The sum of the exterior angles is always 360°.

Q5, **Q6** Students may need to be reminded that these calculations are the same for regular and irregular polygons.

Q10 This question is carefully constructed to give practice in finding angles in polygons of increasing complexity.

Support

Exterior angles of polygons Q2c Students may need to be reminded of the symbol representing a right angle and that a right angle is 90°.

Interior angles of polygons Q1 Make sure students count the number of triangles correctly. Ask if they can see a pattern relating the number of sides of the polygon and the number of triangles.

Purposeful practice

Purposeful practice 1 gives practice in finding interior and exterior angles of polygons in two different ways and seeing which method students prefer.

Purposeful practice 2 gives practice in finding interior and exterior angles of irregular polygons

Reflect

Core

Write the steps for finding the sum of the interior angles of any polygon. You might begin with, 'Step 1: count the number of sides'.

Do your steps work to find the interior angles of one of the regular polygons in **Q8**? If not, rewrite your steps for any regular polygon.

Now write the steps to find the exterior angle of any regular polygon.

This task uses the language of interior and exterior angles, polygons and regular polygons; and asks students to make explicit the steps they take to find interior angles of any polygon. Students must also demonstrate their understanding of regular polygons by writing additional steps for finding their interior and exterior angles.

Support

Write one new thing you learned about polygons in this lesson.

Compare and discuss with a partner.

Students' own answers, but could include reference to sums of exterior angles, similarities and differences between regular and irregular polygons, and to the fact that the sum of the interior angles of a polygon is always a multiple of 180.

Depth

Why do all polygons have an exterior angle sum of 360°?

Students' own answers, for example:

'The exterior angle measures the turn from one side around to the next side, and since you return to the same place it will be as though you have completed a full turn.'

Purposeful practice Reflect and reason

Purposeful practice 1

Which method did you find easier for finding

a the angle sum of a regular polygon

b the size of one interior angle of a regular polygon?

*Students' own answers. For example, the method to find the sum of the interior angles in **Q1** is easier, because it is just one calculation (provided you remember the formula). For the size of one interior angle, it is easier to find the exterior angle and subtract from 180°, as in **Q2**.*

Purposeful practice 2

Why can't you use the method from **Q2** in Purposeful practice 1 to work out the size of an exterior angle of an irregular polygon?

Because the exterior angles of an irregular polygon are not equal. Although they sum to 360°, you cannot 'share' 360 equally to find the size of one exterior angle.

Objectives

Core objectives

- Solve geometric problems, showing reasoning
- Solve problems involving angles by setting up equations

Support objectives

- Solve geometric problems, showing reasoning
- Solve problems involving angles by setting up equations

Depth key concepts

- Solving geometric problems may involve using angles in parallel lines, properties of triangles, quadrilaterals and polygons, and that there is often more than one way of solving a problem
- Understand when it is helpful/appropriate to write and solve an equation to solve angle problems (and when it is not)

Key points

- You can solve a problem by writing an equation and solving it.

Common errors and misconceptions

- Students may not give enough reasons for their calculations. Point out that although they may get the correct answer, they may lose marks because they have not presented a reasoned argument.

Question guidance

Core

Q6 Make sure students don't make incorrect assumptions about the triangle BDE.

Q7 Students need to calculate each angle using substitution and check their totals.

Q7e Yes, because the triangle is equilateral $x + 7° = 60°$. Other equations are $3(x + 7°) = 180°$ and $x + 7° + x + 7° + x + 7° = 180°$

Q10 Refer students to the Key point on p173 to remind them of the properties of an isosceles trapezium.

Support

Writing and solving equations for geometric problems Q1 This question requires setting up and solving a one-step equation.

Writing and solving equations for geometric problems Q2 This question requires setting up and solving a two-step equation, giving reasons.

Purposeful practice

Purposeful practice 1 questions are carefully designed to give practice in writing and solving equations of increasing complexity.

Purposeful practice 2 gives practice in finding values with and without equations.

Reflect

Core

Q6 had a hint to sketch a diagram. Did you sketch the diagram? Did it help you?

Explain.

This task encourages students to think about how sketching a diagram can help them with solving geometric problems. Students may recognise that it helps them to visualise the problem, and/or that it saves them having to hold lots of information in their heads

Support

When solving geometric problems can you always find the missing angle straight away or do you sometimes need to find other angles first? How do you work out which angles to calculate?

You sometimes need to find other angles first. You work out which angles to calculate by using the different properties of shapes and angles.

Depth

a Is there more than one way to approach **Q8**?

b Which route to the answer did you prefer?

Did you find it easier to find other routes once you had found the first route?

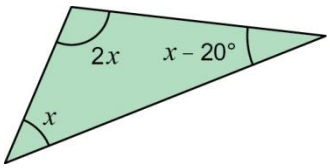
a *Yes, there are lots of ways.*

b *Students' own answers.*

Purposeful practice Reflect and reason

Purposeful practice 1

Danny is asked to work out the size of the angles in this triangle.



He writes

$4x - 20° = 180°$

$4x = 200°$

$x = 50°$

Has Danny solved the problem? Explain.

He hasn't fully solved the problem. He has correctly found the value of x, but he now needs to substitute that value into x, 2x and x – 20° to find the sizes of the angles (50°, 100°, 30°).

Purposeful practice 2

Did you need to write and solve an equation to find the angle in every question?

When would it be easier to work out the angle without an equation?

*Students' own answers. For example **Q1**, **Q3** and **Q5** would be easier without an equation because the equations are simple (one-step) addition. Use equations for two-step or more complex calculations with division or with more than one operation.*

Check, Strengthen and Extend

Core Check up

Q1, **5** These questions test whether students can use the properties of triangles and quadrilaterals to help solve a problem.

Q1, **4**, **5** These questions check that students give appropriate reasons for their calculations.

Q7e This question test that students realise that alternate angles can be obtuse.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

This Strengthen lesson covers these topics

- solving geometric problems

- parallel lines
- interior and exterior angles.

Which topic did you find easiest? Write one thing about this topic you fully understand and you are sure about.

Which topic did you find hardest? Write one thing about this topic you still do not understand or you are not sure about.

This task asks students to identify the topic they found easiest, and what they are confident about, along with the topic they found hardest, and what they still do not understand or are unsure about. Encourage students to ask for help on anything they noted which they still do not understand. You may try to match students for peer learning, so that those who have noted that they fully understand something, explain to those who have noted the same thing, but are still unsure of it.

Extend

Look back at the questions you answered in this Extend lesson. Find a question that you could not answer straight away, or that you really had to think about.

While you worked on this question:

- How did you feel?
- Did you keep trying until you had an answer? Did you give up before reaching an answer, and move on to the next question?
- Did you think you would get the answer correct or incorrect?

Write any strategies you could use to help you stay calm when answering tricky maths questions. Compare your strategies with other people's.

This task aims to help students cope any negative thinking that has the potential to impede students' progress. This is particularly important when answering difficult questions, which can cause anxiety and require perseverance. Having identified the nature of any negative experiences, students are asked to write down some strategies for staying calm and positive in such situations. Students may compare their strategies with others to get some alternative ideas.

Depth

Extend

What strategies do you apply when you are first looking at a new angle problem?

Students' own answers, for example:

'List the angle facts and look through the problem to see where I might use them.'

End of Unit 7 tests: Lines and angles

Pearson published resources

Support/Core

- Year 8 End of Unit 7 Support/Core test
- Year 8 End of Unit 7 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 7 Core/Depth test
- Year 8 End of Unit 7 Core/Depth mark scheme

Revision

End of Term tests: Units 5 to 7

Pearson published resources

Support/Core

- Year 8 Spring End of Term Support/Core test
- Year 8 Spring End of Term Support/Core mark scheme

Core/Depth

- Year 8 Spring End of Term Core/Depth test
- Year 8 Spring End of Term Core/Depth mark scheme

Unit opener: Calculating with fractions

Pearson published resources

Objectives

Core objectives

- Identify fractions as more than $\frac{1}{2}$ or less than $\frac{1}{2}$
- Order fractions

Support objectives

- Identify fractions as more than $\frac{1}{2}$ or less than $\frac{1}{2}$
- Order fractions

Depth key concepts

- Understand how to use a fraction benchmark when ordering fractions

Key points

- When ordering fractions, it can be useful to compare them with $\frac{1}{2}$.
- Fractions with the same denominator can be described as having a common denominator.
  For example, $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$ have a common denominator of 12.
- When ordering fractions, it can be useful to compare their equivalent fractions, all written with a common denominator.

Common errors and misconceptions

- When ordering unit fractions, students may get confused by ascending and descending order and the size of the denominator. Use pictorial representations to show that $\frac{1}{2}$ is larger than $\frac{1}{4}$ and reinforce the idea that for unit fractions the larger the denominator, the smaller the fraction.
- When ordering fractions with different denominators and numerators, students may only use the numerator. Use pictorial representations to show that $\frac{1}{2}$ is larger than $\frac{3}{8}$, even though the numerator of $\frac{1}{2}$ is smaller.

Question guidance

Core

Q8 Encourage students to write all the fractions with denominator 13 up to the fraction they think is greater than $\frac{1}{2}$

Q9c, **d** In these questions, students have to remember how to order negative numbers as well as fractions. If they are struggling, try to identify which skill they are having difficulty with. A number line may help.

Q12 Make sure students don't just try to guess the order, or just use the numerators for their ordering. Encourage students to read the hint for part a.

Support

Fractions more than $\frac{1}{2}$ or less than $\frac{1}{2}$ Q3 In this question students are encouraged to see that in all fractions equivalent to $\frac{1}{2}$ the numerator is half the denominator.

Using equivalent fractions to order fractions Q1 Make sure students can work out the multiplier for the numerator and denominator for each fraction, and that they apply the multiplier to both parts of the fraction.

Purposeful practice

Purposeful practice 1 emphasises the relationship between numerator and denominator and the concept of equivalence

Purposeful practice 2 Q1, **Q2** gives practice in finding equivalent fractions with fractions with an increasing range of denominators.

Reflect

Core

Carol says, 'The larger the denominator, the smaller the fraction.'

Is Carol correct?

Fred says, 'Carol is correct, but only when the numerator is 1.'

Is Fred correct?

Carol is incorrect. Fred is correct. This question is designed to get students thinking about the two different parts of a fraction and about how the numerator and the denominator play a part in determining the size of the fraction.

Support

In this lesson you have used two different methods to order fractions: comparing them to $\frac{1}{2}$ and finding a common denominator. Which method did you find easier? Why?

Students' answer will vary. Some students will have difficulty in working out when a fraction is greater or less than $\frac{1}{2}$ - remind them to divide the denominator by 2.

Depth

When ordering fractions, you can:

- find a common denominator
- find a common numerator
- compare the fractions with an 'easy' fraction like $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$.

When is it best to use each of these methods?

It is best to find a common denominator or numerator when there is an easy lowest common multiple to aim for.

It is better to compare with an 'easy' fraction when there is not a simple lowest common multiple or when they are fractions that you are familiar with and can recognise their relative sizes.

Purposeful practice Reflect and reason

Purposeful practice 1

Look carefully at the fractions you listed under the different headings in **Q1**. What do you

notice about the denominator compared to the numerator in the fractions in each list?

Why isn't it possible to write the fractions $\frac{3}{6}, \frac{10}{20}, \frac{6}{12}$ in descending order?

In 'more than $\frac{1}{2}$' the denominator is less than twice the numerator.

In 'equal to $\frac{1}{2}$' the denominator is exactly twice the numerator.

In 'less than $\frac{1}{2}$' the denominator is more than twice the numerator.

You can't write $\frac{3}{6}, \frac{10}{20}, \frac{6}{12}$ in descending order because they are equivalent fractions.

Purposeful practice 2

In **Q1**, why wasn't it possible to order the fractions by comparing the other two fractions to $\frac{1}{2}$?

In Purposeful practice 2, what method did you use to order the fractions?

Not possible because both fractions were more than $\frac{1}{2}$, or both fractions were less than $\frac{1}{2}$. Therefore, an equivalent fractions method was used.

Objectives

Core objectives

- Add and subtract fractions with any size denominator

Support objectives

- Add and subtract fractions with any size denominator

Depth key concepts

- Understand the addition and subtraction of fractions with any size denominator, where one of more fraction is negative, or the answer is a negative fraction.

Key points

- To add or subtract fractions, they must have a common denominator.

Common errors and misconceptions

- Students may not make the fractions have equal denominators before calculating.
- Students may simply add numerators and denominators together. Use pictorial representations such as two semicircles to show that $\frac{1}{2} + \frac{1}{2} = 1$, and not $\frac{2}{4}$

Question guidance

Core

Q9 This question allows students to develop their own explanations for why adding denominators is not correct. They can use the bar models to see that increasing the denominator makes things *smaller*.

Then they should use their *own* explanation and method to answer part b

Q13e Pressing the S/D button on a calculator changes the way it displays a number. Do some calculations $\frac{{\rm{3}}}{{\rm{4}}}{\rm{ + }}\frac{{\rm{1}}}{{\rm{2}}}$ like (i.e. not recurring fractions) to begin with.

Support

Adding and subtracting fractions – changing one fraction only Q3 If students struggle encourage them to draw bar models to illustrate the calculations.

Adding and subtracting fractions – changing both fractions Q1 Students may need to be reminded how to find the lowest common multiple of two numbers.

Purposeful practice

Purposeful practice 1 gives practice in adding fractions with increasingly complex answers from simple fractions to improper fractions and mixed numbers.

Reflect

Core

Look back at **Q5**, **Q6** and **Q7**.

What steps did you take to work out these calculations?

You might begin with, 'Step 1: I looked at the denominators and noticed ...'

Do your steps work for **Q10**?

If not, rewrite your steps so that they work for all the questions in this lesson.

*This task encourages consolidation of the steps needed to add and subtract fractions. Students begin by writing the steps where one denominator is a factor of another. When considering **Q10**, students may need to rewrite their steps to incorporate finding the lowest common multiple of denominators too.*

Support

In the first section of this lesson, you changed only one fraction before adding or subtracting. In the second section, you changed both fractions before adding or subtracting.

Discuss with a partner how you decide whether to change one fraction or both fractions when adding or subtracting fractions.

Students should be able to identify that only one fraction needs to be changed if the denominator of one is a factor of the denominator of the other, and that both fractions need to be changed if the two denominators have no common factors.

Depth

In **Q7**, how did you know if the missing fractions were positive or negative?

What was the same and what was different about the calculation you did to work out missing positive and missing negative fractions?

Students' own answers, for example, 'If I needed to get a smaller answer than one of the two numbers I was adding, I knew that the missing number had to be negative. If I was looking for a missing positive number, I subtracted the lower brick from the higher. If I was looking for a missing negative number, I subtracted the higher brick from the lower and then knew that my answer would be negative.'

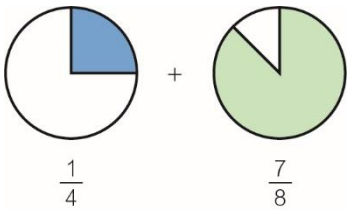
Purposeful practice Reflect and reason

Purposeful practice 1

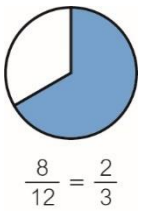
Alex writes $\frac{{\rm{1}}}{{\rm{4}}}{\rm{ + }}\frac{{\rm{7}}}{{\rm{8}}}{\rm{ = }}\frac{{\rm{8}}}{{{\rm{12}}}}{\rm{ = }}\frac{{\rm{2}}}{{\rm{3}}}$

What mistake has Alex made?

Copy these fractions of circles and use them to explain why Alex is wrong.

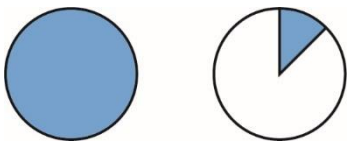


does not give



Alex has added the numerators and denominators.

The correct answer is



$\frac{{\rm{1}}}{{\rm{4}}}{\rm{ + }}\frac{{\rm{7}}}{{\rm{8}}}{\rm{ = }}\frac{{\rm{2}}}{{\rm{8}}}{\rm{ + }}\frac{{\rm{7}}}{{\rm{8}}}{\rm{ = }}\frac{{\rm{9}}}{{\rm{8}}}{\rm{ = 1}}\frac{{\rm{1}}}{{\rm{8}}}$

Purposeful practice 2

How can you use Purposeful practice 1 **Q2** parts **e** and **f** to answer Purposeful practice 2 **Q1** parts **e** and **f**?

Purposeful practice 1 ***Q2e*** *$\frac{{\rm{1}}}{{\rm{9}}}{\rm{ + }}\frac{{\rm{2}}}{{\rm{3}}}{\rm{ = }}\frac{{\rm{7}}}{{\rm{9}}}$* ***f*** *$\frac{{\rm{2}}}{{\rm{15}}}{\rm{ + }}\frac{{\rm{2}}}{{\rm{3}}}{\rm{ = }}\frac{{\rm{8}}}{{\rm{15}}}$*

so Purposeful practice 2 ***Q1e*** *$\frac{{\rm{7}}}{{\rm{9}}}{\rm{ }} - {\rm{ }}\frac{{\rm{2}}}{{\rm{3}}}{\rm{ = }}\frac{{\rm{1}}}{{\rm{9}}}$ and* ***Q1f*** *$\frac{{\rm{8}}}{{\rm{15}}}{\rm{ }} - {\rm{ }}\frac{{\rm{2}}}{{\rm{5}}}{\rm{ = }}\frac{{\rm{2}}}{{\rm{15}}}$*

Purposeful practice 3

What is the same and what is different about the method you used to work out the calculations in Purposeful practice 3, compared to Purposeful practice 1 and Purposeful practice 2?

Same: finding common denominator

Different: only had to find one equivalent fraction in Purposeful practice 1 and Purposeful practice 2. In Purposeful practice 3 both fractions needed to be changed.

Objectives

Core objectives

- Multiply integers and fractions by a fraction
- Use appropriate methods for multiplying fractions

Support objectives

- Multiply integers and fractions by a fraction
- Use appropriate methods for multiplying fractions

Depth key concepts

- Understand the multiplication of fractions, with any size denominator, where one or more fraction is negative, or the answer is a negative fraction
- Apply BIDMAS to fraction calculations, involving the multiplication of fractions

Key points

- Finding a fraction of an integer is the same as multiplying a fraction and an integer. For example, $\frac{3}{4}$ of 100 is the same as $\frac{3}{4} \times 100$
- To multiply two fractions, multiply their numerators and multiply their denominators.
- Sometimes you can rearrange fractions so they can be simplified before multiplying.

Common errors and misconceptions

- Students may confuse rules for multiplying and adding fractions.
- Students may not simplify before and after multiplying fractions. Always encourage students to simplify their answers.
- Students may add the numerators when multiplying by a unit fraction.

Question guidance

Core

Q12 When multiplying two unit fractions, watch out for students getting confused. Remind them to multiply the 1s in the numerator.

Q13 Take this question further by asking if they think this proportion is fair. *Can we tell what fraction of the council is women? Why?*

Support

Multiplying a fraction by an integer Q1a Students should be able to answer this by looking at Molly's sequence, discuss how multiplying is simply repeated addition.

Multiplying a fraction by an integer Q2b You may wish to look at this as

$\frac{3}{8}$ $\frac{3}{8} + \frac{3}{8} = 2 \times \frac{3}{8}$ $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} = 3 \times \frac{3}{8}$, and so on

Multiplying a fraction by an integer Q7 Some of these need to simplified after writing them as mixed numbers.

Multiplying a fraction by a fraction Q2 In these questions the numerators are always 1, so students only need to shade one of the smaller pieces.

Purposeful practice

Purposeful practice 3 gives practice in multiplying fractions with increasing difficulty. Some students may struggle to find the appropriate equivalent fractions

Reflect

Core

Look back at **Q10**. How did you learn how to multiply fractions? What is good about learning a new mathematics skill this way?

Explain your answer.

Is anything not so good?

What other ways do you like to learn new mathematics skills?

This task asks students to think about how they learnt the new skill of multiplying fractions, and the advantages and disadvantages of learning this way. For example, students may recognise that following the worked example has the advantage of allowing them to be independent learners, but has the disadvantage of not allowing them to ask questions. Students are also asked their preferred means for learning new mathematical skills. They may state that they like to learn from their friends or peers, so that they can discuss their understanding.

Support

Work out these multiplications

$\frac{{\rm{1}}}{{\rm{4}}}{\rm{ }} \times {\rm{ }}\frac{{\rm{3}}}{{\rm{5}}}$
$\frac{{\rm{3}}}{{\rm{4}}}{\rm{ }} \times {\rm{ }}\frac{{\rm{1}}}{{\rm{5}}}$
$\frac{{\rm{3}}}{{\rm{5}}}{\rm{ }} \times {\rm{ }}\frac{{\rm{1}}}{{\rm{4}}}$
$\frac{{\rm{1}}}{{\rm{5}}}{\rm{ }} \times {\rm{ }}\frac{{\rm{3}}}{{\rm{4}}}$

What do you notice? Can you explain why this is?

The answer is the same every time $(\frac{3}{20})$ *because the numerator pairs and denominators pairs are the same in each case, and because the order of multiplication doesn't matter* ($\frac{{\rm{1}}}{{\rm{4}}}{\rm{ }} \times {\rm{ }}\frac{{\rm{3}}}{{\rm{5}}}$ *is the same as* $\frac{{\rm{3}}}{{\rm{5}}}{\rm{ }} \times {\rm{ }}\frac{{\rm{1}}}{{\rm{4}}}$)

Depth

Hassan says, 'When you multiply a positive integer by a fraction less than 1, your answer will always be less than the number you started with'.

a Is Hassan correct? Explain why.

b Is Hassan's statement correct if you are multiplying a negative integer by a fraction less than 1?

a *Yes, because you are finding a fraction of a positive number.*

b *No, because the answer will be closer to 0, so it will be greater than the original number.*

Purposeful practice Reflect and reason

Purposeful practice 1

How did your answers to **Q1** help you to answer **Q2** parts **a–d**?

Once you've found $\frac{1}{3}$*, then you can multiply to find* $\frac{2}{3}$*; once you've found* $\frac{1}{4}$*, then you can multiply to find* $\frac{3}{4}$ *etc.*

Purposeful practice 2

When multiplying a fraction less than 1 by another fraction less than 1, do you always get an answer smaller than both of the original fractions? Give examples.

Yes, but only when both fractions are positive. If one is negative and one is positive, the answer lies between the two fractions. If both fractions are negative, the answer is larger than either of the fractions multiplied.

Purposeful practice 3

Why is it sometimes useful to simplify fractions in the working when multiplying fractions?

It makes the multiplications easier to work out because the numbers are smaller.

Objectives

Core objectives

- Find the reciprocal of a number
- Divide integers and fractions by a fraction
- Use strategies for dividing fractions

Support objectives

- Find the reciprocal of a number
- Divide integers and fractions by a fraction
- Use strategies for dividing fractions

Depth key concepts

- Apply the inverse relationship of multiplication and division to fraction calculations
- Apply BIDMAS to fraction calculations, involving the division of fractions

Key points

- The reciprocal of a fraction is the 'upside down' fraction.
  A number multiplied by its reciprocal is always 1. The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$
- You can write integers (whole numbers) as fractions with a denominator of 1.
  Dividing by 1 doesn't change the number. For example, $\frac{3}{1} = 3$.
- Dividing by a fraction is the same as multiplying by its reciprocal.

Common errors and misconceptions

- Students will often assume that division always makes things smaller. Work through examples to show this is not always the case.

Question guidance

Core

Q16 Students are combining at least three key areas of knowledge here: reciprocals; multiplying fractions; negative number rules. If they struggle, check which particular area it is that is causing problems.

Q17 This question expects students to draw out that as the number you multiply by gets smaller, the answer gets smaller. As the number you divide by gets smaller, the answer gets bigger.

Support

Reciprocals Q3 Students may be challenged by the idea that an integer can be written as a fraction. Remind students that $\frac{7}{1}$ means 7 divided by 1 = 7.

Dividing by a fraction Q1 Students may initially struggle with the idea that dividing an integer by a fraction less than 1 gives a larger answer.

Purposeful practice

Purposeful practice 2 gives practice in applying reciprocals.

Reflect

Core

Q10 used bar models to show dividing by fractions.

Did the bar models help you to understand? Explain your answer.

It can be difficult for students to have an image of dividing by a fraction. Students should recognise that the bar models help them to 'visualise' dividing by fractions, as you can actually 'see', for example, two divided into quarters, or a half divided into sixths.

Support

What is the same and what is different when multiplying and dividing fractions?

Which is easier? Explain.

Same: both require a step where the numerators are multiplied together and the denominators are multiplied together.

Different: Dividing fractions requires an additional initial step, finding the reciprocal of the dividing fraction.

Student may find multiplying fractions easier than dividing as fewer steps are required.

Depth

Multiplication and division are inverse operations.

How did you use your knowledge of multiplying fractions to answer questions in this lesson on dividing fractions? Give examples.

Students' own answers, for example, 'I multiplied by the reciprocal instead of working out a division of fractions.'

Purposeful practice Reflect and reason

Purposeful practice 1

Copy and complete the sentence four times using words from these boxes.

an improper fraction a proper fraction a unit fraction an integer

The reciprocal of _____ is _____.

*The reciprocal of **an improper fraction** is **a proper fraction**.*

*The reciprocal of **a proper fraction** is **an improper fraction**.*

*The reciprocal of **a unit fraction** is **an integer**.*

*The reciprocal of **an integer** is **a unit fraction**.*

Purposeful practice 2

What type of numbers are the answers to calculations in **Q1** parts **a**, **d**, **g**, and **h**?

Explain why you get these numbers.

Do you get these numbers when the numerator of the fraction is not 1? Explain.

Square numbers, as the denominator of the fractions multiplying the integer are the same as the integer. This is only when the numerator of the fraction is 1. For example, ${\rm{8 \div }}\frac{{\rm{1}}}{{\rm{8}}}{\rm{ = 64}}$ (square number), ${\rm{8 \div }}\frac{{\rm{2}}}{{\rm{8}}}{\rm{ = 32}}$ (not square number).

Purposeful practice 3

Look at the calculations and answers in **Q1** parts **a**–**d** and **Q1** parts **e**–**h**. What do you notice about dividing one fraction by another where the numerators are the same?

Did you use a different method for **Q3** parts **a**–**d** and **Q3** parts **e**–**h**? If so, explain how your method for **Q3** parts **e**–**h** made the calculations easier.

Where numerators are the same, the denominator of the dividend is the denominator of the fraction in the answer; the denominator of the divisor is the numerator of the fraction in the answer. Where the denominator of the dividend is larger than the denominator of the divisor the answer is a proper fraction; where it is smaller the answer is an improper fraction. Students may have simplified before multiplying to make the calculations easier.

Objectives

Core objectives

- Write a mixed number as an improper fraction
- Use the four operations with mixed numbers

Support objectives

- Write a mixed number as an improper fraction
- Use the four operations with mixed numbers

Depth key concepts

- Understand the four operations with mixed numbers, where one or more mixed number is negative, or the answer is a negative mixed number
- Apply inverse relationships to mixed number calculations
- Apply BIDMAS to mixed number calculations

Key points

- You can add or subtract mixed numbers by adding or subtracting the whole numbers first, then writing the fraction parts with a common denominator to add or subtract them.
- It can be easier to write mixed numbers as improper fractions before subtracting.
- Write mixed numbers as improper fractions before multiplying or dividing.

Common errors and misconceptions

- Students may view the mixed number as two separate numbers. Use cube models to show that mixed numbers and improper fractions can be used interchangeably.

Question guidance

Core

Q8 Before carrying out the calculation, students should look at the numbers and check whether they expect a positive or negative answer.

Q16-20 Students have to decide whether to use multiplication or division. Encourage them to draw diagrams or bar models to help decide.

Q18 Students may divide both parts of $1\frac{3}{4}$ by 2 and want to write the answer as $\frac{{\rm{1}}}{{\rm{2}}}{\rm{ + }}\frac{{{\rm{1}}{\rm{.5}}}}{{\rm{4}}}$. Students may need to be told not to mix decimals within fractions.

Support

Writing a mixed number as an improper fraction Q1 Students often muddle what to multiply by and what to add – by drawing bar models they understand why they multiply.

Purposeful practice

Purposeful practice 3 gives practice in multiplication and division using fractions of increasing difficulty.

Reflect

Core

What is the same when you calculate with fractions and with mixed numbers?

What is different?

This task asks students to consider what is the same or different about calculating with fractions and calculating with mixed numbers. Students should consider all the methods they have used throughout the entire unit. For example, students may recognise that adding fractions and adding mixed numbers can involve the same method of finding a common denominator, but only after the mixed number is converted to a fraction. Similarly, when multiplying fractions and multiplying mixed numbers, the same method of multiplying numerator and denominator is used, but, again, only after the mixed number is converted to a fraction.

Support

Will you always write mixed numbers as improper fractions before adding or subtracting, or will you sometimes use Helena's method (Q4)? Discuss with a partner when one method seems more suitable than the other.

Students' answers will vary. Some students will feel more comfortable with Helena's method because it reduces (or potentially even removes) the need to work with improper fractions.

Depth

In **Q9**, how did you use:

- inverse operations
- improper fractions
- your knowledge of negative numbers?

Students' own answers, for example, 'I used inverse operations to work backwards through the calculations to find the original number. I used improper fractions to make the calculations easier. I used my knowledge of negative numbers to know when a calculation would give a negative answer.'

Purposeful practice Reflect and reason

Purposeful practice 1

Rebecca and David are working out ${\rm{1}}\frac{{\rm{3}}}{{\rm{4}}}{\rm{ + 3}}\frac{{\rm{2}}}{{\rm{3}}}$. The start of their working is shown.

Rebecca

${\rm{1}}\frac{{\rm{3}}}{{\rm{4}}}{\rm{ + 3}}\frac{{\rm{2}}}{{\rm{3}}}{\rm{ = 1 + 3 + }}\frac{{\rm{1}}}{{\rm{6}}}{\rm{ + }}\frac{{\rm{2}}}{{\rm{3}}}$

${\rm{ = 1 + 3 + }}\frac{{\rm{1}}}{{\rm{6}}}{\rm{ + }}\frac{{\rm{4}}}{{\rm{6}}}$

${\rm{ = 4 + }}\frac{{\rm{5}}}{{\rm{6}}}$

David

${\rm{1}}\frac{{\rm{3}}}{{\rm{4}}}{\rm{ + 3}}\frac{{\rm{2}}}{{\rm{3}}}{\rm{ = }}\frac{{\rm{7}}}{{\rm{6}}}{\rm{ + }}\frac{{\frac{{\rm{11}}}{{\rm{3}}}}}$

${\rm{ = }}\frac{{\rm{7}}}{{\rm{6}}}{\rm{ + }}\frac{{\frac{{\rm{22}}}{{\rm{6}}}}}$

Copy and complete each of their working to show they get the same answer.

Use each of their methods to work out ${\rm{1}}\frac{{\rm{3}}}{{\rm{4}}}{\rm{ + 3}}\frac{{\rm{2}}}{{\rm{3}}}$. Which method do you prefer? Explain.

Rebecca: ${\rm{4}}\frac{{\rm{5}}}{{\rm{6}}}$

David:$\frac{{\frac{{\rm{29}}}{{\rm{6}}}}{\rm{ = 4}}\frac{{\rm{5}}}{{\rm{6}}}$

Students' preferences will vary. Encourage students to explain their preferences to their classmates.

Purposeful practice 2

Here is Isla's working for ${\rm{6}}\frac{{\rm{1}}}{{\rm{5}}}{\rm{ }} - {\rm{ 1}}\frac{{\rm{7}}}{{\rm{10}}}$

${\rm{6}}\frac{{\rm{1}}}{{\rm{5}}}{\rm{ }} - {\rm{ 1}}\frac{{\rm{7}}}{{\rm{10}}} = {\rm{6 }} - {\rm{ 1 }} - {\rm{ }}\frac{{\rm{1}}}{{\rm{5}}}{\rm{ }} - {\rm{ }}\frac{{\rm{7}}}{{\rm{10}}} = {\rm{5 }} - {\rm{ }}\frac{{\rm{2}}}{{\rm{10}}}{\rm{ }} - {\rm{ }}\frac{{\rm{7}}}{{\rm{10}}} = {\rm{5 }} - {\rm{ }}\frac{{\rm{9}}}{{\rm{10}}} = {\rm{4}}\frac{{\rm{1}}}{{\rm{10}}}$

Write the mixed numbers as improper fractions and then subtract to check Isla's working.

What did Isla do wrong?

Isla subtracted $\frac{{\rm{1}}}{{\rm{5}}}$ instead of adding it. She should have done:

${\rm{6}}\frac{{\rm{1}}}{{\rm{5}}}{\rm{ }} - {\rm{ 1}}\frac{{\rm{7}}}{{{\rm{10}}}} = {\rm{6 }} - {\rm{ 1 }} + {\rm{ }}\frac{{\rm{1}}}{{\rm{5}}}{\rm{ }} - {\rm{ }}\frac{{\rm{7}}}{{{\rm{10}}}} = {\rm{5 }} + {\rm{ }}\frac{{\rm{2}}}{{{\rm{10}}}}{\rm{ }} - {\rm{ }}\frac{{\rm{7}}}{{{\rm{10}}}} = {\rm{5 }} - {\rm{ }}\frac{{\rm{5}}}{{{\rm{10}}}} = {\rm{5 }} - {\rm{ }}\frac{{\rm{1}}}{{\rm{2}}} = {\rm{4}}\frac{{\rm{1}}}{{\rm{2}}}$

Purposeful practice 3

Write one thing that is the same and one thing that is different when multiplying and when dividing a mixed number by a mixed number.

Same: Mixed numbers are written as improper fractions.

Different: The reciprocal of the second improper fraction is used when dividing. The reciprocal is not used when multiplying.

Check, Strengthen and Extend

Core Check up

Q4 This question checks students understanding of subtracting from 1 and adding up fractions to get to 1.

Q5 This question checks students understanding of using rules for multiplying directed fractions.

Q8b This question addresses a common error. The reciprocal of $\frac{1}{a}$ is $\frac{a}{1}$ but should be simplified to a.

Q12c This question addresses the common error of reversing order of subtraction when the second number is larger than the first when working with fractions

Challenge Q1 In this question, students are encouraged to think about what kind of numbers have an integer product. Most will start with an integer greater than 10 and a fraction less than 1 (often assumed that multiplying by a fraction makes things smaller). Could also look at e.g. ${\rm{6 \times 1}}\frac{{\rm{2}}}{{\rm{3}}}$

Students should use what they know about multiplication and division relationships to find as many pairs as possible.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

List these tasks (A–F) in order from easiest to hardest.

A Adding and subtracting fractions

B Multiplying fractions

C Dividing fractions

D Adding and subtracting mixed numbers

E Multiplying mixed numbers

F Dividing mixed numbers

Look at the first task in your list (the easiest).

What made it easiest?

Look at the two tasks at the bottom of your list (the hardest).

What made them hardest?

Write hints to help you with the two tasks you found hardest.

This task encourages students to consider the kinds of calculations with fractions and mixed numbers that they find easiest or hardest, so it highlights the particular skills that they may need more practice on.

*For example, if students say that their easiest task is **B Multiplying fractions**, perhaps this is because they do not have to manipulate the fractions in any way, as they are simply required to multiply the numerators and then the denominators. If students state that their hardest tasks are **E** and **F** involving mixed numbers, they may find it difficult to convert to improper fractions. Students are asked to write a hint for the tasks they find the hardest.*

Extend

The word fraction is used in lots of ways. Here are two examples:

- In everyday English, a fraction means 'a small amount'. When hanging a picture you might 'move it up a fraction' or you might ask someone to 'budge up a fraction' so you can sit beside them.
- In chemistry, the fractionating process separates a mixture into its components.

Write a definition, in your own words, of 'fraction' in mathematics.

What do you think 'fractional ownership' means? When might it be a good idea?

This task encourages students to recognise that 'fraction' is a term and concept used in subjects and contexts other than mathematics. Students are asked to give a mathematics

definition, and then think about what 'fractional ownership' may mean. This is where several parties own a share (fraction) of an asset. Usually it is a high-value asset, such as a holiday home, yacht or jet.

This may appeal to people who do not have the money to invest in the whole asset, and also who wish to share the risk if the asset goes down in value.

Depth

Extend

Is it more difficult, the same, or easier to work with

a proper fractions or improper fractions? Explain.

b improper fractions or mixed numbers? Explain.

c positive fractions and mixed numbers or negative fractions and mixed numbers?

Explain.

Students' own answers, for example,

a *'The same, as they work in the same way'.*

b *'Improper fractions, because they work like fractions; with mixed numbers you have to check that the fraction part of the calculation works'.*

c *'Positive fractions and mixed numbers, because you don't have to think about the sign of your answer'.*

End of Unit 8 tests: Calculating with fractions

Pearson published resources

Support/Core

- Year 8 End of Unit 8 Support/Core test
- Year 8 End of Unit 8 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 8 Core/Depth test
- Year 8 End of Unit 8 Core/Depth mark scheme

Unit opener: Straight-line graphs

Pearson published resources

Objectives

Core objectives

- Recognise when values are in direct proportion with or without a graph
- Plot graphs and read values to solve problems

Support objectives

- Recognise when values are in direct proportion with or without a graph
- Plot graphs and read values to solve problems

Depth key concepts

- Understand when one (or more) part of a graph shows quantities in direct proportion, but another part does not
- Understand when quantities may sometimes be in direct proportion and sometimes not

Key points

When two quantities are in direct proportion

- plotting them as a graph gives a straight line through (0,0)
- when one variable is zero, the other variable is also zero
- when one variable doubles, so does the other.

Common errors and misconceptions

- Students may not be able to use information on the graph to find values beyond the graph's scale: practice making up larger multiples from key points shown on the graph (10×, 5×, 1×, 2×)
- Students may think that if a graph shows a straight line, then it must be in direct proportion. Address this misconception by using bar models to show that when one value doubles, the other does not, or that if one value is 0, the other is not.
- The importance of accuracy in plotting graphs is not always understood. Students may produce graphs with uneven intervals or incorrectly marked scales, or with poor or no labelling. Address this by showing a graph with a number of exaggerated errors for students to identify and discuss.

Question guidance

Core

Q2 Explain that all they have to look for in a graph is whether the line is straight and whether it goes through the origin.

Q9 Interpret the scales carefully, especially intervals between marked ones. The scale on the y-axis is quite tricky, so ensure students can work out the smaller in-between intervals.

Q10 *Which variable will you plot on each axis? What scale is sensible? What is the largest value? How many squares high can the graph be, allowing for titles and labelling? How can we work out how much each square needs to be worth? Which key intervals will you label? Remember to give a title to your graph and to label each axis with its unit of measure.*

Q12 Students may not be able to see clearly that this is not actually a straight line, but the fact that 5 posters cost more than half of the 10, should show that, because of the postal cost, this does not show direct proportion.

Q14 Suggest that 25ml, 50ml, 75ml may be the easiest to plot even though these are unusual sizes for perfume. Being able to show 50p clearly on the graph is important for this question.

Q16 It might help if students think about these as graphs: **A**1 inch = 2.54 cm; **B** Think about what happens to temperature during a normal day – *does it keep on increasing at a constant rate? What would its graph look like?* **C** Give current rate e.g. £1 = $1.25; **D** Look back at **Q12**. Encourage them to look back at the key point at the beginning of the lesson to check whether all the rules apply or not

Challenge Students will need to plot the points (1,100), (2,160) and (3,220). Explain where the second point comes from, i.e. after two hours the distance will be 100 + 60, after 3 hours 100 + 2 × 60

Support

Reading from a graph Q1 Students may get confused about reading up and reading across. Students might read up from 16 km in **Q1a**, but not know where to go after that, or they may begin on the wrong axis.

Recognise direct proportion on a graph Q5e Some students may not recognise that 20 is double 10, or that 12 is double 6. Get them to sense-check their answer to **part ii** – it must be bigger than the answer to **b iv** (and they may recognise it should be the sum of the answers to **b i** and **b iv**).

Purposeful practice

Purposeful practice 1 Students often forget the graph must also pass through the origin.

Reflect

Core

a List all the things you have to do to interpret graphs (as in **Q7**, **Q8** and **Q9**).

You may start your list, 'Read values'.

b List all the things you have to do to draw graphs (as in **Q10**, **Q11** and **Q12**).

You may start your list, 'Draw a table of values'.

c Which are you better at, interpreting or drawing graphs? Explain why.

This task asks students to consider everything they do when interpreting and drawing graphs. Then it asks them which they are best at. Their lists should help them to identify the particular skills that make them better or worse at interpreting than drawing. This should help them to work out where they may need further practice.

Support

Without looking at these pages, try to remember all the things you have to look for to decide if two quantities are in direct proportion. Check with a classmate that you have remembered them all.

Students' own answers, but should include reference to the graph being a straight line, going through the origin, and the x- and y-values changing at the same rate (e.g. both doubling).

Depth

Describe a situation where two quantities are sometimes, but not always, in direct proportion.

Students' own answers, and could include an example such as 'When ordering things to be delivered, after a certain point you do not have to pay postage, so they are in direct proportion after that point but not before'.

Purposeful practice Reflect and reason

Purposeful practice 1

Dan says, 'All straight-line graphs show quantities in direct proportion to each other.'

Is Dan correct? Explain.

*No, Dan is not correct as the graphs must be straight lines **and** pass through (0, 0).*

Purposeful practice 2

When two quantities are in direct proportion, what methods can you use to work out values not shown on a graph (for example, in **Q1b iii**)?

Extend the graph; use values you know to work out larger values by multiplying.

Purposeful practice 3

Angelo only looked at the first three rows of data in each table.

He says, 'This is enough to decide if the quantities are in direct proportion.'

Is Angelo correct? Explain.

No, it is not enough, because the fourth row could reveal that the quantities are not in direct proportion.

Objectives

Core objectives

- Plot a straight-line graph and work out its gradient

Support objectives

- Plot a straight-line graph and work out its gradient

Depth key concepts

- Understand the relationship between two quantities in direct proportion (increasing or decreasing at the same rate) and the gradient of the graph when the quantities are plotted against each other.

Key points

- The steepness of a line or graph is called the **gradient**.
  To find the gradient, work out how many units the line or graph goes up for every one unit across to the right. If the line or graph goes down for every one unit across to the right, then the gradient is negative.

Common errors and misconceptions

- Students may get the formula for the gradient the wrong way round , thinking the gradient is found by dividing the change in x by the change in y.
  ${\rm{Gradient = }}\frac{{{\rm{change\ in\ vertical}}}}{{{\rm{change\ in\ horizontal}}}}$
  Discuss ways of remembering the correct way round, such as deciding a phrase to aid memory may help: 'up over along' or 'down over along'.
- It may always be apparent which points to use to find the gradient. To address this, identify and mark on a line all the points that lie where grid lines cross. Choose different pairs of points to work out the gradient – they should all simplify to the same number.
- Students may initially struggle with negative and fractional gradients. To address this, get students to make a language association - feeling 'up' is associated with a positive mood (positive gradient); feeling 'down' with a negative mood (negative gradient). For fractional gradients, compare gradients with the $x = y$ and $x = -y$ graphs. Steeper gradients will be > 1, less steep < 1.

Question guidance

Core

Q2 Suggest labelling graphs roughly with 1–5 hours and 0–100 km to find the speed for 1 hour to compare the steepness of the first two graphs and use that information to order the others.

Q4c The lines should be parallel and straight**.** If not, students need to check their table of values.

Q11b needs students to be able to convert between simple fractions and percentages. Suggest they convert to equivalent fractions in hundredths if they are not sure

Q12aiii Remind students that negative × negative = positive and that you expect a negative coefficient of x to make a line that sloped down from left to right.

Q12b, **c** For each graph in Q12 ask**:** *what is the gradient of the line; what is the number* □ is multiplied by (the **coefficient** of x); *by how much does the value of* □ *increase each time?*

Test their understanding by asking questions such as: *What is the gradient of the line* □ = 5□ + 2*? Give me the equation of a graph that has gradient 3; how much does the value of* □ *increase in the equation* □ = □ − 6*?*

Repeat for a negative gradient and a fractional gradient:

What is the gradient of the line $y = -5x + 2$? $y = \frac{3}{4}x - 2$; $y = -\frac{2}{5}x - 1$.

Give me a graph that has gradient $-4; -\frac{1}{2}; \frac{2}{3}; -\frac{4}{5}$

Does the value of □ *increase or decrease in the equation* □ = −2□+ 6*?*

The □*-value increases by 3, what could its equation be? The* □*-value decreases by 1, what could its equation be?*

Show a table of values for $y = 4x - 2$ with only the y values when for $x = 0$ and $x = 2$ filled in.

How could we complete this table of values without having to substitute for □ *every time?*

Support

Plotting straight-line graphs Q1 Remind students that the coefficient of x here is 1

(i.e. $y = (1 \times x) + 3$), so the gradient is 1.

Plotting straight-line graphs Q1, Q2 Encourage students to investigate the relationships between the coefficients of x and the numbers of squares the lines go through.

Gradients Q2 Remind students:${\rm{gradient = }}\frac{{{\rm{change\ in\ vertical}}}}{{{\rm{change\ in\ horizontal}}}}$

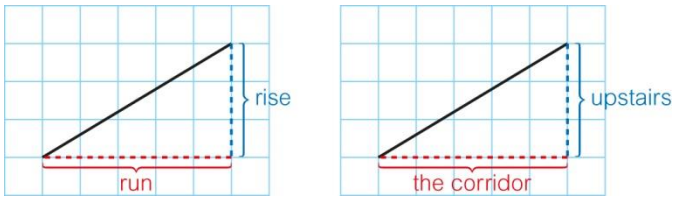
Purposeful practice

Purposeful practice 3 gives practice in calculating positive and negative gradients.

Reflect

Core

Some people think of finding the gradient as 'rise over run' or 'upstairs above the corridor'.



Do you like these ways of remembering how to find the gradient?

Can you think of your own way to remember it?

What else do you use to remember mathematics facts? Compare with your classmates.

This task encourages students to find ways to remember mathematics concepts, in this case, how to find the gradient of a line. They may like 'rise over run' or 'upstairs above the corridor', or they may choose one of their own. They should have other examples of ways to remember mathematics concepts, for example the rhyme 'a litre of water's a pint and three quarters'. By comparing with classmates, students may find more ways to remember key mathematics knowledge and skills.

Support

Look back at **Plotting straight-line graphs Q4**. List all the things you have to do to draw a straight-line graph.

Students' answers will vary, but should include: completing the table of values by substituting values in the equation of the straight line; writing the correct coordinate pairs in the right order (□-value first); choosing the correct values on the axes; plotting the points and joining the plotted points; labelling the line with the equation of the line.

Depth

You have seen lines like this on graphs:

a Which of these lines could show a direct proportion relationship?

b Which line has the gradient closest to 0?

c What is different about the gradients of A and B?

a A *because it could go through the origin as well as showing if you multiply one value by a number, you would need to multiply the other value by the same number to stay on the line.*

b C*, it has a gradient of 0.*

c A *has a positive gradient whereas* ***B*** *has a negative gradient.*

Purposeful practice Reflect and reason

Purposeful practice 1

Which graphs in **Q1** have the same gradient?

What do you notice about the equations of graphs with the same gradient?

$y = x + 1$ *and* $y = x + 3$ *have the same gradient. Their equations both include* x*.*

$y = 2x + 1$ *and* $y = 2x + 3$ *have the same gradient. Their equations both include* $2x$*.*

$y = x + 1$ *and* $y = x + 3$ *are the least steep.*

$y = 2x + 1$ *and* $y = 2x + 3$ *are the most steep.*

Purposeful practice 2

Compare graphs in Purposeful practice 1 **Q1** parts **a** and **b** with the graphs in Purposeful practice 2 **Q1** parts **a** and **b**.

What do you notice about the graphs? And what do you notice about their equations?

The graph slopes up from left to right for a positive gradient and down from left to right for a negative gradient.

$y = x + 1$ *and* $y = -x + 1$ *are reflections of each other in the* x*-axis, as are* $y = x + 3$ *and* $y = -x + 3$*;*

$y = 2x + 1$ *and* $y = -2x + 1$*; and* $y = 2x + 3$ *and* $y = -2x + 3$*. These equations have a change of sign before the term in* x*.*

Purposeful practice 3

Which line is the least steep? And which is the most steep?

Line A is the least steep

Line I is the most steep

Objectives

Core objectives

- Plot the graphs of linear equations
- Write the equations of straight-line graphs in the form $y = mx + c$

Support objectives

- Write the equations of straight-line graphs in the form $y = mx + c$

Depth key concepts

- Identify reflective symmetry between related graphs with different equations.

Key points

- The **y-intercept** is where a graph crosses the y-axis.
- A **linear equation** generates a straight-line (linear) graph.
  The equation for a straight-line graph can be written as $y = mx + c$ where m is the gradient and c is the y-intercept.
- The **origin** on a coordinate grid is the point (0, 0).

Common errors and misconceptions

- Students may struggle when dealing with negative values of x when substituting to complete a table of values. Students may need to be reminded about multiplying with negative signs. Show two negative signs turning into a positive sign and encourage student to learn: two of the same signs = positive; two different signs = negative. Practise all four operations with combinations of two negatives and one negative and one positive.

Question guidance

Core

Q11 It may be helpful to ensure they all know which are positive and which are negative gradients before they begin. Remind students to mark two whole number coordinates on each line to find the gradient: ${\rm{gradient = }}\frac{{{\rm{change\ in\ vertical}}}}{{{\rm{change\ in\ horizontal}}}}$ for the first part of the equation, then use the y-intercept to complete it. Identifying the two parallel lines will save some work.

Q13 Remind students to extend the line in both directions so that it meets the axes to work out *c*. *If a line goes through the origin what is the value of* □ *in the equation* □ = □□ + □*?*

Q14 Sensitivity needed when talking about mass, as this clearly shows that the heavier you are the more force you put on a bike frame.

Support

The gradient of straight-line graphs Q1 Remind students: ${\rm{gradient = }}\frac{{{\rm{change\ in\ vertical}}}}{{{\rm{change\ in\ horizontal}}}}$

The y-intercept of straight-line graphs Q1 The question asks for the coordinates, so make sure students give the coordinate pair, and not just the value of the y-intercept. Ask them what they notice about the value of the x-coordinate.

The equation of straight-line graphs Q3 Encourage students to check their answers by looking at the steepness of the lines and whether the gradients are positive or negative.

Purposeful practice

Purposeful practice 1 gives practice in completing tables of values and plotting associated straight lines.

Reflect

Core

Write down what you think 'linear' means.

$y = mx + c$ is a linear equation.

Write, in your own words, what m and c stand for.

Write hints for how to remember what m and c stand for.

*Students often use words and symbols in mathematics without thinking about what they mean. Therefore, this task asks students to define 'linear' (meaning straight line), and then □ and □ in a linear equation of the form □ = □□ + □. It gives a hint on how '□' may have come to be used for gradient, and asks students to write themselves hints for how to remember □ and □ (e.g. cli**m**b and inter**c**ept).*

Support

You are looking at a straight-line graph on a grid. What steps do you need to take to find the equation of the line? List as many as you can.

Students' answers will vary, but they must identify

- *find the coordinates of two points on the line*
- *find the change in the vertical coordinates*
- *find the change in the horizontal coordinates*
- *calculate the gradient correctly, complete with the correct sign*
- *find the □-intercept correctly.*

Some students may comment on the gradients of horizontal and vertical lines.

Depth

Look back at **Q8**. Write a sentence explaining how you decided if each statement was true or false. Compare your explanations with others in your class.

Students' own answers, but should demonstrate knowledge and understanding of the different parts of the equation, identifying the gradients and being able to compare gradients; identifying the □-intercept; identifying the effect of a reflection.

Purposeful practice Reflect and reason

Purposeful practice 1

What is the same and what is different about the equations and graphs in **Q1**?

What is the same and what is different about the equations and graphs in **Q2** parts **i–iii**?

*In **Q1**, □-coefficients and gradients are the same constants and □-intercepts are different.*

*In **Q2**, □-coefficients and gradients are the same for **i** and **ii** but different for **iii** and **iv**. Constants and □-intercepts are the same for **ii**, **iii** and **iv** but different for **i**.*

Purposeful practice 2

Antony says, 'To answer this question, I looked at where each graph crossed the y-axis first then matched its gradient.'

Judith says, 'To answer this question, I looked at graphs with positive gradients first and compared their steepness.'

Whose first step do you prefer? Explain why.

What was your first step to answer this question?

Students' own answers, but students should recognise that they can use either the gradient or the value of the □-intercept to narrow down their choices.

Check, Strengthen and Extend

Core Check up

Q1–2 These question test students' ability to work with negative coordinates, to complete a table of values and use it to plot a linear graph.

Q3 This question is designed to test students to see if they can work out a simple gradient and recognise a negative gradient.

Q4–5 In these questions, students will need to recognise and use the fact that in the equation of the straight-line graph $y = mx + c$, the gradient is m and the y-intercept is c.

Q6–8 These question test students' ability to interpret direct proportion shown as a line graph and to plot a graph from a table of data, explaining whether it shows direct proportion or not.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Look back at the questions you got wrong in the Check up test.

Now look back at the Strengthen questions you answered.

Write down one thing you now understand better.

Is there anything you still need help with? Ask a friend or your teacher to help you with it.

This task asks students to undertake an error analysis, and consider what topic they struggled with most in the formative assessment (Check up) and what they have since learned about this topic in the Strengthen lessons. Students are encouraged to seek help on something they still do not understand or are unsure of.

Extend

Larry enters pairs of coordinates in a spreadsheet like this.

	A	B
1	x	y
2	4	7
3	3	5
4		

In cell B4, he types in the formula **=slope(B2:B3, A2:A3)**

Use what you know about gradients to explain what this spreadsheet formula does.

What answer does Larry get? Test it for yourself in a spreadsheet.

*This task encourages students to consider the concept of gradient, in the context of a spreadsheet formula. Students should comment on the use of the word 'slope', as well as recognise that **B2:B3** represents the difference in y-values, and **A2:A3** represents the difference in x-values. Then, these are divided. Larry should get the answer 2. If possible, students should test this for themselves in a spreadsheet. You may also ask them to test gradients of other straight-line graphs they have worked on in the extend lessons.*

Depth

Extend

Why do you think it is important to know if two quantities are in direct proportion, or not?

How might you use this outside of the classroom?

Students' own answers, and could include reference to the fact that you will know how one thing changes as another changes if they are in direct proportion. Outside the classroom, examples could include 'In a science experiment or when planning for a business.'

End of Unit 9 tests: Straight-line graphs

Pearson published resources

Support/Core

- Year 8 End of Unit 9 Support/Core test
- Year 8 End of Unit 9 Support/Core mark scheme

Core/Depth

- Year 8 End of Unit 9 Core/Depth test
- Year 8 End of Unit 9 Core/Depth mark scheme

Unit opener: Percentages, decimals and fractions

Pearson published resources

Objectives

Core objectives

- Change time to decimal hours
- Recall equivalent fractions and decimals
- Recognise recurring and terminating decimals
- Order fractions by converting them to decimals or equivalent fractions

Support objectives

- Change time to decimal hours
- Recall equivalent fractions and decimals
- Recognise recurring and terminating decimals
- Order fractions by converting them to decimals or equivalent fractions

Depth key concepts

- Understand what is the same and what is different about a terminating decimal with repeating numbers and a recurring decimal with the same repeating numbers
- Recognise where fractions of time result in a recurring decimal

Key points

- A terminating decimal ends after a definite number of digits, for example 0.22 or 0.519.
- The line in a fraction means 'divide by'. You can use written division to write fractions as decimals. For example, to write $\frac{3}{4}$ as a decimal, work out ${\rm{8}}\left){\vphantom{1{\rm{3}}}}\right. \!\!\!\overline{\,\,\,\vphantom 1{{\rm{3}}}}$
- A recurring decimal contains a digit, or sequence of digits, which repeats itself for ever.
- In a recurring decimal, a dot over a digit shows that it recurs; a dot over the beginning and end of a sequence shows that it recurs. For example, ${\rm{0}}{\rm{.\dot 7}}$ = 0.777 777 777... ${\rm{0}}{\rm{.1\dot 7}}$ = 0.177 777 777... ${\rm{0}}{\rm{.\dot 10\dot 7}}$ = 0.107 107 107...

Common errors and misconceptions

- Students may not notice that a decimal is recurring if the repeated pattern is very long, e.g. for $\frac{1}{7}$. Students should increase the digits displayed by their calculators or scroll the display to find the first repetition.
- Students may read hours and minutes as a decimal, e.g. 1 h 48 m = 1.48 h. Remind students that time is not measured using the decimal system. Demonstrate using a simple example such as 30 minutes = $\frac{1}{2}$hour = 0.5 hour.

Question guidance

Core

Q10 Write each fraction with 10 or 100 as denominator. Cancel where possible.

Q14c Students may place a dot above the 4. Emphasise that the first dot is placed above the **beginning** of the repeating pattern, which may not be the first digit. Give other examples, e.g. 0.223 434 343 4...

Q19 Remind students to find the lowest common denominator.

Support

Changing time to fractions of an hour Q4 This question reinforces the idea that an hour does not have 100 minutes, student should be familiar with $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$ of an hour.

Fractions and decimals Q2 Students may need reminding of how to simplify a fraction.

Fractions and decimals Q3 These can all easily be written as decimals and some are familiar fraction to decimal conversions that student should know.

Purposeful practice

Purposeful practice 1 gives practice in converting fractions and decimals of increasing complexity.

Purposeful practice 2 gives practice in dealing with terminating and recurring fractions of increasing complexity.

Reflect

Core

Write a short definition, in your own words, for

terminating decimals recurring decimals

unit fractions equivalent fractions and decimals.

Give an example with each definition.

This task emphasises some new terminology (terminating decimals, recurring decimals) and some terminology students may have met before (unit fractions, equivalent fractions and decimals).

It asks students to write their own short definition, accompanied by an example. It also asks students for any other terminology they have learned that relates to decimals and fractions.

They may think of rounding to decimal places, or improper fractions.

Support

When converting fractions to decimals you used two different methods:

Method 1: You wrote the denominator as a power of 10 (10, 100 or 1000).

Method 2: You used short division.

Which method did you prefer? Why?

Students' answers will vary. This question will help identify students' confidence in dividing either by powers of 10 or by 1- or 2-digit numbers, and they may suggest that the different methods can be used for different fractions.

Depth

Use 1.0909, $1.\dot{0}\dot{9}$ and $1.0\dot{9}$ to explain the difference between a terminating decimal with repeating digits and a recurring decimal.

1.0909 *is a terminating decimal; all the decimal places after the last 9 are 0.*

$1.\dot{0}\dot{9}$ *is a recurring decimal and has infinitely many repeating digits that alternate between 0 and 9.*

$1.0\dot{9}$ *only repeats the 9 infinitely.*

Purposeful practice Reflect and reason

Purposeful practice 1

Simon says, 'The decimal in **Q2f** is a recurring decimal.'

Is he correct? Explain.

No, Simon is not correct as there are a definite number of digits. If recurring, the 9s would continue forever.

Purposeful practice 2

How could you use the answer to **Q2a** to work out the answer to **Q2b**?

How could you use the answer to **Q2c** to work out the answers to **Q2** parts **d**–**h**?

$\frac{2}{3} = 2 \times \frac{1}{3}$ *and* $0.\dot{6}$ *is* $2 \times 0.\dot{3}$

$\frac{2}{11} = 2 \times \frac{1}{11}$ *and* $0.\dot{1}\dot{8}$ *is* $2 \times 0.\dot{0}\dot{9}$

$\frac{3}{11} = 3 \times \frac{1}{11}$ *and* $0.\dot{2}\dot{7}$ *is* $3 \times 0.\dot{0}\dot{9}$

$\frac{3}{3} = 1$ *not* $0.\dot{9}$ *Jenny has not taken account of the recurring nature of the decimal*

Purposeful practice 3

When are fractions of hours terminating decimals?

Terminating when multiples of 3, as all multiples of 3 as a fraction of 60 give a decimal number of twentieths. Twentieths can be written as hundredths, so this is always terminating. Students should notice pattern in ***Q1*** *is +0.05 so 2 hours 21 minutes is 2.35. Students should notice approximately doubling each time and predict* ${\rm{2}}{\rm{.5\dot 2}}$ *or* ${\rm{2}}{\rm{.5\dot 3}}$.

Objectives

Core objectives

- Recall equivalent fractions, decimals and percentages
- Use different methods to find equivalent fractions, decimals and percentages
- Use the equivalence of fractions, decimals and percentages to compare two proportions

Support objectives

- Recall equivalent fractions, decimals and percentages
- Use different methods to find equivalent fractions, decimals and percentages

- Use the equivalence of fractions, decimals and percentages to compare two proportions

Depth key concepts

- Understand proportions involving large numbers
- Know how to deal with proportions that involve decimals
- Compare and interpret more than two proportions

Key points

- A positive mixed number is greater than 1, so the decimal equivalent is greater than 1 and the percentage equivalent is greater than 100%. For example, ${\rm{1}}\frac{{\rm{3}}}{{\rm{4}}}$ = 1.75 = 175%.
- A proportion of a whole can be written as a fraction, decimal or percentage.
- Sometimes you might need to use a denominator of 1000 when you convert between fractions, decimals and percentages.

Common errors and misconceptions

- Students may calculate a proportion as though it is a ratio, e.g. 2 out of 5 calculated as 2 ÷ 7. Write the fraction $\frac{{\rm{2}}}{{\rm{5}}}$ and say 2 *out of* 5 as you point to the numerator and denominator. Refer students to the definition of a proportion in the Key point. Use a bar model to demonstrate a proportion. 2 out of 5 means 2 parts out of a total of 5 parts.

Question guidance

Core

Q4 Explain that to convert any number (fraction or decimal) to a percentage, multiply by 100, and vice versa.

Q5 100% represents the previous profits, so an increase of 200% mean the profits have increased by twice the previous profits, making them now three times the previous profits.

Q7a Point out that the totals are given. Students will find it easiest to write the fraction first, then percentage (change denominator to 100) then decimal.

Q7b *Does it matter whether you compare fractions, decimals or percentages?* (no) *Which is easiest?*

Challenge Students have to arrange number and symbol cards into sets containing equivalent fractions, decimals and percentages. For example, $\frac{{\rm{1}}}{{\rm{5}}}$, 0.2, 20%

Support

Converting between fractions, decimals and percentages greater than 100% Q1 This question supports student who can't remember the key fraction, decimal and % equivalents.

Comparing proportion Q1, Q2 Students often struggle with the word proportion. These questions link proportion to fractions, decimals and percentages.

Comparing proportion Q3 Students should see that although a larger area is shaded red the proportions are equivalent.

Purposeful practice

Purposeful practice 1 gives practice in converting fractions, decimals and percentages of increasing complexity.

Purposeful practice 2 Q3 gives practice in calculating percentages from proportions given in words.

Reflect

Core

Which is easier?

- Writing a fraction as a decimal and a percentage (as in **Q9**)
- Writing a percentage as a decimal and a fraction (as in **Q19**)

Look at the task you chose as easier. What made it easy?

Look at the task you chose as harder. What made it hard?

Write a hint to help you with the task you found harder.

This task asks students to compare conversions between, fractions, decimals and percentages. It should highlight to students the multiplicative calculations that they find easiest and hardest. For example, if their hardest task is writing a fraction as a decimal, perhaps they find it difficult to identify the multiplier to make the denominator of a fraction a power of 10. However, if their hardest task is writing a percentage as a decimal, perhaps they find it difficult to accurately divide by 100.

Support

Kathi says, 'You can't have a percentage greater than 100%.'

Leo says, 'You can't have a percentage that is a decimal.'

Use what you have learned in this lesson to explain to a partner why both Kathi and Leo are incorrect.

Kathi is wrong because you can have a percentage of a number greater than 1, e.g. ${\rm{1}}\frac{{\rm{3}}}{{\rm{4}}}$ = 1.75 = 175%.

Leo is wrong because a percentage is a different way of expressing decimal fractions, and if the decimal fraction has more than 2 digits, the percentage will have a decimal fraction, e.g. 0.275 = 27.5%

Depth

What method do you use when comparing and interpreting more than two proportions?

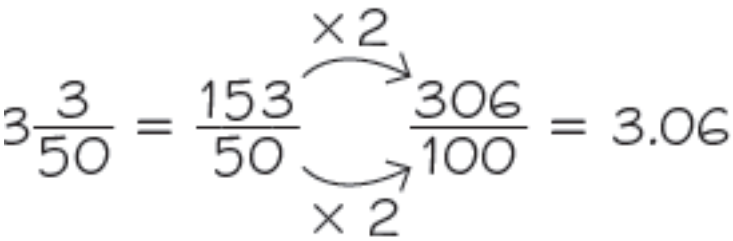
Is it the same or different from your method for comparing and interpreting exactly two proportions? Explain.

Students' own answers, for example, 'Write all the proportions as fractions with the same denominator, then compare the numerators. This would also work if you were comparing exactly two proportions. Or write all the proportions as percentages.'

Purposeful practice Reflect and reason

Purposeful practice 1

Holly uses this method to write mixed numbers as decimals.



First she writes the mixed number as an improper fraction.

Is this a good method? Explain.

Do you have to convert to improper fractions to write a mixed number as a decimal?

If not, show a different method.

When does it help to convert mixed numbers to improper fractions?

No, it is not a good method – separating the whole and fraction part and then converting the fraction to a decimal results in easier calculations and is more efficient.

You don't have to convert to an improper fraction. Students may show $3\frac{3}{50} = 3\frac{6}{100} = 3.06$

Converting to improper fractions can help when subtracting, multiplying or dividing mixed numbers.

Purposeful practice 2

Why are these answers the same: **Q2b** and **Q2e**; **Q2d** and **Q2g**?

$\frac{{\rm{2}}}{{\rm{125}}} = \frac{{\rm{4}}}{{\rm{250}}}$,
$\frac{{\rm{3}}}{{\rm{250}}} = \frac{{\rm{6}}}{{\rm{500}}}$ *The decimal equivalents of each pair are the same*

Purposeful practice 3

Jon writes

13.5% is the decimal 1.35 and the fraction $\frac{{{\rm{135}}}}{{{\rm{100}}}}{\rm{ = }}\frac{{{\rm{27}}}}{{{\rm{20}}}}{\rm{ = 1}}\frac{{\rm{7}}}{{{\rm{20}}}}$

What mistake has Jon made? What is the correct answer?

Jon has divided the percentage by 10 instead of 100.

The correct answer is 0.135 and $\frac{{{\rm{27}}}}{{{\rm{500}}}}$

Objectives

Core objectives

- Express one number as a percentage of another when the units are different
- Work out an amount increased or decreased by a percentage
- Use mental strategies to solve percentage problems

Support objectives

- Express one number as a percentage of another when the units are different
- Work out an amount increased or decreased by a percentage
- Use mental strategies to solve percentage problems

Depth key concepts

- Understand how to express one measure as a percentage of another where the proportion involves large measures (e.g. the proportion of pence out of £1000s) and the units are not adjacent (e.g. the proportion of a measure given in mm out of a metre measure)
- Investigate mental strategies for solving problems involving decimal percentages, and make decisions about most efficient method to use for different problems

Key points

- To express one amount as a fraction or percentage of another, they must be in the same units.
- To increase an amount by a percentage, work out the percentage of the amount, then add it to the original amount.
- Interest is money charged for borrowing or earned for saving. Simple interest is the interest calculated only on the original amount of money. It is the same amount each year.

- To decrease an amount by a percentage, work out the percentage of the amount, then subtract it from the original amount.

Common errors and misconceptions

- Students may not convert quantities to the same unit before comparing. Compare 1 mm with 1 cm if necessary. Advise students that converting to the smallest unit is usually easiest.
- Students may have difficulty increasing or decreasing by complex percentages, e.g. 3%, 1.5%. *Find 1% first, then you can work out any number of percent.*

Question guidance

Core

Q10a *What is 10% of £15?* (£1.50). *So what is 20%?* (2 × £1.50 = £3) *What is the price including VAT?*

Q12 *What is 1% of £25?000?* (£250). *So what is 2%?* (£500)

Q14c Work out 1%, then multiply the answer by 8

Q23 Decrease the cost of the shoes by 20% (work out 10% first) and add the cost of a jumper. *How much is left out of £60 spent?*

Support

Expressing one number as a percentage of another when the units are different
Q1 Students need to convert units of measure before writing one number as a percentage of another.

Purposeful practice

Purposeful practice 1 gives practice in expressing amounts of a quantity as a percentage in ml and litres (**Q1**), g and kg (**Q2**) and p and £ (**Q3**).

Purposeful practice 2 gives practice in using percentages to calculate increases and decreases in amounts.

Purposeful practice 3 applies percentages to interest calculations in bank accounts.

Reflect

Core

What was the same and what was different about the way you worked out your answers to **Q12** and **Q20**?

Write yourself three tips or hints for working out amounts increased or decreased by a percentage. Compare your hints and tips with others in your class.

*Students' answers will vary, but they should recognise that in both **Q12** and **Q20**, you have to multiply the original amount by the percentage, and that to find Leela's new salary (**Q12**) you add, and to find the new budget (**Q20**) you subtract.*

Support

In **Q2** you subtracted and in **Q3** you added, to work out the answers. Would you add or subtract, to work out the answer to each of these questions? Explain how you know.

a A car starts with 5 litres of fuel. The driver uses 10% of the fuel. How much fuel is left?

b A shopkeeper adds 25% to the price of a £1 drink. What is the new price of the drink?

a *You would subtract. Some of the fuel has been used so the answer must be smaller.*

b *You would add, because the new price will be bigger.*

Depth

In **Q11**, for each percentage, which of your two different methods do you prefer?

Explain why.

Students' own answers. Students should recognise that 25% = $\frac{1}{4}$ *and 5% =* $\frac{1}{20}$*, so calculations can be made easier using these fractions.*

Purposeful practice Reflect and reason

Purposeful practice 1

What do you notice about your answers to **Q2** parts **g** and **h**, and **Q3** parts **f** and **g**? Explain.

They are all greater than 100%. This is because the amount in grams is greater than the amount in kilograms in ***Q2g*** *and* ***Q2h****, and the amount in pence is greater than the amount in pounds in* ***Q3h****.*

Purposeful practice 2

What do you notice about the amount you were subtracting for a decrease of 5%, then 10%, then 15% of each mass?

Predict 600 g decreased by 20%. Write a calculation to check your prediction.

Subtracting the same amount each time. Students should predict 600 g decreased by 20% is 510 g subtract 30 g, i.e. 480 g.

Purposeful practice 3

What do you notice about your answers to **Q1i** and **Q2ii**? Explain.

The answers are the same. This is because adding 1% is the same as adding 2 × 0.5%.

Objectives

Core objectives

- Use a multiplier to calculate amounts increased or decreased by a percentage
- Use the unitary method to solve percentage problems

Support objectives

- Use a multiplier to calculate amounts increased or decreased by a percentage
- Use the unitary method to solve percentage problems

Depth key concepts

- Understand how to use a repeated multiplier to work out an amount that has undergone more than one percentage change
- Understand how to use the unitary method to work out an original amount where there has been more than one percentage change (e.g. a decrease of a given percentage and then an increase of a given percentage; or a decrease of a given percentage and then another decrease of a given percentage).

Key points

- To **decrease** an amount by a percentage, you can **subtract** the percentage from 100%, then work out the multiplier.
- To **increase** an amount by a percentage, you can **add** the percentage to 100%, then work out the multiplier.
- Sometimes you want to find the original amount after a percentage increase or decrease. You can use the unitary method.

Common errors and misconceptions

- Students may have difficulty seeing that a multiplier of 0.85 is used for a decrease of 15%. Explain that the original amount is always 100%. Decreasing this by 15% gives 100% − 15% = 85% which is equivalent to the decimal 0.85
- Students may not know whether to multiply or divide by a multiplier. Encourage students to check that their answer makes sense, e.g. a smaller amount after a reduction and remind them that dividing by a decimal fraction less than 1 will give a bigger answer.

Question guidance

Core

Q11d Multiplying by 1.2 is the same as increasing by 20%. Multiplying by 0.8 is the same as decreasing by 20%.

Q18 *What percentage is £6.30?*

Q19 Students should give their answer to a sensible degree of accuracy, e.g. to the nearest £10 000.

Support

Using a multiplier Q4 This question leads in to reducing a value by a percentage using a multiplier.

The unitary method Q4 This question leads students into the Core questions on finding the original value from word problems.

Purposeful practice

Purposeful practice 1, 2 give practice in calculating and using multipliers to increase and decrease amounts.

Reflect

Core

a Write a definition of a multiplier in your own words.

b Write the steps you take to use a multiplier to calculate

i a percentage increase **ii** a percentage decrease.

Give examples.

This task draws out students' understanding of the important concept of using a multiplier for finding percentages. Students are asked to write a definition of a multiplier, and then guided in writing the steps taken when using a multiplier to find percentage change.

Support

In this lesson you have worked with percentages

a using a multiplier

b using the unitary method.

Discuss with a partner when you would use each method to solve a problem involving percentages.

Students' answers will vary, but they should recognise that the different methods might be easier for different questions.

Depth

a Write the steps you would use to find the original amount after a decrease of 5% and then an increase of 10%.

b What would be the same, and what would be different, when finding the original amount after an increase of 5% and then a decrease of 10%?

a *First you would undo the increase of 10% by knowing you had 110% and using that to find 100%. Then you would undo the 5% decrease by knowing your answer was 95% of the original and using that to find100% of the original.*

b *You would still go back in reverse order, but you would start with 90% not 110% and then you would have 105% of the original amount.*

Purposeful practice Reflect and reason

Purposeful practice 1

Renie says, 'The multiplier to increase and decrease an amount by 50% is the same.'

Is Renie correct? Explain.

No, because to increase an amount by 50%, you multiply by 1.5; to decrease by 50% you multiply by 0.5.

Purposeful practice 2

What do you notice about your answers to **Q2** parts **a** and **b**; parts **c** and **d**; parts **e** and **f**; and parts **g** and **h**?

*The answers to **Q2a** and **b**, **Q2c** and **d** add to $60, which is the original amount; the answers to **Q2e** and **f**, **Q2g** and **h** add to 200 ml, which is the original amount. This is because the percentages in each pair add to 100% and so give the original 100% amount when added.*

Purposeful practice 3

Jay and Sabina work out **Q1b** differently.

Jay divides by 20 to find 1%, then multiplies by 100 to find 100%.

Sabina notices that 20% × 5 = 100%, so she multiplies by 5 to find 100%.

Which method do you prefer? And do both their methods work when you're given 18% of a distance and asked to find the original? Explain your answer.

Students may prefer Sabina's method because it is shorter and more efficient. However, it only works when the given percentage is a factor of 100.

Sabina's method doesn't work with 18% as 18 is not a factor of 100.

Check, Strengthen and Extend

Core Check up

Q5a This question can be used to check that students understand how to calculate proportions and express the proportions as fractions and percentages.

Q5b This question is designed to test whether students can use comparisons of proportions in questions comparing amounts.

Q9 This question tests whether students can work out that a multiplier of 0.85 is used for a decrease of 15%.

Q9 This question is designed to test whether students understand whether to multiply or divide by a multiplier.

Based on their results and confidence in the Check up, students can decide whether to Strengthen or Extend their learning.

The content in Strengthen is divided into topics so that students can select specific areas for improvement.

Reflect

Core

Strengthen

Julie says, 'Working with fractions, decimals and percentages is all about multiplying and dividing.'

Look back at the questions you answered in this Strengthen lesson.

Write two questions where you had to multiply to find an answer.

Write two questions where you had to divide to find an answer.

Write two questions where you had to multiply and divide to find an answe